interference, by interacting with a wanted signal of high amplitude, intermodulation products might be produced parametrically and which do interfere. The relevant variables which might determine this are the amplitude of the time domain interference and whether the resulting intermodulation product is of the time duration of the original wanted signal.

Blocking, which is an overload of the RF amplifiers which increases their nonlinearity and results in gain reduction, is only possible with large interfering signals.

It is worthwhile to consider the frequency modulation (FM) approach, here, because it involves (i) frequency tuning, (ii) amplitude limiting; (iii) frequency discrimination and conversion to AM; (iv) rectification of the AM to audio. Step (ii) rids the signal of unwanted amplitude variations. The question is, from the time domain point of view, whether step (ii) removes any time domain signal from interfering with the FM signal. A second question concerns a possible hindrance to time domain signal use, in that in an FM system signal-to-noise is increased if the widest bandwidth is used. Now the limiter, (ii) above, converts the interfering signal into a phase modulation of the required signal. The result is that for an interfering signal of amplitude x, the instantaneous frequency is $\omega_0 - x\omega_i\cos[\omega_i t]$, with a maximum deviation of $x\omega_i$, at a modulating frequency of ω_i, where ω_0 is the frequency of the wanted signal and ω_i is the difference between ω_0 and that of the interference. Thus the interfering output is proportional to the difference frequency, ω_i, so the wider the bandwidth of the FM system, the *less* the time domain system will interfere. It follows that for a wideband FM system, that is, one with a high modulation index, the interference results in a *smaller voltage swing*. This is because, due to the amplitude limiting, the interference is proportional to not only its amplitude, x, but also the frequency difference, ω_i. It should also be appreciated that this interference must first pass through the IF stage. Therefore it appears that the chances of interfering with FM systems are smaller than with AM systems.

It should be apparent by now that the designers of present-day radio receivers have been following a design philosophy exactly the opposite to that required for time domain receiver design. The following quotation reveals this:

"The great majority of radio receivers use the superheterodyne principle because the fixed intermediate frequency or frequencies can lead to designs with high selectivity. Selectivity is important in the realisation of the basic idea of a receiver - that it should produce an (audio) output only from an RF signal at the correct channel frequency."[24]

That being the case, a major question is whether intermodulation products can be produced in these high frequency-selective designs. In order not to obtain intermodulation products from interference, the RF amplifier must be as linear as possible. This requirement is at odds with the requirement, mentioned above, to convert to an IF signal, because this conversion uses a nonlinear element to which is fed the signal and the local oscillator signal. Therefore, the mixer cannot be a linear unit.

The overwhelming majority of most receiver-demodulators concern narrowband signals. The two exceptions, and for which there is the possibility of interference by a time domain transmitter, are (i) television signals and (ii) spread spectrum signals. These two kinds of signals are the most important kind to consider for interference.

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[24]Howson, D.P., RF and mixer stages, pp. 51-74, in W. Gosling (ed) *Radio Receivers*, Peter Peregrinus, London, 1986.

Demodulators recover the baseband signal transmitted. In the past, the baseband signal was digital, but presently, and in the future, it will be digital. For digital reception, the demodulator reduces the incoming signal to baseband analog form and then this is passed to a data modem. The demodulation will depend on the type of transmission defined with reference to the following generic equation:

$$y(t) = A \cos[\omega_c t + \phi].$$

If A is a time-dependent variable, then the signal is AM. If either ω_c or ϕ are time dependent, then the signal is FM. Variation of both A and ω_c or ϕ will generate other modulations including single sideband AM (SSBAM) and single sideband FM (SSBFM).

In the case of AM signals, the description is:

$$A(t) = d_0 + d_1 z(t) = d_0[1 + \{d_1/d_0\}z(t)],$$

where z(t) is the baseband modulating function and d_0, d_1 are parameters. d_1/d_0 is the modulation index, m. The classification of AM signals is then[25]:

(i) if $m<1$, A(t) is non-zero and positive for all t. This is full carrier double-sideband AM.

(ii) if $m = 1$, A(t) is positive except for values of t at which it instantaneously falls to zero. This is also double-sideband AM but at 100%. The instantaneous frequency is always constant and equal to ω_c.

(iii) if $m > 1$, A(t) changes sign for different values of t. This is *double-sideband diminished carrier* (DSBDC).

(iv) if $m \rightarrow \infty$ ($d_0 \rightarrow 0$), A(t) is identical in sign with z(t). This is *double-sideband suppressed carrier* (DSBSC). In the present context it is known as *binary phase-shift keying* (BPSK).

In the case of (iii) and (iv), the sign of A(t) can change during the modulation cycle. When this occurs, there is an instantaneous change in the angle π as A(t) passes through zero. This change in phase is also an infinite impulse function in the frequency domain. Therefore, for (iii) and (iv) the instantaneous frequency can become infinite at zeros of A. These phase "jumps" could be a cause of interference to time domain communications. However, as they would not be correlated with the pulse train of an operating time domain systems, there is no reason to suppose that the jumps would affect time domain information transmission.

If z(t) is expanded:

$$z(t) = \Sigma_1^n a_r \cos[\omega_r t + \phi_r],$$

then:

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[25]Gosling, W., Demodulators, pp. 153-180 in W. Gosling (ed) *Radio Receivers*, Peter Peregrinus, London, 1986.

$$y(t) = d_0\{1 + m\Sigma_1^n a_r \cos[\omega_r t + \phi_r]\}\cos[\omega_c t + \phi],$$

or

$$y(t) = a_0\cos[\omega_c t + \phi] + md_0\Sigma a_r\cos[(\omega_c + \omega_r)t + \phi + \phi_r] + md_0\Sigma_1^n\ a_r\cos[(\omega_c + \omega_r)t + \phi + \phi_r].$$

The first component is the carrier, the second is the upper sideband and the third is the lower sideband.

A major observation is that if ω_r is a function of r, and if the maximum modulating component is ω_n, then the spectrum of y(t) then extends from $(\omega_c - \omega_n)$ to $(\omega_c + \omega_n)$ or $2\omega_n$. Thus AM is not, strictly speaking, narrowband. Secondly, most of the energy is in the carrier, ω_c.

In the case of frequency modulation (FM),

$$y(t) = A\cos[\omega t + \phi],$$

where

$$\omega = \omega_c + \Delta w\ z(t) - 1 < z < +1.$$

The spectrum is of infinite extent and symmetrical, but the sidebands diminish rapidly in amplitude away from ω_c, so there is no interference to neighboring channels. Again, if the maximum frequency component in z(t) is ω_n, then the practical bandwidth is (i) $2\omega_n$, if $\Delta\omega << \omega_n$ (narrowband FM); (ii) $2\Delta\omega$ for $\Delta\omega \geq \omega_n$.

A simple kind of AM demodulator is a *diode circuit*, which, for all its simplicity, is nonetheless nonlinear and cannot generate IF harmonics and spurious receiver responses. It remains to be seen whether a time domain impulse signal together with a wanted AM signal will together generate IF harmonics. The amplitude of the time domain signal would be, of course, critical.

Another demodulator is the *AM product demodulator*. This demodulator has the unsatisfactory property of continuing to demodulate in the absence of a signal because the limiter amplifier limits fully on noise. It remains to be seen whether a time domain impulse signal in noise would generate an unwanted signal. In this case, the duration of the time domain signal would be critical.

The FM *pulse counting discriminator* demodulator operates on the rising or falling IF y(t). The hard limiting and digital signals could possibly interfere with the reception of time domain signals, but not the information transmission if those signals were train encoded.

