Appendix 4. Some Information on Parabolic Antennas

$$\Theta \approx \frac{65\lambda}{D}$$

$$G_D \approx \frac{2.00x10^4}{\Theta^2}$$ and solving for D,

$$D \approx \frac{1}{42.1}\sqrt{G_D}$$

Θ = Beamidth
λ = Wavelength = .0517 meters at 5.8 GHz
D = Antenna Diameter
G_D = The antenna gain ratio.

D (meters, inches)	G_D	$10Log_{10}\ G_D$	note
0.25, 9.8	110.8	20.4 dB	1/4 meter
0.305, 12	164.6	22.2 dB	1 foot
0.50, 19.7	443	26.5 dB	half meter
6 feet	3259	35.1 dB	

This shows that gains in excess of 20 dBi are achievable with very small parabolic antennas. Table 5 indicates that the average gain is negative for parabolic antennas of about 10 inch diameter of more.

ATTACHMENT B

AVERAGE ANTENNA GAIN OF PART 15 DEVICES AS SEEN BY A LOW EARTH ORBIT SATELLITE

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Lucent Technologies/Bell Labs
December 4, 1996

ABSTRACT

This paper addresses the issue of average antenna gain for the NII/SUPERNet devices, as seen by a satellite associated with the Mobile Satellite Service (MSS). This analysis is intended to answer the question of whether the NII/SUPERNet power output limit should be specified as a maximum EIRP, or as a maximum RF power level into the antenna terminals. MSS interests have raised the concern that the latter would allow high EIRP, and that if high-gain antennas are systematically directed horizontally (as would be expected), the EIRP as seen by a satellite at a low elevation angle could cause harmful interference to the MSS forward link.

The results show that such a concern is unjustified. In fact, the opposite is true. The average gain seen by the satellite is actually lower for high gain antennas. Three NII/SUPERNet antenna types are compared: (1) the antenna pattern introduced by AirTouch in its Reply Comments, with a range of beamwidths; (2) a parabolic dish with various diameters; and (3) a half-wave dipole. It was found that for types (1) and (2), the higher the maximum gain, the lower the average gain. For antennas with any significant directivity, the average gain was found to be less than 0 dBi. Of all the antennas studied, the half-wave dipole exhibited the highest average gain (about 0.8 dBi). This is because the high-gain antennas direct most of their energy below the 10° minimum elevation angle of the satellite.

It is concluded that it is transmit power, not EIRP, that should be regulated in the Commission's Rules, since it is the maximum transmit power that determines the potential for interference to the MSS forward link. With a transmit power limit, the interference to MSS is likely to be *less* than with an EIRP limit, because a transmit power requirement would provide some incentive for designers to use directive antennas, for which the average power to the satellite would be less than for a non-directive antenna with the same input power.

INTRODUCTION

The band 5150-5250 MHz is allocated to feeder uplinks in the Mobile Satellite Service (MSS). The FCC has proposed to also allow unlicensed operation of low-power "NII/SUPERNet" devices in that band under Part 15. A concern has been raised about the potential for interference from the Part 15 devices to the satellites. To accurately assess the restrictions that need to be applied to the Part 15 devices to avoid such interference, the impact of the Part 15 antenna pattern on the power received by the satellite needs to be understood. The basic question is whether power limit for Part 15 devices should be expressed in terms of total RF power output (i.e., into the antenna

terminals), or effective isotropic radiated power (EIRP), which limits the product of the power output and the maximum antenna gain.

The purpose of this paper is to address the question of whether the total power from the Part 15 devices as seen by the satellite depends on the power output or the EIRP.

GEOMETRY AND NOTATION

Figure 1 shows the geometry of the situation.

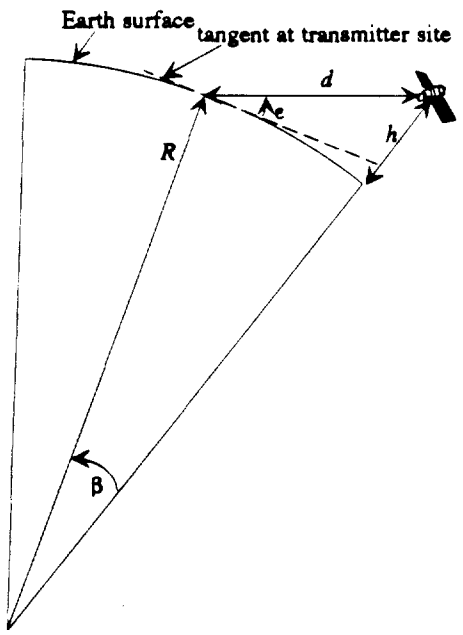

Figure 1

R is the effective radius of the Earth, allowing for atmospheric diffraction. If r is the actual radius (3963 mi), then $R = Kr$, where $K = 4/3$ represents normal diffraction, and will be used in the calculations in this paper. The satellite is h miles above the surface; in this case, $h = 879$ mi. The elevation angle of the satellite as seen by a device on the surface is ε, and the line-of-sight distance from the device to the satellite is d. The angle subtended by the surface arc between the device and the point on the surface directly below the satellite is β.

Figure 2 shows the coordinates used in the analysis. The antenna boresight (direction of maximum gain) is oriented parallel to the surface. The orientation of the satellite relative to the boresight is described by the elevation angle ε and the azimuth angle α. For an antenna with a pattern that is a figure-of-revolution (such as the parabolic dish shown in Fig. 2), the gain is a function of ϕ, which is the polar angle between the boresight and the satellite, with $\cos\phi = \cos\alpha \cdot \cos\varepsilon$. For completeness, the angle θ represents revolution about the boresight.

