While we agree that there is a free-rider problem, which we attempt to address, we caution the FCC against buying in to proofs by numerical example, when there is no underlying structure that guarantees the generality of the examples. We think that a more realistic model is one that allows for the possibility of resale, does not assume that all bidder valuations are known, and recognizes that there is a large common value element in the value of these licenses.

(2ii) Response to Professor McAfee's second round reply comments.

Turning to Professor McAfee's second round of reply comments, we are troubled by his interpretation of the results in Appendix B of our first report: "In [Bulow and Nalebuff's] discussion, they make two errors. First, they *assume* that a national licenses is the only aggregation that has any value. ... Second, even if a national license were the only important aggregation, their proposal promoted inefficient aggregation, which they ignore. Indeed, they describe an example in which it is assumed that bidders both know and choose to bid their value for the license. As is well established in the auction literature, bidders in an auction will act strategically to maximize profits, which involves bidding less than their maximum willingness to pay, contrary to the discussion in the report of Professors Bulow and Nalebuff."

This interpretation of Appendix B is surprising to us in light of what we wrote. We had no intention to restrict attention to national combinatorial bidding. We wrote:

> "For simplicity, we begin with the case where combinatorial bidding is restricted to national licenses. *We then show how the arguments extend to fully general combinatorial bidding below* [emphasis added.]"

After presenting the results with only nationwide combinatorial bidding we then wrote:

> "The above discussion focused on only one type of combinatorial bidding for a national license. To extend the auction design to allow combinatorial bidding for any combination of licenses, the FCC would ..."

and we went on to explain the more general result.

McAfee's second criticism was that we were so naive as to miss the strategic component of sealed bid auctions, i.e., that people bid below their true valuations. We did begin by

asking the reader to imagine that people bid their true valuations in order to explain how the allocation mechanism worked. But then we showed that people indeed wanted to bid their true valuation under the proposed payment system. As we wrote:

> "There are two presumptions about this efficient auction technique that remain to be addressed. First, is it possible to give everyone an incentive to reveal their true valuations? ... [Yes.] ... The solution is that each bidder gets a surplus equal to the marginal surplus brought to the system by his or her announced valuations. What the bidder pays is his or her valuations for the licenses received net of this surplus. ... The payments are not the bids but rather the announced valuations minus the difference between the maximum total valuation with the bidder and without the bidder."

Although we are at a loss to explain this confusion, it allows us to restate the main point of the Appendix B. We wrote "Experience has shown that even economics Ph.D. students have trouble understanding the description." We reject the all possible combinatorial bidding approach not because it is flawed in theory, but because it is too difficult to understand.

APPENDIX B

Assumptions: (1) The number of bidders is the same for both the national and regional auctions. (2) The value of a national network is the sum of the regional values; there is no premium for a national network. (3) In each region, the value of a license is identically and symmetrically distributed around its mean value for that region.[B1]

Theorem. *Under Assumptions (1), (2), and (3), the sequential region-by-region auction has the higher expected bid total once the number of bidders is strictly greater that one plus twice the number of licenses to be sold.*

For example, if there is one license to be sold, the sum of the regional auction has the higher expected value once there are more than three bidders. If we combine the two MTA licenses, then the regional auction will have the higher expected value provided there are at more than five bidders. Since we expect there will be a large number of bidders, absent economies of scale and scope, the regional bidders should win the auction. The intuition is that absent economies of scale, it is quite unlikely the highest national valuation will exceed the sum of the highest regional valuations; with more than five bidders, even the second-highest national valuation is expected to be less than the sum of the second-highest regional valuations.

Proof: When there are n bidders and $j-1$ items for sale, bidding stops when the price hits the jth highest valuation.[B2] We are interested in comparing the jth order statistic of the sum (the national bid) to the sum of the jth order statistics (the sum of the regional bids). Since the national bid is based on 51 times the *average* valuation of the licenses, the distribution of the regional sum is a mean-preserving increase in risk compared to the distribution of the national valuations. We now demonstrate that a mean-preserving increase in risk increases the expected value of order statistics

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[B1] For example, the value of a license in region 1 to bidder i equals $\mu_1 + \epsilon_1^i$, where μ_1 is the mean value of a license in region 1 which is common to all bidders and ϵ_1^i is an idiosyncratic value to bidder i for the license in region 1. We assume that the distribution of ϵ_1^i is symmetric around zero and furthermore that the distribution of ϵ_1^i is the same as that of ϵ_1^j.

[B2] This assumes no resale. With resale, regional bidders might by the license to firms interested in aggregating to a national license or a national bidder might purchase the license intending to break it up and resell the parts to individual firms. We look at the expected auction results without resale is to see who is likely to win based on solely on individual valuations, ignoring speculative bidding.

above the median (and lowers the expected value of order statistics below the median). Once the number of bidders is at least $2j$, the jth order statistic will be above the median. Thus, if there are two items for sale, the bidding will stop at the third-highest valuation which is above the median once there are at least 6 bidders. Since the order statistic is above the median, the greater "riskiness" associated with the distribution of winning regional bids implies that the expected value is higher than the expected value of the highest national bid.