In the FM *direct conversion* receiver, the intermediate frequency is reduced to zero, and there is an absence of frequency selectivity. There are two ways in which these

receivers can be suffer interference: (i) spectrum folding: the requirement in this receiver is for negative frequency components to be superimposed on the positive components using, e.g., a Hilbert transform receiver. Thus signals from outside the wanted signal band can appear in the receiver output due to the folding operation; (ii) spurious demodulation: there is no RF selectivity and due to the presence of some nonlinearity, time domain signals could mix with wanted signals producing spurious output. Some approaches to removing this drawback involve using a mixer element which has odd-order symmetry (cubic law) rather than even-order (square law).

Automatic Gain Control (AGC) or *Feedback Automatic Gain Conrol* (FBAGC) might be affected by time domain communications. AGC is a nonlinear closed-loop gain technique, the objective of which is to suppress large mean level variations of a received signal's envelope. AGC prevents the gain-stages in AM-type receivers from being overloaded or undriven and to maintain the output signal level of the receiver at a relatively constant level. These variations are referred to as *fading*, and are of the slow and fast type. The slow type is of no concern to us here. The fast type is the result of multipath and can occur at rates of up to 100 Hz. In order to reject envelope variations, a signal is extracted from the output proportional to the magnitude of the envelope. This is passed through a smoothing filter which removes the high frequency terms (which probably includes time domain signals). This filtered signal is then compared with a fixed DC reference voltage and a second filter extracts low frequency components in the resulting difference signal. The output from this filter is then used to control the gain of a variable gain amplifier (VGA).

However, FBAGC is only partially successful in achieving suppression of fast fading. There are problems in time delay. If the bandwidth of the FBAGC circuit is increased in order to suppress unwanted envelope fading, then complete FBAGC failure can result[26]. Feedforward, rather than feedback AGC systems have been implemented to combat fast fading. This is known as *Feedforward Automatic Gain Control* (FFAGC). In FFAGC, the unwanted fast signal fluctuations are extracted from the input by a temporal filter and then divided into the total signal plus the signal fluctuations line. However, FFAGC has no effect on random frequency variations. This suggests that all types of AGC would function to pursue a futile effort to follow the fading introduced by a time domain communications system. Another technique, that of *FeedForward Signal Regeneration* (FFSR) uses a linear mix-down with a delayed version of the input signal for reduction of ambient noise. The conventional wisdom now is that FBAGC should be used for slow fading and FFAGC and FFSR for fast fading - the former for AM systems and the latter for SSB (single sideband) systems.

In the case of *Direct Sequence* (DS) *Spread Spectrum* receivers for frequency hopping there is a combination of multiplicative mixing and narrowband filter or integration which is equivalent to correlation. Hence the receiver is a correlator system. A correlation system has mathematical correspondences with a matched filter, so the receiver is also a matched filter (although the outputs from a correlator and a matched filter are, of course, different). In a digital system, a modulator applies impulses to a spreader. Unlike a time domain communications system, the bandwidth of the impulse is not transmitted, but only part of its spectrum weighted according to amplitude and phase. The receiver correlator or matched filter reconstitutes the impulse digital form of the transmitted signal. That the frequency bandwidth is used sequentially, instead of simultaneously, as in the time domain communications case, is immaterial to the bandwidth design of the receiver, especially as a

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[26]McGeehan, J.P., Automatic gain control, pp. 181-223 in W. Gosling (ed) *Radio Receivers*, Peter Peregrinus, London, 1986.

number of different transmission channels operate within a given bandwidth allocation. For maximum noninterference between signals on a multiply-used bandwidth, such signals must be mutually orthogonal. Orthogonality can be achieved with Walsh functions, Costas codes, etc. If there is absolutely a necessity for strict orthogonality, then a limited number of codes is obtained (the number depending on the length of the train). If merely a low correlation is all that is required, then many more codes are available.

Spread spectrum codes for frequency hopping are based on a feedback shift register pseudo-random sequence generator. Operation of these codes require knowledge of an initiation of the sequence. I adapted this approach to time domain communications, which are a form of time hopping. The codes I developed also require knowledge of an initiation of the sequence, of course.

Such codes are of the feedback shift-register pseudo-random form. Maximal-length sequences should be cosidered for time domain communications because of their relative non-interference properties. Maximal length shift register sequences (m-sequences) are binary sequences of length $p = 2^m - 1$, where m is the number of shift register stages. An m-sequence has the property that a window of length m slid along an m-sequence will see every $2^m - 1$ nonzero binary m-tuple once. m-sequences are also pseudorandom with respect to the distribution of runs, where a run is a maximal string of identical symbols. In any m-sequence, one half the runs have length 1, one quarter have length 2, one eighth have length 3 and so on, as long as these fractions give and integral number of runs. In each case the number of runs of 0's is equal to the number of runs of 1's.

m-sequences are described by characteristic polynomials which have a binary feedback shift register form. The number of m-sequence codes possible of length $2^m - 1$ is given by Euler's phi function:

$\phi(2^m - 1)/m$,

where the phi function describes the number of positive integers including 1 that are relatively prime to and less than $2^m - 1$. In the case of $m = 8$, $2^m - 1 = 255$. The prime factors of 255 are: 3, 5 and 17, and 128 positive integers are relatively prime to and less than $2^m - 1$. Therefore the number of m-sequence codes for an 8 stage shift register is: $128/8 = 16$ codes.

Regarding the cross-correlation and auto-correlation sidelobes: in the case of an m-sequence code of length 255, other codes will correlate with the matched receiver but at 10 db down. To obtain 40 db down, codes should be used of length 14,200. Using my algorithms I can readily obtain such discriminating codes. However, all these codes require knowledge by the receiver of the initiation of the sequence (i.e., synchronization). Having remarked on the similarities between conventional spread spectrum communications and time domain communications, we turn now to examine spread spectrum receivers.

A *sequence inversion keying* (SIK) or a *phase shift keying* (PSK) correlation receiver has a local oscillator modulated by the same sequence as that of the transmitter. The receiver sequence must be synchronized with the received signal, i.e., knowledge of the initiation of the sequence must be available. In which case, the mixing of the spread local oscillator signal with the received signal gives a narrowband IF modulated by the original bit stream. Finally, the demodulator gives the information bit stream which is decoded to give the analog signal.

The crucial question is as follows: given the *same* frequency allocation bandwidth, would a time domain communications systems interfere with, or receive interference from, a conventional spread spectrum communications system? On first blush, one might say yes, because even although the time domain system's use of the allocated bandwidth is both a simultaneous use (in the case of the monocycle pulse) and sequential (in the case of the coded train), and the conventional spread spectrum's use is always sequential, the spread spectrum direct sequence receiver's antenna does not know the difference (except,. perhaps, to transform the signal by dispersion and differentiation). Having arrived at the front end of the direct sequence receiver, the time domain monocycle should mix with the local oscillator signal. The further question is whether undispersed pulses of 1 nsec. duration will interact for a long enough time for a correlation to take place. Furthermore, there is also the presupposition that the time domain broadband monocycle was also registered as the initiating pulse for a sequence. In effect, for wideband interference - the case for a time domain monocycle train - the interfering energy must be in the intermediate frequency bandwidth. Given that the pulse on-off rate (GHz) for the time domain system is much faster than that of a frequency domain system, nonetheless, the chip rate (MHz) is about the same. Thus, given the *possibility* of a time domain system interference with a spread spectrum receiver because of the chip rate (MHz), the question, again, becomes whether *actually* the time constant of the spread spectrum's mixing and threshold process is fast enough given the pulse on-off rate of the time domain system (GHz). The answer to this question can only be obtained by an examination of spread spectrum receivers on a case-by-case basis. *Specifically, there is the need to know the integration time of the mixing process.*

In the case of telecommunications, it is instructive to consider to which of the three major access techniques to a satellite transponder time domain communications corresponds. The three are[27]:

(a) FDMA (*frequency division multiple access*). As with FDMA the transponder is divided into frequency band segments and each segment assigned to a user, there is no correspondence here with time domain communications.

