Since $v(t)$ is exponentially-distributed with a mean of 1, $r(t)$ is Rayleigh-distributed with a probability density function:

$$f_{\mathrm{r}}(r) = 2re^{-r^2} \tag{A-50}$$

From (A-49),

$$\Phi_{\mathbf{gg}}(f) = 2\Phi_{\mathbf{rr}}(f)\Phi_{\mathbf{ss}}(f_0)B_N \tag{A-51}$$

From (A-34), $\Phi_{\mathbf{gg}}(f) = |H(f)|^2 \Phi_{\mathbf{ss}}(f_0)$ if the filter bandwidth is small compared to the bandwidth of the input signal. Substituting this into (A-51) for $\Phi_{\mathbf{gg}}(f)$ gives:

$$\Phi_{\mathbf{rr}}(f) = \frac{|H(f)|^2}{2B_N}. \tag{A-52}$$

As a sanity check, $\int_{-\infty}^{\infty} \Phi_{\mathbf{rr}}(f)df = \frac{1}{2B_N}\int_{-\infty}^{\infty} |H(f)|^2 df = 1$, as should be the case because the expected value of $v(t) = r^2(t)$ is 1.

Therefore, the correlation properties of the filter output are determined by the filter impulse response. The correlation time will inversely proportional to the filter bandwidth. Thus, the output from a 1-MHz filter will have a correlation time on the order of 1 μsec.

ATTACHMENT B

WIDEBAND DIGITALLY MODULATED SIGNALS EMISSION STATISTICS AND MEASURTEMENT

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Abstract

It is shown that the emissions of wide bandwidth, digitally modulated devices such as those likely to be developed in accordance with the new Rules of FCC Docket 96-102 will behave as a gaussian distributed noise signal in virtually all situations in which they may potentially interfere with other systems. This leads to the conclusion that the principal parameter necessary to be controlled is the mean power level in a bandwidth of about 1 MHz.

However, if only the mean power level is specified and measured, there is a potential for a signal to have relatively short duration high power levels of a few data symbol times in duration which might, in some cases, dominate the potential interference effect. Thus, it is necessary to supplement mean power level restrictions with further restrictions on these short term peaking effects. Specification and measurement of short duration high power levels is difficult because of the statistical nature of all signals, including acceptable signals with smooth envelopes. A means of computing the relative effects of short duration high levels during a burst is developed and some conclusions relative to specifying and measuring this effect are described.

1.0 Summary

The power spectral density of emissions from the wide bandwidth, digitally modulated devices conforming to the new Rules of FCC Docket 96-102 (hereinafter called U-NII devices for Unlicensed NII band) is controlled to enforce spectrum sharing between these systems and other co-channel and adjacent spectrum systems. Typically, the other uses potentially affected by U-NII emissions utilize narrower bandwidths than U-NII digital systems and devices, thus it is necessary to control the narrow bandwidth levels of the U-NII signals. To do this, the regulations control the power spectral density of the U-NII emissions by specifying the level produced in a measurement bandwidth much lower than the spectrum mask bandwidth of the device emissions.

This paper investigates the signal description that results from the above process; that is, from the process of filtering a wide bandwidth digitally modulated carrier signal with a narrow bandwidth bandpass filter. The conclusion is that the U-NII emissions can be best specified by the mean power of the emission while the device is on, supplemented with some further control of potential higher power segments of a few symbols duration.

In all significant cases of potential interference the emission can be treated as a gaussian random noise signal.

More specifically, it is shown that the probability distribution of the voltage level of a 1 MHz bandwidth sample of a 27 Mb/s QAM modulated U-NII signal occupying about 20 MHz emission bandwidth (26 dB bandwidth) is truncated gaussian with a truncation value of at least 2.6σ. It is further concluded that the 1 MHz bandwidth voltage distribution of any U-NII device that results from a digitally modulated single carrier emission with the above bandwidth is also a truncated gaussian distribution with a truncation level in this same range.

The truncation level increases inversely with the square root of measurement or victim bandwidth and the truncation level produced by multiple devices transmitting simultaneously increases with the power ratio of the multiple devices over that of a single device. In virtually all instances of potential interference, the U-NII emission will behave as random noise to devices of 1 MHz bandwidth and less.

It is shown that short term variations of as little as 1.5 μs (which corresponds to about 20 data symbols in the widest bandwidth emissions expected) in the data signal power level can be detected with a 1 MHz bandwidth spectrum analyzer. Thus, a specification on signal level of as few as 20 data symbols can be detected in such a test. However, statistical variations in test results will need to be carefully analyzed.

Measurements of such short term effects should utilize the highest spectrum analyzer resolution bandwidth available.

2.0 A Quadrature Amplitude Modulation Case

A digitally modulated, Quadrature Amplitude Modulation (QAM), wide bandwidth signal can be used as a typical U-NII emission with which to describe the characteristics of a narrow bandwidth signal measured within the spectrum mask of the U-NII signal. The following describes such a QAM signal[1].

$$S_i(t) = r_i x(t) \cos\omega_c t + q_i x(t) \sin\omega_c t \qquad (2\text{-}1)$$

where

r_i and q_i are random variables assuming values of +1 or -1 with equal probability,

x(t) is a unit pulse which exists over a time interval of between 1 and 2 symbol times and

ω_c is the radian carrier frequency.