The expectation of the jth order statistic is

$$S_j = \int_0^1 n\binom{n-1}{j-1} xF(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j} f(x,\epsilon)dx.$$

Integrating this by parts we have,[83]

$$\begin{aligned} S_j = &\int_0^1 (j-1)n\binom{n-1}{j-1} xF(x,\epsilon)^{j-2}[1-F(x,\epsilon)]^{n-j+1}dx \\ &- \int_0^1 (n-j)n\binom{n-1}{j-1} xF(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j}dx \\ &+ \int_0^1 n\binom{n-1}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-(j-1)}dx, \end{aligned}$$

$$= (n-j+1)S_{j-1} - (n-j)S_j + \int_0^1 n\binom{n-1}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-(j-1)}dx,$$

$$= S_{j-1} + \int_0^1 \frac{n}{n-j+1}\binom{n-1}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-(j-1)}dx.$$

$$= S_{j-1} + \int_0^1 \binom{n}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-(j-1)}dx.$$

as $(j-1)\binom{n-1}{j-1} = (n-j+1)\binom{n-1}{j-2}$.

Define k such that $(j-1)+k=n$. Thus position j and k are symmetric around the median. For example, if there are 11 positions, then the 9rd rank and the 3rd rank are symmetric. Without loss of generality, assume that $j \geq k$. Then

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[83] $u = n\binom{n-1}{j-1} xF(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j}, dv = f(x,\epsilon)$

$$S_j - S_k = S_{j-1} - S_k + \int_0^1 \binom{n}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-(j-1)}dx$$

$$= \sum_{i=1}^{j-k} \int_0^1 \binom{n}{j-i} F(x,\epsilon)^{j-i}[1-F(x,\epsilon)]^{n-(j-i)}dx.$$

Taking the derivative with respect to ϵ we have

$$\frac{d(S_j - S_k)}{d\epsilon} = \sum_{i=1}^{j-k} \int_0^1 (j-i)\binom{n}{j-i} F(x,\epsilon)^{j-1-i}[1-F(x,\epsilon)]^{n-(j-i)} \frac{dF(x,\epsilon)}{d\epsilon}dx$$
$$- \sum_{i=1}^{j-k} \int_0^1 (n-(j-i))\binom{n}{j-i} F(x,\epsilon)^{j-i}[1-F(x,\epsilon)]^{n-1-(j-i)} \frac{dF(x,\epsilon)}{d\epsilon}dx$$

$$= -\int_0^1 (n+1-j)\binom{n}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j} \frac{dF(x,\epsilon)}{d\epsilon}dx$$
$$+ \sum_{i=1}^{j-k-1} \int_0^1 \left[(j-i)\binom{n}{j-i} - (n+1-(j-i))\binom{n}{j-1-i}\right] F(x,\epsilon)^{j-1-i}[1-F(x,\epsilon)]^{n-(j-i)} \frac{dF(x,\epsilon)}{d\epsilon}dx$$
$$+ \int_0^1 k\binom{n}{k} F(x,\epsilon)^{k-1}[1-F(x,\epsilon)]^{n-k} \frac{dF(x,\epsilon)}{d\epsilon}dx$$

$$= \int_0^1 \left[k\binom{n}{k} F(x,\epsilon)^{k-1}[1-F(x,\epsilon)]^{n-k} - (n+1-j)\binom{n}{j-1} F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j}\right] \frac{dF(x,\epsilon)}{d\epsilon}dx$$

$$= k\binom{n}{k} \int_0^1 \left[F(x,\epsilon)^{k-1}[1-F(x,\epsilon)]^{n-1-k} - F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j}\right] \frac{dF(x,\epsilon)}{d\epsilon}dx$$

as $(j-i)\binom{n}{j-i} = (n-(j-i))\binom{n}{j-1-i}$.

But, whenever $F(x,\epsilon) \le \frac{1}{2}$,

$$F(x,\epsilon)^{k-1}[1-F(x,\epsilon)]^{n-1-k} - F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j} \ge 0 \text{ and } \frac{dF(x,\epsilon)}{d\epsilon} \ge 0.$$

Conversely, whenever $F(x,\epsilon) \ge \frac{1}{2}$,

$$F(x,\epsilon)^{k-1}[1-F(x,\epsilon)]^{n-1-k} - F(x,\epsilon)^{j-1}[1-F(x,\epsilon)]^{n-j} \leq 0 \text{ and } \frac{dF(x,\epsilon)}{d\epsilon} \leq 0.$$

Hence

$$\frac{d(S_j - S_k)}{d\epsilon} \geq 0.$$

QED

It is possible to determine which bidding type, regional or national, is expected to win given much more complicated assumptions about the number of bidders and the premium placed on having a national network. But it all likelihood, these results will be based on a simulation analysis.