(b) TDMA (*time division multiple access*). A TDMA user has access to a broadband channel of communications and digital transmission is usual, so there is some common ground with time domain communications. However, only one user appears on the transponder at any one given time, so there is no correspondence with time domain communications in this regard.

(c) CDMA (*code division multiple access*). With CDMA coding is used to spread the transmitted signal over the available frequency and temporal bandwidths. Time domain communications certainly uses coding to achieve comparable spreading. However, the short duration monocycle signal is frequency spread to begin with, which is not the case with conventional CDMA.

A reading of the literature on conventional receivers reveals that so prevailing is the mindset concerning frequency selectivity, that, as yet, I have not found any specifications on the transient response time of those receivers. It is important to know those specifications. A narrowband receiver may have a steady state response, say 1 GHZ, while

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[27]Freeman, R.L., *Radio System Design for Telecommunications (1-100 GHz)*, Wiley.

its transient turn-on response is less speedy, say 100 MHz (Fig. 1). In order to predict the interference with the receiver, the transient response must be known.

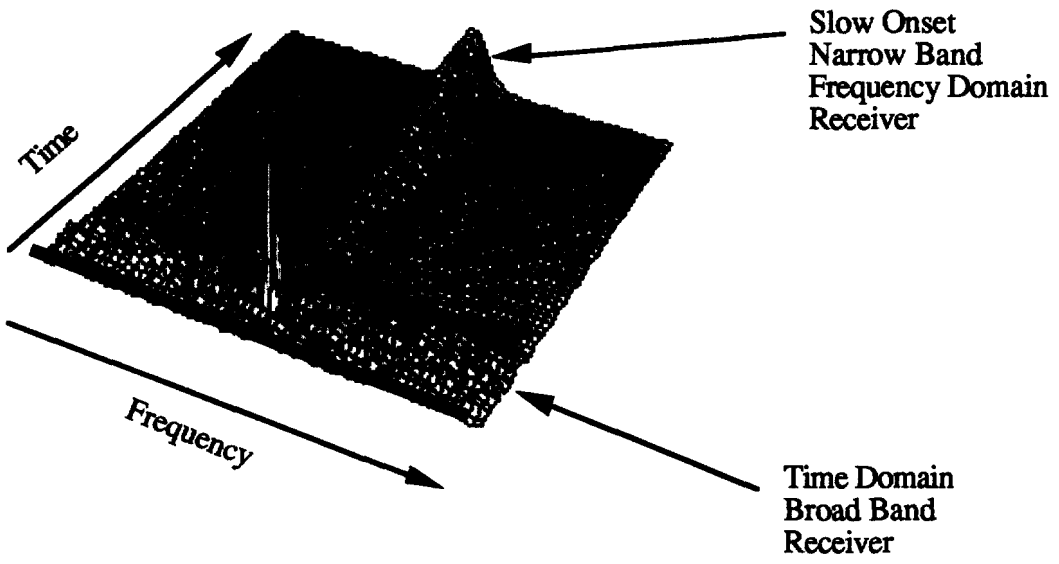

Fig. 1. This is a time-frequency representation of a time domain monocycle pseudorandomly coded pulse train and the receiver for a narrow band sinusoid. The area of narrowband reception does not overlap with the pseudorandomly coded pulse train, but this would not be apparent if viewed only from the vantage of a frequency-amplitude plot even if the narrow-band receiver is tuned to the pulse average frequency. There is a maximum frequency indicated - that of the pulse's average frequency. This would only be seen or sampled and displayed on a television screen which is a form of realm-time reception. The time domain pulse, represented here, is of too short a duration for the turn-on time of the narrow-band receiver, *even although the receiver and the pulse are at the same frequency allocation.*

The major difficulty in characterising the interference to frequency domain receivers is the difference in philosophy underlying the approach to frequency, versus time, domain receivers. In the case of frequency domain narrow band sinusoidal signals of long duration, Δt, any point of discontinuity, dt, e.g., at the beginning and end of the signal, is small compared with the duration Δt. For such signals, *local time methods* relate the *global nature of the signal*, i.e., its envelope, to its spectrum. This procedure is followed in Fourier analysis, as well as in Taylor series and other expansions.

In contrast, in the case of time domain pulse or wave packet signals, any point of discontinuity, dt, is large or of the same order compared with the signal duration Δt. Therefore time domain signals must be treated differently using *real time methods* to

preserve the *local nature, or fine structure, of the signal.* Periodic, or aperiodic (transient), i.e., frequency domain approaches to time domain signal processing, cannot be used due to the fact that for time domain signals: $dt \approx \Delta t$. The implication of this is that time domain signals and their derivatives do not possess discontinuities, indicating their analytical signal properties. Analytic signals are not deterministic or causal but are continuous and require local, or real time methods for characterization. Whereas the frequency domain signal is characterized by its harmonic frequency, the time domain signal is characterized by its instantaneous frequency.

That being the case, what is the required referent for a time domain receiver corresponding to the frequency (phasor) referent of frequency domain receivers? With ω as the radian frequency of a wave and s the phasor, or neper, frequency defining the envelope shape, a *complex signal* is:

$$s_0 = \sigma_0 + i\omega_0,$$

and an *analytic signal* (also known as the *preenvelope function*) is:

$$\mathcal{S}(t) = S(t) + i\hat{S}(t) = \exp[s_0] = A(t)\exp[i\omega_0 t + \phi t] = \exp[i\omega_0]\exp[\sigma_0],$$

with real part

$$S(t) = \exp[\sigma_0]\cos\omega_0 t,$$

and imaginary part:

$$\hat{S}(t) = \exp[\sigma_0]\sin\omega_0 t.$$

$\hat{S}(t)$ is the *Hilbert transform* of S(t):

$$\hat{S}(t) = (1/\pi)\int_{-\infty}^{\infty} S(\tau)/(t - \tau)dt.$$

$$S(t) = -(1/\pi)\int_{-\infty}^{\infty} \hat{S}(t)(t - \tau)d\tau.$$

Therefore, $\hat{S}(t)$ can be considered the output of a quadrature filter with input S(t). The quadrature filter has an impulse response:

$$h(t) = 1/(\pi t)$$

and system function:

$H(i\omega) = -i,\ \omega > 0;\ i,\ \omega < 0.$

The envelope of a wave packet (also known as the absolute value of the preenvelope) is defined as:

$envS(t) = \exp[\sigma_n] = \sqrt{[S(t)^2 + \hat{S}(t)^2]},$

and the carrier, average or midfrequency is defined as:

$\exp[i\omega_n] = [S(t) + i\hat{S}(t)]/[\sqrt{[S(t)^2 + \hat{S}(t)^2]}].$

For example, suppose

$S(t) = A(t)\cos\omega_0 t,$

then using the Hilbert transform the analytic signal is:

$\mathcal{S}(t) = A(t)\cos\omega_0 t + iA(t)\sin\omega_0 t,$

and the envelope of S(t) is:

$envS(t) = \exp[\sigma_n] = \sqrt{[(A(t)\cos\omega_0 t)^2 + (A(t)\sin\omega_0 t)^2]} = A(t).$

Now the power spectra are defined:

$\Phi_{S\hat{S}}(w) = \Phi_{SS}(\omega)H(i\omega) = -i\Phi_{SS}(\omega),\ \omega > 0;\ = i\Phi_{SS}(\omega),\ \omega < 0.$

Therefore:

$\Phi_{\mathcal{S}\mathcal{S}}(\omega) = 2[\Phi_{SS}(\omega) + i\Phi_{S\hat{S}}(\omega)] = 2\Phi_{SS}(\omega)\ ,\ \omega > 0;\ = 0\ \omega < 0.$

This final result is the ultimate reason for considering time domain signals as analytic signals, rather than complex signals in the time doman. The issue of causality for random continuous signals dictates that the power density spectrum be zero for negative frequencies. The analytic signals - together with the Hilbert transform - permits this

condition to be met. The use of the complex signal (i.e., with real and imaginary part) does not, and therefore is not suitable for use in the time domain. Causality conditions require that inverse transforms must be zero for negative t and have to be met in order to preserve the information in the signal. If the real and imaginary parts of a Fourier integral of a function $\mathcal{S}(t)$ satisfy the Hilbert transform relations, then $\mathcal{S}(t)$ is causal.