A sequence of N symbols (2N bits) can then be described as S(t) where

$$S(t) = \sum_{i=1}^{N} S_i(t - iT) \qquad (2\text{-}2)$$

where T is the reciprocal of the symbol rate. It is expected that the value of N for U-NII devices will normally be in the range 200 to 2,000 (50 to 500 octets).

U-NII devices will usually transmit bursts of symbols periodically at low duty cycle, with the power off between bursts. It is the characteristics of the signal while power is on that are of interest. Subsequently when the power or voltage distribution of the measured signal is mentioned it is meant to apply to that which exists while power is on.

The power spectrum of the signal of equation 2-2 is usually approximately flat within about 3 dB for a frequency width of 1/T and the 26 dB emission bandwidth is approximately 1.5/T. For example, for a signaling rate of 27 Mb/s, the symboling rate is 13.5 Mb/s, T is 75 ns and the 3 dB and 26 dB bandwidths are approximately 13 MHz and 20 MHz respectively.

It is shown in annex A that the signal that results from filtering a single symbol of the above wideband digital signal with a narrow bandwidth filter can be expressed in the following form

$$S_{oi}(t) = K\left\{\left[r_i y_o(t) - q_i z_o(t)\right]\cos\omega_m t + \left[q_i y_o(t) + r_i z_o(t)\right]\sin\omega_m t\right\} \qquad (2\text{-}3)$$

where

K is a constant depending on the narrow band filter,

ω_m is the radian center frequency of the measurement filter,

$y_o(t)$ and $z_o(t)$ are signals of relatively long duration and

the other symbols are as defined before.

The complete narrow band sample resulting from the transmission of N random symbols (2N bits) is

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[1] Equation 2-1 actually describes a family of amplitude and phase modulated signals. Many characteristics of the resultant can be controlled by properly shaping x(t).

The signal of equation 2-1 has a constant mean power level over a few symbol times, but will usually have a higher power level over a maximum single cycle (the peak envelope power). Thus, to determine the mean power of the signal it is necessary to average over a few symbol times. This averaging is assumed in the body of the report when referring to the total power level of the signal.

ATTACHMENT B - WIDEBAND DIGITALLY MODULATED SIGNALS

$$S_o(t) = K\sum_{i=1}^{N} S_{oi}(t - iT). \qquad (2\text{-}4)$$

If the wideband signal spectrum bandwidth is W and the resolution bandwidth of the measurement is B_r, then the signals $y_o(t)$ and $q_o(t)$ have a duration proportional to WT/B_r. That is, the duration is on the order of W/B_r symbol times. Annex A shows that for a typical 1 MHz measurement filter and the 27 Mb/s signal described above, the ½ amplitude duration for W = 13 MHz and Br = 1 MHz is about 750 ns, which corresponds to 10 symbol times. Thus, in this typical case the ½ amplitude is about 0.8WT/Br.

Other cases will not vary much from this ratio.

Thus, the summation in (2-4) includes about $0.8W/B_r$ significant contributions from different $S_{oi}(t)$ signals, that is, each value of the narrow band filter output consists of a sum of signals generated by approximately $0.8W/B_r$ symbols [the duration of $y_o(t)$ and $z_o(t)$]. The contribution of each symbol is given by (2-3) translated in time, thus each component is a carrier signal of random phase and amplitude with both the phase and amplitude determined by the random numbers r_i and q_i. Because of the strong dependence on the bit senses (r_i and q_i), each component is essentially statistically independent of the previous and subsequent components.

It can be shown that the average and variance of the sum of independent random variables are given by the sum of the individual averages and variances[2]. Further, the central-limit theorem states that the distribution of the sum of n independent variables approaches the gaussian distribution as n approaches infinity[3]. Thus, if the bandwidth ratio (W/B_r) is high, the distribution of $S_o(t)$ is gaussian with a mean of 0 and a standard deviation equal to the square root of the mean power. The envelope voltage distribution is Rayleigh and the power distribution is exponential. However, the maximum value of $S_o(t)$ is limited, thus the distributions are truncated. Usually the gaussian distribution can be used for estimating levels up to 70 to 80% of the truncation level.

An estimate of the maximum value of $S_o(t)$ (and thus the truncation level) can be made by considering the case where each successive symbol in (2-1) is the complement of the previous symbol. In this case the modulating signal can be represented as a sequence of x(t) unit pulses of alternating polarity. Such a sequence can be represented by a Fourier series with a fundamental component of frequency 1/2T. The spectrum of such a modulated carrier signal has two spectral lines at +/- 1/2T Hz around the center frequency, thus when the measurement filter is centered on one of these lines the output is ½ the total power of the in-band signal. Since this condition can conceivably occur, it can be used to estimate the minimum truncation level.

The mean power output of the measurement filter is approximately the total power times the bandwidth ratio B_r/W. Thus, the peak power to mean power ratio is very nearly equal to $W/2B_r$. For the case postulated above with W = 13.3 MHz and B_r = 1 MHz this ratio is about 6.6. This is approximately $\sqrt{6.6} = 2.6$ standard deviations above the rms voltage. Thus, the gaussian voltage distribution truncates at or above 2.6σ and the peak envelope power can be at least as high as 8.2 dB above the mean power.

Note that the truncation level of the voltage distribution is proportional to the square root of the bandwidth ratio. If B_r is assumed to be 100 kHz, the truncation level is over 8σ. In this case, the signal will be virtually indiscernible from random noise.