Fig. 2. shows the representations of a complex plane representation of the complete wave packet . It is usual in the teaching of electrical engineering to represent a circuit's response on the left side of the complex plane, i.e., the transient decay, but never on the right side, which is the transient response. However, time domain pulses must be represented on both sides and the transient onset response is critical in the understanding of receiver circuitry to such pulses. Unfortunately knowledge of the transient onset response is lacking for frequency domain receivers. The supposition has been that noise would always be frequency domain noise.

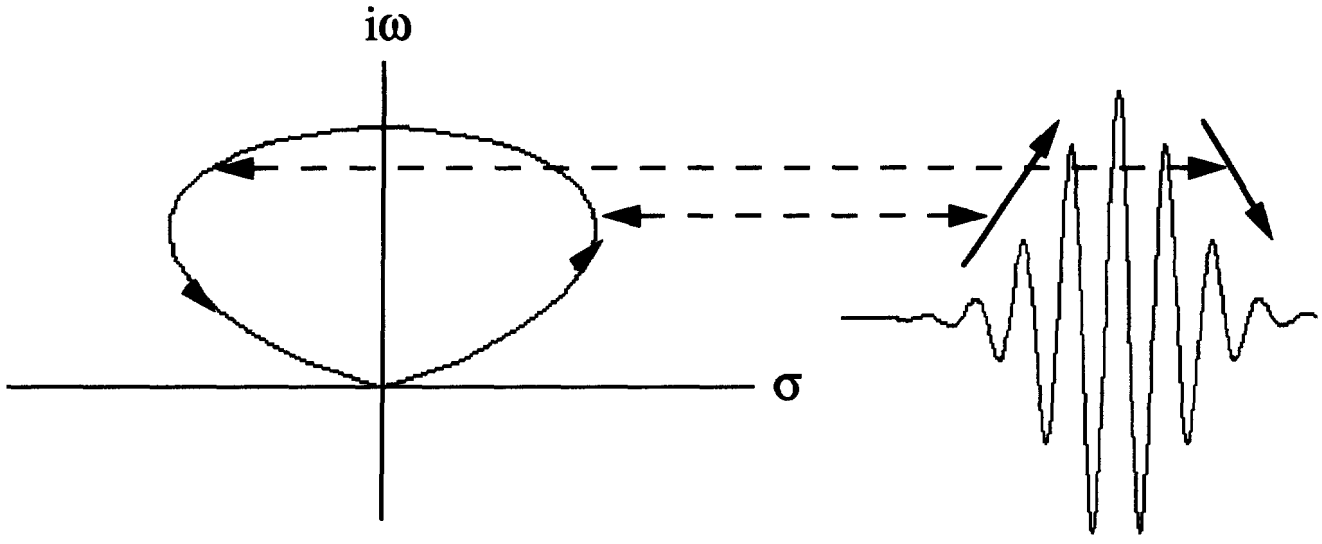

Fig. 2. Representation of a short pulse (left) in the complex plane (right). The onset of the pulse is represented in the right upswing in the complex plane, and the offset is represented in the left of the complex plane.

Returning to Fig. 1., if a cut is made through the figure then temporal response of a time domain pulse can be shown with respect to the transient turnon reponse of representative receivers (Fig. 3). Each synchronous frequency domain receiver is labeled with respect to its transient onset response (100, 50, 25, 10 and 5 MHz) and its steady state reponse (all: 1 GHz). Even for the case of the fastest transient response shown, 100 MHz, the time domain signal is 20 db down.

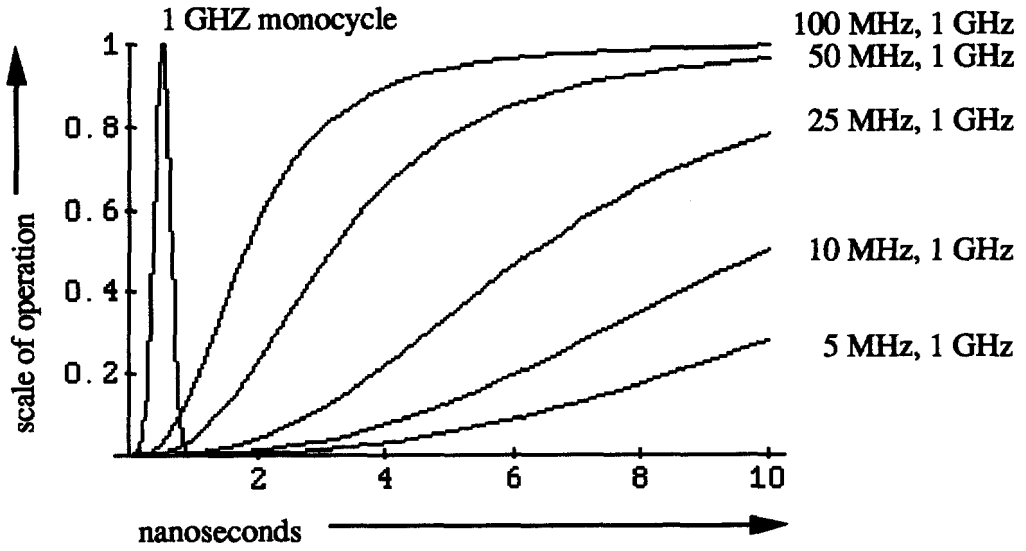

Fig. 3. This is a cut through the three-dimensional Fig. 1., above, and represents the transient onset response of synchronous frequency domain receivers with respect to that of a time domain 1 GHz pulse of 1 nsec. Representative transient responses are shown with the transient onset frequency represented. All receivers shown have a steady state response of 1 GHz.

The final question considered, here, is whether conventional signals can cause interference *to* time domain communications. This question can be examined from two points of view: (i) because the monocycle's receive antenna is broadband, there should be many conventional sources, including spark ignitions from automobiles, etc., which should cause noise to a *microsignal (at GHz rates)* in a time domain system; (ii) on the other hand, as the pulse train is sequentially coded, there should be sufficient processing gain that the *macrosignal (at MHz rates)* is recoverable from the noise. In other words, given that each individual pulse is vulnerable to interference, can the sequential coding provide enough processing gain offsetting the individual pulse's noise interference? The present conclusion is that the processing gain provided by the coded sequence offsets any individual pulse interference.

Appendix C.

Abstract: The types of codes necessary for minimum interference between users of impulse radio are examined. The types of codes proposed which give minimum interference are modifications of frequency-hopping Costas codes to time hopping.

The coding prolem can be thought about in the following way. The pulse train sequence can be considered in the time hop format (or Barrett-amended Costas codes), in which we have a code of length 10 and 10 dither slots within a frame (10 x 10, because 256 x 256 is too tedious, although, of course, it can be done). An illustrative coding scheme is shown in Fig. 1, below.

As an exercise, we can look at the possible interference between two Costas codes amended for time domain hopping. Then:

(1) n is the number of T = 1 nsec. pulses in a pulse train. n must be one less than a prime number, p, i.e., $n = p - 1$. We shall address $n = 10$, and thus $p = 11$, which is a prime number.

n is also the number of dithered intervals, δt, so we have an n x n matrix, i.e., a 10 x 10 matrix.

(2) We can define a <u>coincidence function</u> C(r,s) as the number of coincidences of marks between an original array and its translation. The parameters r and s define the amount of translation; r is the number of integer shifts to the right or left (translated by columns), and s is the number of integer shifts up or down (translated by rows). The coincidence function is a discrete analog of the (unnormalized) ambiguity function. Similarly, the <u>cross-coincidence function</u> is a discrete analog of the cross-ambiguity function. That is, With two n x n arrays, A and B, for each integer pair, (r,s), their cross-coincidence CAB(r,s) is defined as the number of coincidences of marks between A and translated versions of B, which has been shifted to the right by r and up by s.

(3) A Costas array is an array (i) consisting of n rows and n columns which has exactly one mark in each row and column; (ii) with a coincidence function satisfying:

$$C(r,s) \leq 1 \qquad (1)$$

for all integer pairs $(r,s) \neq (0,0)$ with $|r| \leq n - 1$, $|s| \leq n - 1$. In other words, a Costas array is a permutation matrix where the vector between each pair of elements in the array (matrix) is unique.

(4) We now run into a problem. If we require the following:

"Find two n x n Costas arrays, A and B, such that $C_{AB}(r,s) \leq 1$ for all integer pairs with $|r| \leq n - 1$, $|s| \leq n - 1$."

then it is known, that it is not possible to find two such arrays if $n \geq 4$. An n x n array for $n = 4$ is too small a pulse sequence to work with.

Fortunately, we can bound the bandwidth of the dithering to be a small portion of the total bandwidth of each dithered pulse train. The problem is now constrained and can be reworded thus:

"Given an integer m ($0 \le m \le n - 1$), find two n x n Costas arrays, A and B, such that $C_{AB}(r,s) \le 1$ for all integer pairs with $|r| \le n - 1$, $|s| \le m$."

(5) Finding two arrays which minimally correlate implies that the signals based on those arrays minimally correlate, i.e., that the maximum value of the cross-ambiguity function between signals f(t) and g(t) given by:

$$\left|\int_{-\infty}^{+\infty} f(t)g^*(t)\exp[-i2\pi\phi t dt\right|^2 \qquad (2)$$

is small compared with the peak value of the ambiguity function.
The problem stated in (4) can be solved but with the following restrictions:

$$n = p - 1, \qquad (3)$$
$$m \le (n - 2)/2, \qquad (4)$$
$$m \le (n - q)/q \text{ or } (m + 1)q \le n, \qquad (5)$$

where q is the number of Costas arrays with the required noninterference properties. What this means is: if the amount of dither (m) is set, and the number of pulses in the pulse train (n) is set, then the number of noninterfering Costas codes (q) is determined. Alternatively, if it is decided that a certain number of noninterfering Costas codes (q) is required, then the number of pulses in the pulse train (n) and the amount of dither (m) are determined. In a word: there is a set relation between the number of noninterfering codes to be obtained, the number of pulses in the pulse train, and the amount of dither.

(6) We now procede to construct Costas codes for n = 10 (i.e., p = 11). This approach can be the basis for an algorithm to construct codes of any n. First, some more definitions:

GF(p) is a <u>Galois field</u> in arithmetic modulo, p. (Galois fields are finite fields containing q $< \infty$ elements).
a is a <u>primitive element</u> of the Galois field.

$$d_j = (m + 1)(j - 1) + 1, \; j = 1,\ldots,q \text{ are integers in the} \qquad (6)$$
$$A_j = [a^{dj}, a^{dl+1},\ldots,a^{dj+n-1}], \; j = 1,\ldots q \text{ arrays.} \qquad (7)$$

where q satisfies the inequality (5) and the powers in (7) are evaluated modulo p. A_1 is called a <u>Welch array</u>.

(7) We commence with n = 10, p = 11, m = 2, q = 3 (satisfying (5)). The primitive element of GF(11) is a = 2.
Using (5), $d_j = (2 + 1)(1 - 1) + 1 = 1$, for j = 1.
Using (6), we obtain the following numbers: 2,4,8,5,10,9,7,3,6,1, by

$2 = 2^1$
$4 = 2^2$
$8 = 2^3$
$5 = 2^4 - 11 = 16 - 11$
$10 = 2^5 - 22 = 32 - 22$
$9 = 2^6 - 55 = 64 - 55$
$7 = 2^7 - 121 = 128 - 121$
$3 = 2^8 - 253 = 256 - 253$
$6 = 2^9 - 506 = 512 - 506$
$1 = 2^{10} - 1023 = 1024 - 1023$

This is the A_1 array or a Welch array. We can plot this as a coding matrix in Fig. 1.

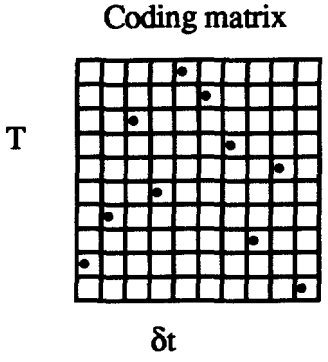

Fig. 1. Coding Matrix for an amended Costas sequence (the A_1 array or the Welch array for n = 10, p = 11, m = 2, q = 3). $\alpha = 2$ is a primitive element of GF(11). A_1 is 2,4,8,5,10,9,7,3,6,1.

(8) The next step is to extend the Welch amended Costas array horizontally to the right by periodic extension with period n. From the periodically extended array (upper in Fig. 2) an n x n subarray is taken starting at (m + 2)- nd column (shaded area in Fig. 2). This is A_2.

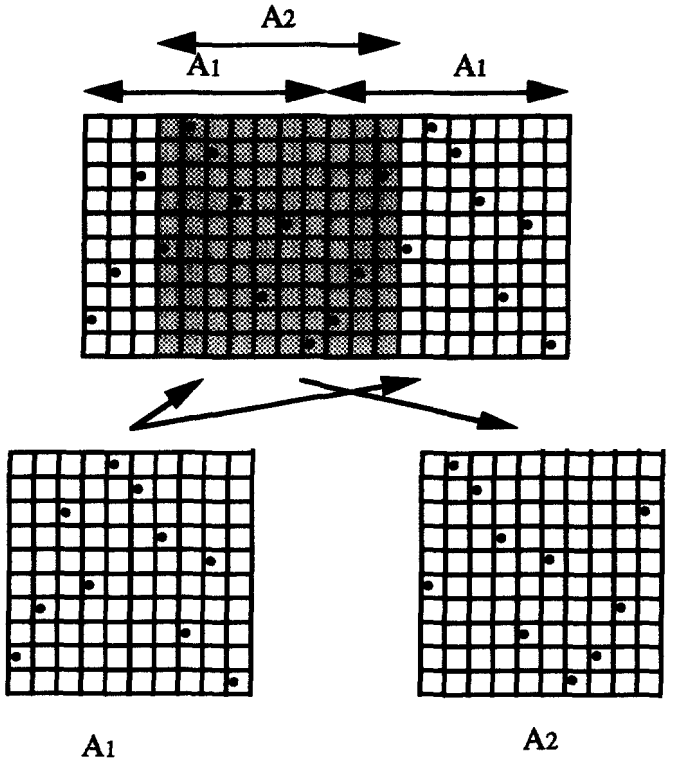

Fig. 2. Generation of the A_2 array from the A_1 Welch array for n = 10, p = 11, m = 2, q = 3. $\alpha = 2$ is a primitive element of GF(11). A_1 is 2,4,8,5,10,9,7,3,6,1; A_2 is 5,10,9,7,3,6,1,2,4,8.

(9) The next step is to generate A_3. A similar procedure is followed as in step (8) above, but starting the subarray at the $(m + 1)(j - 1)$ column for $m = 2$, $j = 3$. This is A_3 (Fig. 3).