However, occurrence of a level near the truncation level depends on a specific long sequence of bits occurring in the modulating signal. It is not likely that a pattern used for testing will have such a sequence when the bandwidth ratio W/B_r is high; and any test for peak envelope power is likely to be sensitive to the

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[2] See Emanuel Parzen, Modern Probability Theory and Its Applications, John Wiley and Sons, 1960, chapter 8, section 4

[3] See Mischa Schwartz, Information Transmission, Modulation and Noise, McGraw-Hill, 1959, section 7.11.

testing sequence used. Also, even if the modulating pattern is truly random for a very long interval, the waiting time for the occurrence of a signal near the truncation level will likely exhaust the patience of the person performing the test.

3.0 The General Case

In general, any bandwidth limited carrier modulated signal in which the bandwidth is much less than the carrier frequency can be represented by a signal of the form

$$Y(t) = u_1(t)\cos\omega_m t + u_2(t)\sin\omega_m t \qquad (3\text{-}1)$$

where ω_m is the radian frequency at which the signal is observed and $u_1(t)$ and $u_2(t)$ are baseband signals with spectral bandwidths near the spectral width of the emission. If this signal is filtered by a bandpass filter of resolution bandwidth B_r, the resulting signal can be expressed by

$$Y_o(t) = u_{o1}(t)\cos\omega_m t + u_{o2}(t)\sin\omega_m t \qquad (3\text{-}2)$$

where $u_{o1}(t)$ and $u_{o2}(t)$ are signals formed by filtering $u_1(t)$ and $u_2(t)$ by a low pass filter of bandwidth $B_r/2$ as described as $H_1(f)$ in annex A.

As in the case of QAM, Y(t) consists of the delayed sum of components of the form

$$Y_i(t) = u_{i1}(t)\cos\omega_m t + u_{i2}(t)\sin\omega_m t \qquad (3\text{-}3)$$

For all wideband digitally modulated signals in which a baseband signal is impressed upon a single carrier with amplitude or phase modulation, $u_{i1}(t)$ and $u_{i2}(t)$ are functions, in general, dependent on the magnitude and sign of the combinations of bit senses comprising a symbol and they normally have a duration slightly longer than a symbol time (T).

For QAM case, the precise dependence of $u_{o1}(t)$ and $u_{o2}(t)$ on bit polarities is given in the preceding section. In the general case, the exact form of the dependence on symbol polarity and amplitude may differ from the QAM class, but they are nevertheless random variables dependent upon random bit polarities. Thus, the probability distribution of $Y_o(t)$ is truncated gaussian. And this applies to any narrow bandwidth sample of the digitally modulated signal spectrum - either in-band or out-of-band. The level at which the probability distribution is truncated may be different in the general case, but it isn't likely to vary very much from the QAM case.

4.0 Interference Potential of Wide Bandwidth Emissions

Devices and systems occupying spectrum adjacent to U-NII systems or sharing spectrum with them will generally have receiver bandwidths in the order of 100 kHz to 1 MHz. Thus, it is reasonable to specify the emission levels with measurement bandwidths in this range and to investigate the effect U-NII emissions might have on systems with bandwidths in this range. This section shows that U-NII emissions can be expected to conform to the gaussian distribution in virtually all cases in which there is potential interference with other systems.

The peak envelope power of gaussian noise is undefined since a gaussian random variable can, in theory, assume any value. Thus, the power level of such an emission is best defined as the mean power as opposed to the peak envelope power. Interference specifications for potential victim receivers reference the mean power of such emissions and are based on the assumption of a gaussian distribution. Once the mean is known, the statistical properties of the emission are well known and the effect on system performance of a given mean noise power level is known.

The mean power of a number of simultaneous emissions is equal to the sum of the mean power of the individual emissions and even if the individual emissions have lower variability than the gaussian, the sum tends to a gaussian distribution. Further, the sum of the mean power of the narrowband components of a wideband transmission is equal to the mean power of the overall wideband power. For example, if the spectrum is flat over a bandwidth B and the power measured in a bandwidth B_r within B is P_r then the total

power within B is BP_r/B_r. It is very important to note that this is not necessarily true of the peak power, even if the peak is limited; the overall transmission signal envelope can have a virtually constant envelope while the low bandwidth samples will tend to be gaussian distributed. Thus, the sum of the peak envelope power of the narrowband components will usually be considerably higher than the peak envelope power of the overall transmission. The sum will always be higher than the mean power of the transmission.

The assumption of a gaussian distribution for an interference signal is a worst case assumption when high signal variations from the mean are necessary to create interference effects. In other words, a truncated gaussian signal will have less impact on the interfered system than a pure gaussian signal with the same average power.

It was shown in the previous sections that the signals that receivers of 1 MHz and lower will experience from a single U-NII device transmission have essentially a truncated gaussian voltage distribution. The conditions under which the truncation level is high enough for the gaussian approximation to be valid is considered next.

In most cases where U-NII devices have any likelihood of interfering, the potential interfering signal must be the composite of signals from a large number of such devices. It is shown below that if the composite power from a number of devices transmitting simultaneously is as much as 9 dB above that of the highest level received from a single device, then for all practical purposes, the probability distribution of the voltage is gaussian.

The average power level from a number of devices transmitting simultaneously is the sum of the average power levels which they would produce individually. Since it is theoretically possible for all voltage levels to add at peak values, it can be deduced that the truncation level of a transmission which produces a power level K times the power level of the device producing the largest of the levels will be larger than the truncation level of the single highest level device by a factor of the square root of K.