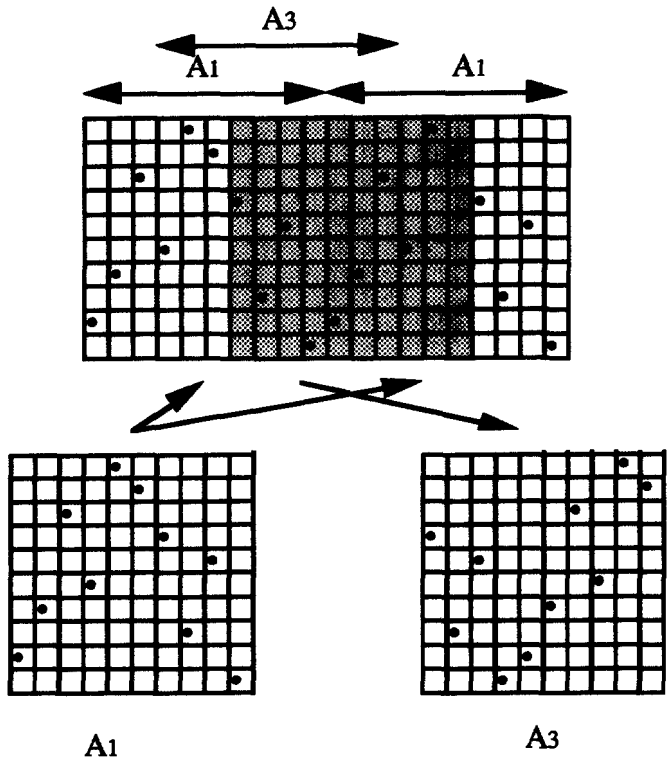

Fig. 3. Generation of the A_3 array from the A_1 Welch array for $n = 10$, $p = 11$, $m = 2$, $q = 3$. $\alpha = 2$ is a primitive element of GF(11). A_1 is 2,4,8,5,10,9,7,3,6,1; A_3 is 7,3,6,1,2,4,8,5,10,9.

(10) Difference matrices can then constructed for the A_1, A_2 and A_3 sequences (Fig.s 4-6).

a_j	2	4	8	5	10	9	7	3	6	1
i=1	2	4	-3	5	-1	-2	-4	3	-5	
=2	6	1	2	4	-3	-6	-1	-2		
=3	3	6	1	2	-7	-3	-6			
=4	8	5	-1	-2	-4	-8				
=5	7	3	-5	1	-9					
=6	5	-1	-2	-4						
=7	1	2	-7							
=8	4	-3								
=9	-1									

Fig. 4. Difference Matrix for A_1.

a_j	5	10	9	7	3	6	1	2	4	8
i=1	5	-1	-2	-4	3	-5	1	2	4	
=2	4	-3	-6	-1	-2	-4	3	6		
=3	2	-7	-3	-6	-1	-2	-7			
=4	-2	-4	-8	-5	1	2				
=5	1	-9	-7	-3	5					
=6	-4	-8	-5	1						
=7	-3	-6	-1							
=8	-1	-2								
=9	3									

Fig. 5. Difference Matrix for A_2.

a_j	7	3	6	1	2	4	8	5	10	9
i=1	-4	3	-5	1	2	4	-3	5	-1	
=2	-1	-2	-4	3	6	1	2	4		
=3	-6	-1	-2	7	3	6	1			
=4	-5	1	2	4	8	5				
=5	-3	5	-1	9	7					
=6	1	2	4	8						
=7	-2	7	3							
=8	3	6								
=9	2									

Fig. 6. Difference Matrix for A_3.

The difference triangle can be viewed as follows. The first row (i=1) corresponds to a δt dither of one time slot. Therefore the first element (=2) of this row means that two shifts upward will yield a coincidence between the transmitted pulse, T_1, and the transmitted pulse, T_2, i.e., they will be ambiguous. Two shifts upward corresponds to two steps in the pulse train. Therefore an ambiguity diagram, or sidelobe diagram can be created, in which the "2" in the first row will correspond to a "1" in the sidelobe matrix in the row that corresponds to a shifted T of 2, and in the column that corresponds to a δt of 1. This sidelobe matrix for the A_1 array of Fig. 1 is shown in Fig. 7, for the A_2 array in Fig. 8 and for the A_3 array in Fig. 9.

Sidelobe Matrix

Pulse Number	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
9					1														
8						1								1					
7			1				1								1				
6							1	1				1	1						
5					1				1		1			1		1			
4				1		1			1		1	1						1	
3		1					1	1	1		1		1		1				
2				1		1		1	1		1	1	1				1		
1	1			1		1		1	1			1	1		1		1		
0										10									
-1			1		1		1	1			1	1		1		1			1
-2			1				1	1	1		1	1		1		1			
-3					1		1		1		1	1	1					1	
-4		1						1	1		1			1		1			
-5				1		1			1		1				1				
-6							1	1				1	1						
-7					1								1				1		
-8						1								1					
-9															1				

Dither Delay Interval

Fig. 7. Sidelobe Matrix for the A_1 array.

Sidelobe Matrix

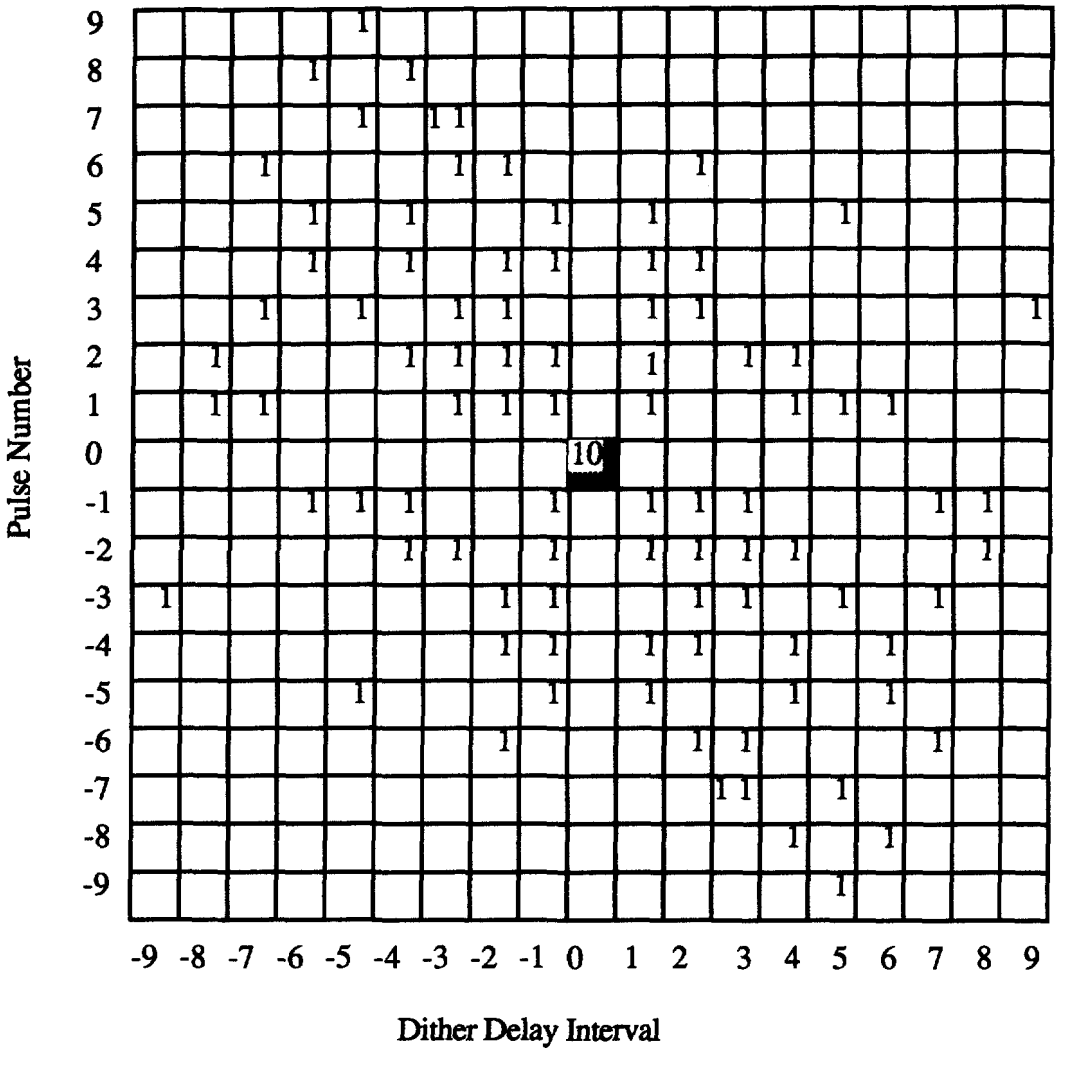

Fig. 8. Sidelobe Matrix for the A_2 array.