The following table illustrates this relationship on the assumption that the gaussian truncation level of a single device transmission is 2.6σ, as was the case for the QAM class of modulation structures and a 1 MHz bandwidth for the measurement or victim receiver.

Composite power/maximum device power (dB)	Truncation Level/σ
0	2.6
3	3.7
6	5.2
9	7.3
12	10.3

It can be expected that the distribution will follow the gaussian to within 70 t0 80% of the truncation level. The probability of a gaussian variable exceeding 5σ is about 3×10^{-7} per independent sample, thus, this will be taken as a threshold level. When the power level is increased above the highest single device level by about 9 dB due to multiple simultaneous transmissions, the 1 MHz bandwidth truncation level is greater than 7σ and the gaussian distribution can be expected to be accurate to over 5σ.

If the victim receiver bandwidth is 100 kHz the single device truncation level is at least 8σ and the gaussian distribution can be used even for single device emissions when the data signal bandwidth is 20 MHz.

5.0 Slow Amplitude Variations

In the above sections it was assumed that a signal described by equation 3-1 exists for the full duration of all bursts.

$$Y(t) = u_1(t)\cos\omega_m t + u_2(t)\sin\omega_m t \qquad (3\text{-}1)$$

If the power level of this signal is measured by averaging the envelope over a few digital symbol times, the measured level will be virtually constant over the full burst duration. However, it is possible that the signal level will vary slowly as

$$Y_2(t) = A(t)[u_1(t)\cos\omega_m t + u_2(t)\sin\omega_m t] \qquad (5\text{-}1)$$

where A(t) is a slowly varying function of time compared to the symbol rate.

If only the mean power of the burst is specified in the regulations, then it is possible that power control could manipulate A(t) and create an interference potential that is higher than the mean power would indicate. On the other hand, there are legitimate modulation techniques which have power variations over a few symbol times and these techniques should be permitted without penalty. Thus, some control of the mean power over an interval shorter than a burst duration but longer than a single symbol is necessary.

If $A^2(t)$ has a mean equal to 1, then the mean power of 5-1 will be the same as that of 3-1. For example, A(t) could vary considerably over a burst duration, yet be relatively constant over several symbol times. If this is the case, the interference potential of the transmission may be greater than it would be without such variations in the envelope. If the average value of $A^2(t)$ over a transmission burst is 1, the maximum voltage level is A_{max} and the mean power of the burst is P_b then the maximum value of power during the burst is

$$\text{Maximum burst power} = A_{\max}^2 P_b .$$

This condition only exists for a short period of time if the value of A_{max} is high. Thus, if a large number of simultaneous device transmissions are necessary to create the interference potential, the likelihood of simultaneous peaks is low and the maximum interference can best be estimated by ignoring the single peak effect. However, in cases where interference can be caused by one or a few simultaneous transmissions (for example interference with other U-NII devices) the peaking effect needs to be controlled. In these cases, control over short time intervals is necessary.

If the potential victim receiver bandwidth is B_r and B_r is lower than the data spectrum width, then the short bursts of high power will have little effect on the victim, providing the duration is much less than $1/B_r$. Legitimate data modulation techniques produce envelope variations with autocorrelation times of about the data symbol duration and thus will have no effect on potential victim with bandwidths of 1 MHz or less.

Note: In fact, data signal envelope variations at the symbol rate will have little effect on wider bandwidth receivers as well, but this subject is beyond the scope of this paper.

Consider A(t) as a function that assumes either of two values, a normally low value and a higher value which it assumes and maintains for a short duration t_1, where $t_1 > 1/B_r$. If $H_1(f)$ is a low pass filter of low pass bandwidth $B_r/2$, as described in annex A, in which the response to A(t) is $A_o(t)$, then $Y_o(t)$ can be approximated by

$$Y_{2o}(t) = A_o(t)\left[u_{o1}(t)\cos\omega_m t + u_{o2}(t)\sin\omega_m t\right] \qquad (5\text{-}2)$$

where $u_{o1}(t)$ and $u_{o2}(t)$ are the responses to a filter $H_1(f)$ as defined in 3-2. The waveform will be equal to $Y_{2o}(t)$ after a transient period following the magnitude change[4].

[4] The duration of the transient period is approximately $1/2B_r$. The signal will not have the exact form of equation 5-2 during the transient period, but the rise time will be approximately the same as that of $A_o(t)$.

The envelope of this signal is

$$E_o(t) = A_o(t)\sqrt{u_{o1}^2(t) + u_{o2}^2(t)} \qquad (5\text{-}3)$$

This envelope can be detected, captured, displayed and processed by a spectrum analyzer. In this case, B_r is the resolution bandwidth of the spectrum analyzer. As stated earlier, a standard measuring bandwidth of 1 MHz is appropriate to represent the worst case potential victim bandwidths, thus the autocorrelation time of $E_o(t)$ is about 1 μs.

Most spectrum analyzers have a 1 MHz resolution bandwidth capability. The question arises whether measurements can be made using a 1 MHz resolution bandwidth that will isolate the slow envelope variations [A(t)] from the legitimate variations of the quantity under the radical in 5-3, and what parameters should be specified in the regulations to control this.