Sidelobe Matrix

Pulse Number	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
9															1				
8														1		1			
7													1		1		1		
6							1					1	1					1	
5						1			1		1			1	1				
4								1	1		1	1		1		1			
3					1				1		1	1	1				1	1	
2			1				1	1			1	1		1		1			1
1					1		1	1	1		1	1	1	1		1			
0										10									
-1				1		1	1	1	1		1	1	1		1				
-2	1			1		1		1	1			1	1				1		
-3		1	1				1	1	1		1				1				
-4				1		1		1	1		1	1							
-5					1	1			1		1			1					
-6		1					1	1					1						
-7			1		1		1												
-8				1		1													
-9					1														

Dither Delay Interval

Fig. 8. Sidelobe Matrix for the A_3 array.

We may now procede to answer the : What is the interference between two of these codes? Taking the sidelobe matrix plots of arrays A_1, A_2 and A_3, the composite superimposed matric plot is shown in Fig. 9. This indicates considerable overlap, indicated by the 2's and 3's, but also considable uniqueness.

Sidelobe Matrix

Pulse Number	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
9					2										1				
8				1		2								2		1			
7			1		1		3						1		2		1		
6			1				3	2				2	3					1	
5				1	1	2			3		3			2	2	1			
4				2		2		2	3		3	3		1		1		1	
3		1	1		2		2	2	2		3	2	2		1		1	1	1
2		1	1	1		2	2	3	2		3	2	2	2		1	1		1
1	1	1	1	1	1	1	2	3	3		2	2	2	2	2	2	1		
0										30									
-1			1	2	2	2	2	2	2		3	3	2	1	1	1	1	1	1
-2	1		1	1		2	2	2	3		2	3	2	2		1	1	1	
-3	1	1	1		1		2	2	3		2	2	2		2		2		
-4		1		1		1		3	3		3	2		2		2			
-5				1	2	2			3		3			2	1	1			
-6		1					3	2				2	3				1		
-7			1		2		1						3		1		1		
-8				1		2								2		1			
-9					1										2				

Dither Delay Interval

Fig. 10. Sidelobe matrix A_1, A_2 and A_3 superimposed.

In order to show this uniqueness pictorially, i.e., that Costas-Barrett signals can be noninterfering, the data of Fig. 10 are replotted and coded in Fig. 11. This Figure shows that any square not black, which indicates hits of 2 and 3, is unique to the codes from the A_1, A_2 and A_3 arrays.

Sidelobe Matrices A1, A2 and A3: n = 10, p = 11, m = 2, q = 3. $\alpha = 2$ is a primitive element of GF(11

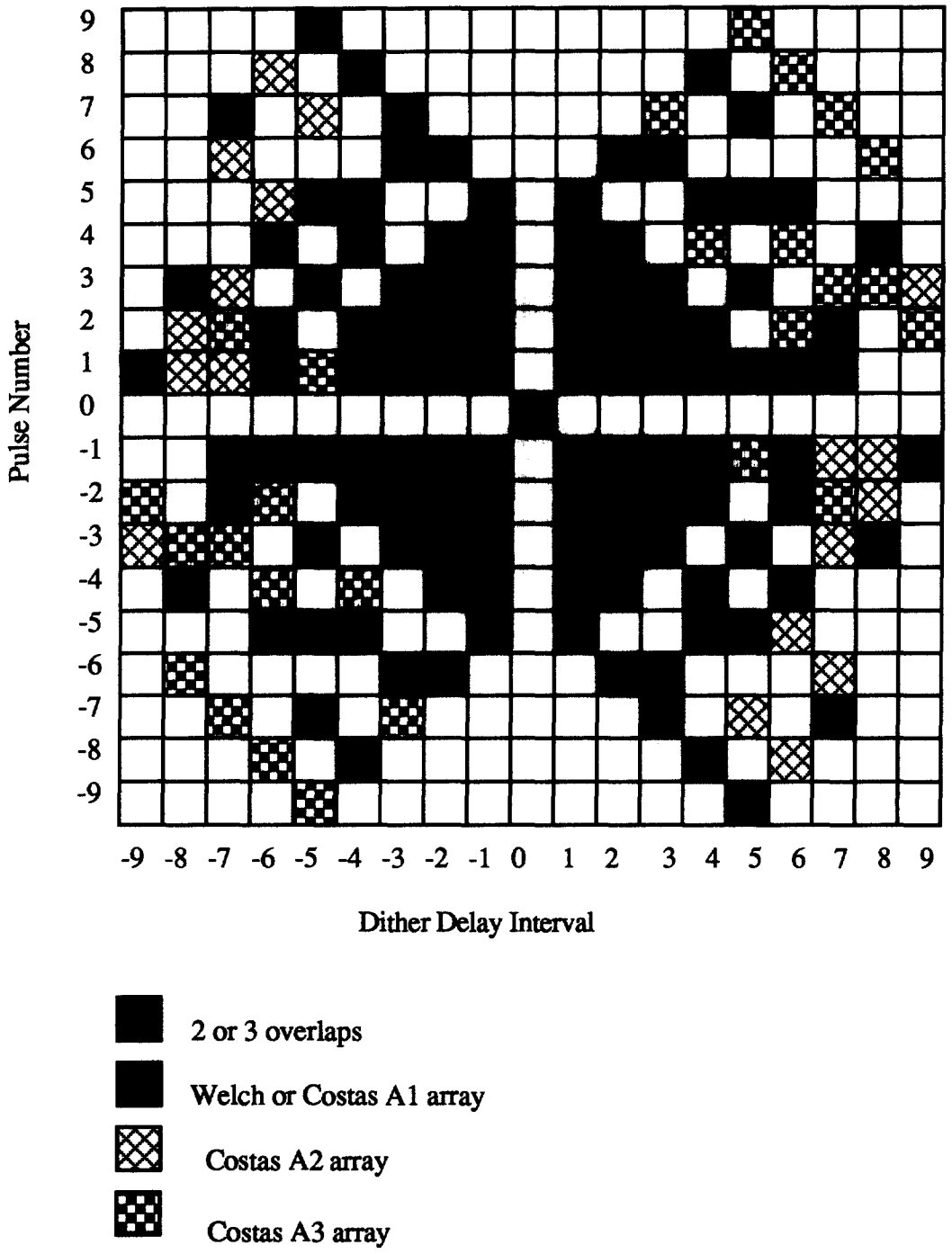

Fig. 11. Sidelobe matrix A_1, A_2 and A_3 superimposed and uniquely labeled.

The result and answer is clearcut for only two codes: unique, and therefore noninterfering, time-frequency patterns are obtained for these codes and the reader is reminded that this analysis is based on only a 10 x 10 array. (However, all of the above reasoning applies to arrays of any size - all that is required is scale-up).