It was established in previous sections that the signal under the radical sign in 5-3 can be treated as a narrow bandwidth noise signal of spectrum width B_r for wide bandwidth digital modulation. For convenience, this will be referred to as v(t) below. Then $v^2(t) = p(t)$ is the envelope power. The signal v(t) has a Rayleigh voltage distribution and p(t) has an exponential distribution.

$$v(t) = \sqrt{u_{o1}^2(t) + u_{o2}^2(t)} \qquad (5\text{-}4)$$

$$p(t) = u_{o1}^2(t) + u_{o2}^2(t)\,. \qquad (5\text{-}5)$$

Some properties of the probability distribution of these signals are given in annex B, and annex C addresses the effect of variations of $A_o(t)$ in terms of an effective increase in the noise power of over that of a signal with $A_o(t) = 1$.

In summary, annex C shows:

1. A single burst containing short high power segments of duration 1.5 μs or more can be detected with a spectrum analyzer of bandwidth 1 MHz or more with limited assurance of detection.
2. The assurance of detection can be enhanced to an arbitrarily high level by analyzing multiple bursts.
3. The spectrum analyzer bandwidth for testing for these conditions should be as high as the instrument is capable of achieving.

ATTACHMENT B - WIDEBAND DIGITALLY MODULATED SIGNALS

Annex A - Computation of the Output of a Bandpass Filter

This will sketch the relationship between wideband signal x(t) of equation 1 and the narrow bandwidth signals $y_o(t)$ and $q_o(t)$ of equation (2-3).

Equations 2-1 and 2-3 are repeated here for convenience.

$$S_i(t) = r_i x(t)\cos\omega_c t + q_i x(t)\sin\omega_c t \qquad (2\text{-}1)$$

$$S_{oi}(t) = K\{[r_i y_o(t) - q_i z_o(t)]\cos\omega_m t + [q_i y_o(t) + r_i z_o(t)]\sin\omega_m t\} \qquad (2\text{-}3)$$

Rowe[5] shows the technique of computing the response of a bandpass filter to any signal such as (2-3) which can be expresses in the form given by equation 3-1

$$Y(t) = u_1(t)\cos\omega_m t + u_2(t)\sin\omega_m t\,. \qquad (3\text{-}1).$$

In (2-3) and (3-1), ω_m is the center frequency of the measurement filter which is, in general, offset from the carrier frequency (ω_c). If $\omega_m = \omega_c + \Delta\omega$, then the right side of equation 2-1 can be set equal to the right side of equation 3-1 and with the use of the trigonometric identities for the cosine and sine of the sum of 2 angles the following result can be established.

$$r_i x(t)\cos\omega_c t + q_i x(t)\sin\omega_c t = [u_1(t)\cos\Delta\omega t + u_2(t)\sin\Delta\omega t]\cos\omega_c t + [-u_1(t)\sin\Delta\omega t + u_2(t)\cos\Delta\omega t]\sin\omega_c t$$

By setting the coefficients of the $\mathrm{Cos}\omega_c t$ and $\mathrm{Sin}\omega_c t$ terms equal and solving the resultant 2 simultaneous equations the following results

$$u_1(t) = r_i x(t)\cos\Delta\omega t - q_i x(t)\sin\Delta\omega t \qquad (A1)$$

$$u_2(t) = q_i x(t)\cos\Delta\omega t + r_i x(t)\sin\Delta\omega t\,. \qquad (A2)$$

Then equation 2-1 can be expressed in the form

$$S_i(t) = u_1(t)\cos\omega_m t + u_2(t)\sin\omega_m t\,.$$

Now set

$y(t) = x(t)\cos\Delta\omega t$ and

$z(t) = x(t)\sin\Delta\omega t$ then

$$S_i(t) = [r_i y(t) - q_i z(t)]\cos\omega_m t + [q_i y(t) + r_i z(t)]\sin\omega_m t \qquad (A3)$$

Note that the coefficients of the sine and cosine terms in these equations are multiples of x(t) and are thus limited in duration to slightly longer than T, the symbol time.

Rowe[5] treats the mathematics of computing the response of a bandpass filter to signals of the type of equation A3. If the bandpass filter is asymmetrical about ω_m, the solution is complex and involves Hilbert transforms, however, the bandpass filter is usually symmetrical about ω_m and there will be little error in the simpler solution. Thus, symmetry is assumed here.

Rowe calls the transfer function of the bandpass filter H(f) and defines a low pass filter $H_1(f)$ as follows.

[5] H. E. Rowe, Signals and Noise in Communication Systems, D. van Nordstrand Company, 1965, section 3.4.

$$H_1(f) = \begin{cases} \dfrac{H(f+f_c)}{H(f_c)} & 0 < f < f_c \\[2ex] \dfrac{H(f-f_c)}{H^*(f_c)} & -f_c < f < 0 \end{cases}$$

$H_1(f)$ is a low-pass filter with a double-sided bandwidth equal to the bandpass measurement filter bandwidth, or a single sided bandwidth of ½ the measurement filter bandwidth.

Now let

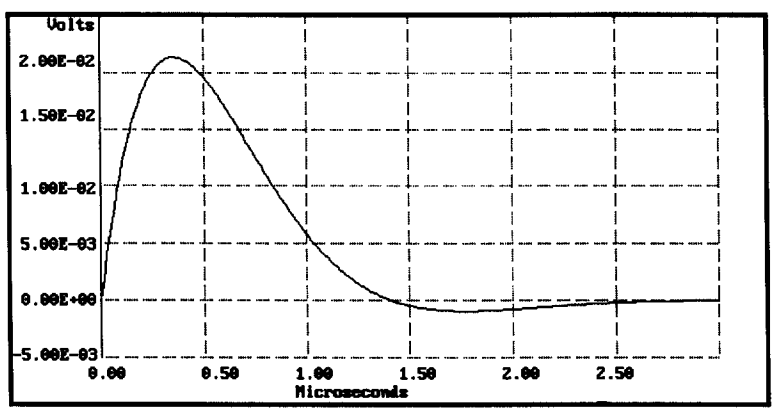
$y_o(t)$ = the response of $H_1(f)$ to $y(t)$ and

$z_o(t)$ = the response of $H_1(f)$ to $z(t)$.

The response of $H_1(f)$ to an input of either $y(t)$ of $z(t)$ is the convolution of the input and the impulse response of $H_1(f)$ which can be called $I(t)$ for convenience. The following graph shows $I(t)$ for a low-pass, 2-pole Butterworth filter of 0.5 MHz bandwidth, which corresponds to a bandpass measurement filter of 1 MHz 3 dB bandwidth.

For a QAM signaling speed of 27 Mb/s, which is achievable with a 20 MHz 26 dB emission bandwidth, the duration of the symbols is slightly higher than 75 ns and the symbol repetition period is 75 ns. Since $I(t)$ is much longer than this, the duration of $y_o(t)$ and $z_o(t)$ is about the same as the duration of $I(t)$.

The ½ amplitude width of the typical impulse response shown below is approximately 750 ns which is 10 symbol times. Thus, at least 10 equivalent symbols contribute to the response defined in equation 3 for a Butterworth measurement filter of 1 MHz bandwidth.



Impulse Response of a 2-Pole Butterworth Low-Pass Filter of 3 dB Bandwidth = 0.5 MHz.

Rowe shows that if $y_o(t)$ and $z_o(t)$ are inserted into the right side of equation A3 for $y(t)$ and $z(t)$ then the result is the output of the measurement filter. That is, the result is given by equation 2-3.

Annex B - Envelope Probability Distribution

The envelope of a narrow bandwidth sample of a wide bandwidth digitally modulated signal behaves approximately as the envelope of narrow band noise. This envelope of a digitally modulated signal was represented in section 5 as

$$v(t) = \sqrt{u_{o1}^2(t) + u_{o2}^2(t)} \tag{5-4}$$

$$p(t) = u_{o1}^2(t) + u_{o2}^2(t) \tag{5-5}$$

where v(t) is the instantaneous envelope voltage and p(t) is the instantaneous envelope power.

With v_m the mean voltage of a narrow band noise envelope, the probability density function is

$$f\left(\frac{v}{v_m}\right) = \frac{\pi}{2}\frac{v}{v_m^2}\varepsilon^{-\frac{\pi}{4}\left(\frac{v}{v_m}\right)^2} \quad v > 0. \tag{B1}$$

The density is 0 for v < 0. This is the Rayleigh density function.

The mean of v/v_m is 1 and the variance about the mean is $(4-\pi)/\pi$.

The square root of the mean power is the rms voltage or v_r. This can be determined as follows: The mean of the squared value of v/v_m is the variance about the mean plus the square of the mean, and the rms voltage is the square root of the variance. Thus

$$Var(x) = \int_{-\infty}^{\infty} x^2 f(x)dx = E(x^2) \text{ and evaluating B1,}$$

$$Var\left(\frac{v}{v_m}\right) = E\left(\frac{v}{v_m}\right)^2 = \frac{4-\pi}{\pi}$$

$$E\left(\frac{v}{v_m}\right)^2 + 1 = \frac{4}{\pi}$$

$$\frac{v_r}{v_m} = \sqrt{\frac{4}{\pi}} = 1.128 \tag{B2}$$

This is 1.049 dB, thus, the rms voltage is 1.049 dB higher than the average envelope voltage. When the envelope is filtered by a low pass filter, the mean of the output tends toward the envelope voltage mean for low filter bandwidth. Thus, the limit displayed on a spectrum analyzer that results from narrow bandwidth video filtering tends to be 1.049 dB less than the mean power level.

The standard deviation of a Rayleigh distribution with a mean of 1 volt is 0.522 volts. The signal B(t) has an autocorrelation time of approximately $1/B_r$. Thus n independent samples will be collected in a time n/B_r. The sample mean of n samples has the following statistical properties

$M_n = 1$

$$\sigma_n = \frac{\sigma_1}{\sqrt{n}} = \frac{0.5227}{\sqrt{n}} \text{ for the envelope voltage.} \tag{B3}$$

Thus, if $E_o(t)$ of section 5 is evaluated by averaging n samples and it is assumed that A(t) =1, then it takes about n/B_r seconds to collect the samples and the variation about the mean is inversely proportional to the number of independent samples collected.

If the video filter has a bandwidth of B_{vid}, then the output can be considered the average of approximately $B_r/2B_{vid}$ samples and the standard deviation of the output is approximately

$$\sigma_v \approx \frac{\sigma_1}{\sqrt{\frac{B_r}{2B_{vid}}}} = 0.74\sqrt{\frac{B_{vid}}{B_r}} \,. \qquad \text{(B4)}$$

The mean is approximately 1.05 dB below the rms.