The maximum number of intersections and the maximum value of the discrete cross-correlation function, R(k,n), between two Costas arrays is bounded by the maximum of the relative powers l and m between the analogous primitive elements defining the arrays, i.e., $R(k,n) \leq \max(l,m)$. ($l = m = 10$ for the 10 x 10 matrices considered here).

The sidelobe arrays studied above are digital representations of the corresponding ambiguity functions. That is, the ambiguity problem is "discretized" by these arrays. This correspondence is established as follows:

The energy in the T = 1 nsec.-long duration transmitted pulses can be normalized. If A is the constant amplitude of a sine wave present in each segment of length T/N seconds, then the energy of each segment is A^2T. To normalize the energy of a rectangular pulse requires:

$$A = 1/\sqrt{T} \tag{8}$$

and each pulse is:

$$s_k(t) = (1/\sqrt{T})\exp[i(\omega_k + \phi_k)], \text{ for } 1 \leq k \leq N \tag{9}$$

within the 1 nsec. time interval, i.e., within $(k - 1)T/N \leq t \leq kT/N$. If B is the 1 nsec. pulse's signal bandwidth, (so that each pulse occupies a signal time-bandwidth product of 2BT), then the frequency components of the pulse are given by:

$$\omega_k = \omega_0 + y_k(B/N), \tag{10}$$

where the y_k, k = 1,2,...., N are an ordered set of integers. The complete 1 nsec. signal can then be given as:

$$u(t) = \sum_{k=1}^{N} s_k(t) = (1/\sqrt{T})\exp[i\omega_0 t]\sum_{k=1}^{N}\exp[i\phi_k] \times \exp[i(B/N)y_k t]\, p(t - (T/N)k), \tag{11}$$

where p(•) is a rectangular gating pulse given by:

$$p(t) = 1, \text{ if } -T/N \leq t \leq 0;\ 0, \text{ otherwise.} \tag{12}$$

The ambiguity function for the signal is then:

$$A_u(\tau;\omega) = \int_{-\infty}^{+\infty} u(t + \tau)u^*(t)\exp[i\omega t]dt \tag{13}$$

$$= (1/T)\exp[i\omega_0\tau]\sum_{k=1}^{N}\sum_{l=1}^{N}\exp[i(\phi_k - \phi_l)]\exp[i(B/N)y_k\tau]$$

$$\times \exp[-i(T/N)l(\omega - (B/N)(y_k - y_l))]$$

$$\times A_p(\tau - (T/N)(k - l);\ \omega - (B/N)(y_k - y_l)),$$

where * denotes complex conjugation and A_p is the ambiguity function of the rectangular gating pulse p(•). If the phase of each pulse is zero, i.e, $\phi_k = 0$, then Eq. (13) becomes:

$$A_u(\tau;\omega) = (1/T)\exp[i\omega_0\tau]\sum_{k=1}^{N}\sum_{l=1}^{N} A_u^{(k,l)}(\tau;\omega), \tag{14}$$

k=0 l=0

where each $A_u^{(k,l)}(\tau;\omega)$ is given by:

$$A_u^{(k,l)}(\tau;\omega) = \underbrace{\exp[i(B/N)y_k\tau]}_{1.} \; \underbrace{\exp[i(T/N)l\omega]}_{2.} \; \underbrace{\exp[il(BT/N^2)(y_k - y_l)]}_{3.} \times \underbrace{A_p(\tau - (T/N)(k - l);\ \omega - (B/N)(y_k - y_l))}_{4.} \tag{15}$$

Now the terms labeled 1. and 2. are frequency x time functions and term 3. is a multiple of 2π reducing to unity for digital representation. In other words, in the case of digital representation Eq. (15) reduces to term 4. which is the sidelobe matrix.

Having established the relationship between the discrete ambiguity function representation of the sidelobe matrix and the continuous conventional representation, we may procede to plot in "bird's eye" form the conventional ambiguity function for a 10 x 10 Costas-Barrett signal in Fig. 12 using Eq. (15).

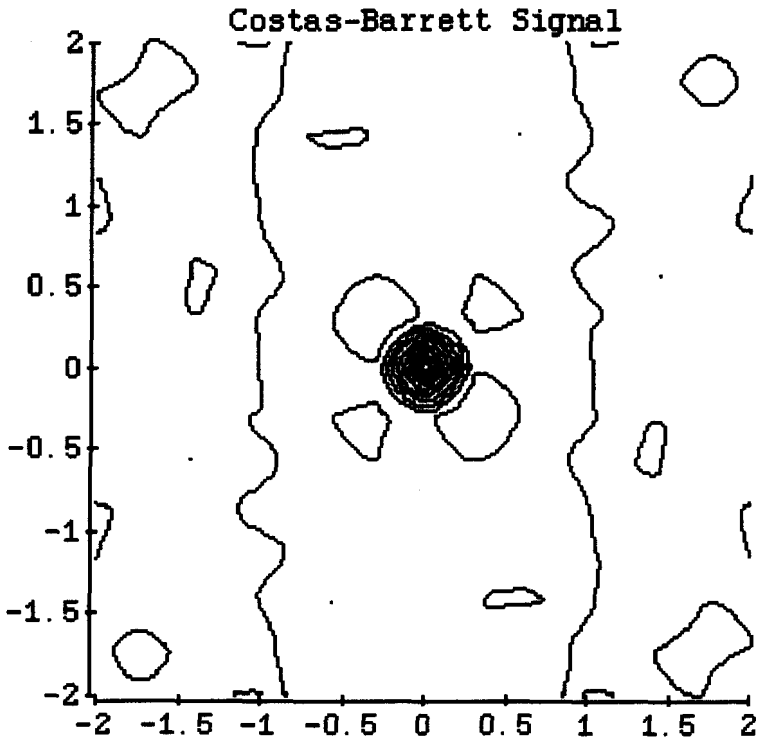

Fig. 12. Contour plot of the ambiguity diagram for a 10 x 10 Costas-Barrett signal.

The above was based on two small 10 x 10 array and their interference overlap. Naturally, a 256 x 256 would be more enlightening, but in a report like this, the argument would be less transparent to a reader. However, needless to say, the same logic applies.

EXHIBIT B

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November 5, 1992

William S. Moorhead, President
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8280 Greensboro Drive, Suite 500
McLean, VA 22102-3807

Dear Mr. Moorhead:

I'm writing to you to report my impressions to date of Pulson Communications' impulse radio technology, especially with regard to the number and quality of nearly orthogonal codes for controlling pulse transmission times. I understand that this letter may be used as part of a filing to the Federal Communications Commission.

As you know, these codes are a critical part of the system because they accomplish two things: (1) they smooth the distribution of energy in the frequency domain, thereby minimizing the potential for interference to and from conventional receivers, and (2) they contribute to the maximization of the impulse radio system's capacity.

In order to develop my opinion of the coding requirements for Pulson Communications' system, I developed a thorough understanding of the system, including all aspects of pulse formation, transmission, and receiver processing. Moreover, I observed demonstrations of signal propagation in a laboratory environment. My conclusions at this time are:

- Good pseudo-random codes for controlling monocycle transmission times do exist, and these codes will tend to spread the transmitted signal power relatively uniformly over a bandwidth on the order of the reciprocal of the monocycle's duration.
- There are a large number of codes that meet the technical requirements of the system. Moreover, these codes can be generated with standard digital circuits.

I believe that the multiple-access impulse radio system is sound and holds the promise of providing service equalling and quite possibly surpassing that of other asynchronous multiple-access technologies. Furthermore, I believe that Pulson's impulse radio technology is by far the widest of the asynchronous wideband RF multiple-access technologies currently being developed for commercial applications. Hence it represents a true pioneering effort that is likely to reveal new communication capabilities and novel system architectures. For example, while observing signal propagation in the laboratory, it was clear that the subnanosecond time resolution of the Gaussian monocycle waveform makes unprecedented multipath resolution possible. Application of Pulson's proprietary receiver technology,