With p_m the mean power of a narrow band noise envelope, the probability density function is

$$f\left(\frac{p}{p_m}\right) = \varepsilon^{-\left(\frac{p}{p_m}\right)} \qquad \text{(B4)}$$

The density is 0 for $v < 0$.

The mean of p/p_m is 1 and the standard deviation about the mean is 1.

If n samples of envelope power of 1 watt are averaged, the mean power of the samples will be 1 watt and the standard deviation will be

$\sigma_n = \frac{1}{\sqrt{n}}$ for the envelope power.

Annex C - The Effective Noise Power of a Non-Uniform Signal Envelope

Section 5 discusses the effect of data signal envelope variations that are slow relative to the symbol rate, but shorter than a burst time. This annex evaluates the approximate effect of such power level variations. These variations are equivalent to a mean power increase in interference level to narrower bandwidth signals. An approximate means of estimating the equivalent increase is developed.

Equation 5-3 is repeated below for convenience

$$E_o(t) = A_o(t)\sqrt{u_{o1}^2(t) + u_{o2}^2(t)}\,. \qquad (5\text{-}3)$$

Further, for convenience let

$y(t) = A^2_o(t)$,

T_b be a burst duration,

$p(t) = E_o^2(t) = y(t)\left[u_{o1}^2(t) + u_{o2}^2(t)\right]$ be the instantaneous power of 5-3 and

the mean value of $\sqrt{u_{o1}^2(t) + u_{o2}^2(t)} = 1$.

Then the probability density of p at time t is

$$f(p,t) = \frac{1}{y(t)}\varepsilon^{\frac{-p}{y(t)}}$$ since y(t) is the mean power at time t.

The composite probability density is thus

$$f(p) = \int_0^{T_b} \frac{1}{y(t)}\varepsilon^{\frac{-p}{y(t)}}dt\,.$$

This leads to a cumulative probability distribution function of

$$F(p,t) = \Pr\{P < p|y(t)\} = \int_0^p \frac{1}{y(t)}\varepsilon^{\frac{-z}{y(t)}}dz = 1 - \varepsilon^{\frac{-p}{y(t)}}$$

where P is a random sample of the power P, and

$$F(p) = \Pr\{P < p\} = 1 - \int_0^{T_b} \varepsilon^{\frac{-p}{y(t)}}dt\,. \qquad (C1)$$

If $A_o^2(t)$ is constant at *a* watts for an interval t_1 then at b watts for the remainder of the burst interval $(T_b\text{-}t_1)$, then F(p) has a simple closed form. Call this $F_{a,b}(p)$ and it can be shown using C1

$$F_{a,b}(p) = 1 - \frac{t_1}{T_b}\varepsilon^{\frac{-p}{a}} - \frac{T_b - t_1}{T_b}\varepsilon^{\frac{-p}{b}} \qquad (C2)$$

$A_o^2(t) = a$ for $0 < t < t1$

$A_o^2(t) = b$ for $t1 < t < T_b$

If $E_o(t)$ has 1 watt average power over a burst then the coefficient b must be

$$b(a,t_1) = b_a = \frac{1 - a\frac{t_1}{T_b}}{1 - \frac{t_1}{T_b}} \qquad \text{(C3)}$$

Thus if a is large, b must be small to compensate and keep the mean burst power the same as it would be for a = b = 1.

Note: If the resolution bandwidth through which the signal is measured is 1 MHz, then the minimum value of t_1 for which this distribution (C2) is valid is about 1 μs. Caution should be observed when applying the results to shorter time intervals.

The probability of an independent sample of power exceeding R times the average power is $1-F_{a,b}(R)$. The value of $1 - F_{1,1}(R) = \varepsilon^{-R}$. Thus

$$Ln\left[1 - F_{1,1}(R)\right] = -R$$

where Ln means the natural logarithm.

If $F_{a,b_a}(R_2)$ is set equal to $F_{1,1}(R)$ then the probability of an independent sample of the power waveform with non-uniform envelope exceeding R_2 is the same as that of the waveform of uniform envelope power exceeding R when each signal has the same average power. R_2 will be less than R, thus a non-uniform signal with power $10\text{Log}(R/R_2)$ less than that of a waveform with a uniform envelope would have the same probability of exceeding a particular high value. In this sense, the effective power of the non-uniform envelope signal is $20 \text{ Log } (R/R_2)$ dB higher than its average power.

When $F_{a,b_a}(R_2)$ is set equal to $F_{1,1}(R)$, the following results

$$Ln[1 - F(R)] = Ln\left[1 - F_{a,b_a}(R_2)\right] = -R$$

$$R = -Ln\left(\frac{t_1}{T_b}\varepsilon^{\frac{-R_2}{a}} + \frac{T_b - t_1}{T_b}\varepsilon^{\frac{-R_2}{b}}\right)$$

and, as stated above, the effective interference power of the non-uniform envelope signal is $10\text{Log}(R/R_2)$ higher than that of a uniform envelope signal of the same average power. In other words, if the non-uniform envelope waveform power is reduced by $10\text{Log}(R/R_2)$ this makes the incidence of high power levels the same as for the uniform envelope waveform at some low incidence rate, that rate being ε^{-R} per independent sample.

The table below shows some computations of the effective power increase for various values of the peak power using the above formula.

ATTACHMENT B - WIDEBAND DIGITALLY MODULATED SIGNALS

	High to Low Power Ratio (dB)	Fraction of Burst at High Power	Peak Power Relative to Mean (dB)	Low Power Relative to Mean (dB)	Equivalent Interference Increase (dB)
1	5	0.001	4.99	-.01	0.001
2	30	0.001	26.99	-3.01	1.14
3	50	0.001	29.96	-20.04	1.14
4	5	0.01	4.91	-.09	0.88
5	30	0.01	19.59	-10.4	2.82
6	50	0.01	20.0	-30.0	2.83
7	5	0.10	4.15	-.85	1.94
8	30	0.10	9.96	-20.0	4.48
9	*50*	*0.10*	*10*	*-40*	*4.49*
10	50	0.10	10	-40	4.487
11	50	0.11	9.59	-40.4	4.494
12	50	0.12	9.21	-40.8	4.490
13	50	0.13	8.86	-41.1	4.476
14	50	0.14	8.54	-41.5	4.455
15	*10*	*0.10*	*7.2*	*-2.8*	*3.51*
16	10	0.20	5.5	-4.5	3.38
17	10	0.30	4.3	-5.7	2.98
18	10	0.40	3.4	-6.6	2.50
19	10	0.50	2.6	-7.4	2.03

Table C1. Equivalent Interference Increase Due to Intermittent High Emission Levels
$R = 9, \quad \varepsilon^{-R} = 1.23 \times 10^{-4}$

The table applies to a non-uniform power envelope with the same average power as a uniform envelope signal.

The first column of the table (20Log(a/b) gives the ratio of the high power to the low power. The third column of the table (20 Log a) shows the actual increase of the envelope power above the mean which occurs for a fraction of the burst duration of column 2. The last column (20 Log k) gives the effective mean noise power increase due to the short high power segment of the burst.

It will be assumed that the U-NII emissions are specified in terms of the power spectral density of the transmissions measured while the power is on. This quantity is defined in terms of the long term mean power. In addition to controlling PSD, it is necessary to control the local peak power measured over intervals of a few data symbols of the wideband waveform. The currently proposed time interval is 30/B

where B is the 26 dB emission bandwidth of the U-NII devices. This corresponds to about 20 symbol intervals. Since the U-NII bandwidth may be as high as 20 MHz, the shortest 30/B interval is about 1.5 μs.

The objective of the specifications and measurement technique is to insure that short term power peaks either do not increase the potential interference possibility in potential "victim" receivers, or that a conformance test technique is available to detect their presence.

Table C1 shows that if a device transmits a high power level for a short period t_1, then virtually turns off for a longer period T_b, a burst power increase above the average of about $10\text{Log}(T_b/t_1)$ is achievable. This is shown by the upper rows of the table (rows 3, 6 and 9). These rows show the equivalent noise effect for various power burst levels. For example

Power burst = mean +	Maximum Equivalent Noise increase =
30 dB	1.1 dB (third row)
20 dB	2.8 dB (6th row)
10 dB	4.5 dB (11th row).

In these instances, the high short term power is achieved by an extremely large high to low power ratio. In these cases, it is obvious that the peaking effect can be detected in the signal out of a resolution filter of bandwidth B_r if t_1 is on the order of $1/B_r$ or greater. The response to such a short peak can be expected to fully register at the output, and the output level will be relatively low during the remainder of the burst. Thus, if the wideband peak power to low power ratio is high (for instance rows 2, 5 and 8 of table C1) the contrast will be detectable in a 1 MHz bandwidth waveform. The condition that is hardest to detect in conformance testing will be when the high power of the segment is relatively low compared to the mean.

Note the lower italicized row of table C1(row 15). In this case, the high to low power ratio is 10 dB and the effective noise increase due to peaking is about 3.5 dB. This can be taken as an example of a condition that is most difficult to detect in a compliance test. Consider a 1 MHz bandwidth resolution filter and a 1 MHz bandwidth potential victim receiver. A high power duration of 1.5μs in the wideband signal will have a duration at full amplitude that is somewhat shorter than 1.5μs in the 1 MHz waveform and will have slightly less than the recorded 3.5 dB effect. If the envelope voltage of this signal is measured through a video filter of about 167 kHz bandwidth, the mean output of the filter during the high power segment of the burst will fully register and will be 10 dB higher than the mean of the low power segment. The standard deviation of the output during each segment will be about 0.23 times the mean by equation B4 of annex B. Thus, intuitively it can be seen that the bursty condition of row 15 can be detected with a high level of assurance on a single burst as the envelope voltage during the high power segment will be considerably higher than that during the low power segment.

Appropriate analysis of multiple bursts should raise the assurance to an arbitrarily high level. Also, the conclusions hold for shorter bursts if multiple bursts are evaluated. Further analysis would be needed to establish the minimum burst duration.

Table C1 applies to a case where the occurrence incidence of the noise is about 1 in 10^4 independent samples. The effective power level increase is higher if the occurrence rate is lowered. For example if the incidence of occurrence is one per 10^6, the highest effective interference increase is 5.8 dB with the power at the high level about 7% of the time during a burst. The power level of the high portion of the burst is about 11.5 dB above the average in this case, so the same conclusions hold.

The resolution bandwidth for measuring the burstiness effect should be higher than 1 MHz if the spectrum analyzer has the capability. For example, the above conclusions considering a 1.5 μs high level duration would hold for 0.75 μs if the resolution bandwidth of the measurement is 2 MHz.