ANTENNA GAIN PATTERN

Consider the *radiation intensity* $U(\alpha,\varepsilon)$ in watts/steradian emanating from an antenna at the center of a sphere. Clearly, integrating $U(\alpha,\varepsilon)$ over the surface of the sphere (4π steradians) should yield the total radiated power P (the power into the antenna terminals, minus losses):

$$\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} U(\alpha,\varepsilon)\cos\varepsilon \, d\varepsilon \, d\alpha = P \ . \qquad (1)$$

Therefore, for an isotropic antenna, $U_i(\alpha,\varepsilon) = U_i = P/4\pi$.

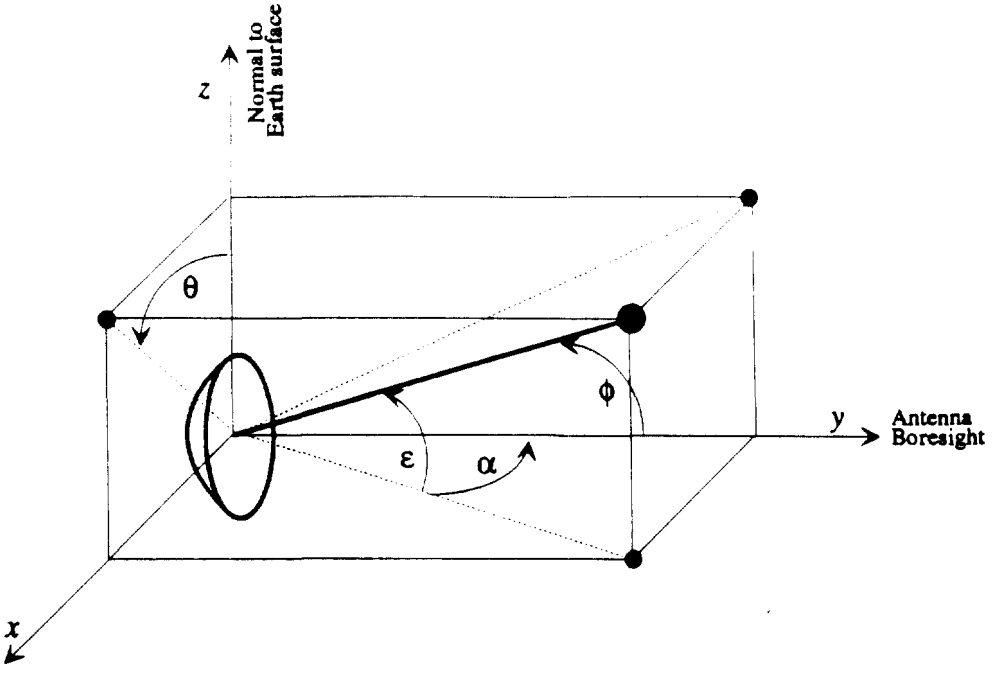

Figure 2: *Spherical coordinates*

The antenna gain pattern $G(\alpha,\varepsilon)$ gives the radiation intensity in the (α,ε) direction relative to that which would result from an isotropic antenna with the same total radiated power. Thus, $G(\alpha,\varepsilon) = U(\alpha,\varepsilon)/U_i = 4\pi U(\alpha,\varepsilon)/P$, and

$$\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} G(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon\, d\alpha = 4\pi . \tag{2}$$

The *effective power* radiated in the (α,ε) direction, relative to that from an isotropic antenna, is $P\cdot G(\alpha,\varepsilon)$. The maximum value of $G(\alpha,\varepsilon)$ is the "gain" or "directivity" of the antenna. The EIRP is normally taken to mean $P\cdot G(\alpha,\varepsilon)_{\max}$.

Of interest here is the average antenna gain, as seen by the satellite, of a terrestial transmitter with some anisotropic antenna pattern. The azimuth angle of the transmitter is assumed to be uniformly-distributed between $-\pi$ and π radians. The elevation angle ε can vary between some minimum ε_0 and $\pi/2$. For the case of interest here, $\varepsilon_0 = 10°$ ($\pi/18$ radians). However, the distribution is not uniform, because some elevation angles are more likely than others. Let $f_\varepsilon(x)$, $\varepsilon_0 \le x \le \pi/2$, be the probability density function (pdf) of the elevation angle, with

$$\int_{\varepsilon_0}^{\pi/2} f_\varepsilon(x)dx = 1 \tag{3}$$

The average gain as seen by the satellite is:

$$G_{av} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{\varepsilon_0}^{\pi/2} G(\alpha,\varepsilon) f_\varepsilon(\varepsilon)d\varepsilon d\alpha \tag{4}$$

Note that for an isotropic antenna, $G_{av} = 1$.

THE AIRTOUCH ANALYSIS

In its Reply Comments in ET Docket 96-102, AirTouch attempted to compute the average gain for an antenna with a gain pattern:

$$G_{AT}(\alpha,\varepsilon) = \frac{27000}{B_\alpha B_\varepsilon} 10^{-\frac{1}{2}\left[\left(\frac{\varepsilon}{B_\varepsilon}\right)^2 + \left(\frac{\alpha}{B_\alpha}\right)^2\right]} + 1 \tag{5}$$

It is clear from inspection that this is not a valid antenna pattern, because it violates (2). As shown in Appendix A,

$$\begin{aligned}\frac{1}{4\pi}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} G_{AT}(\alpha,\varepsilon)\cos\varepsilon\, d\alpha\, d\varepsilon &\cong 1 + \frac{27000(\pi/180)^2}{2\ln 10} e^{-B_\varepsilon^2/2\ln 10} \\ &= 1 + 1.786 e^{-B_\varepsilon^2/2\ln 10}\end{aligned} \tag{6}$$

This has been verified with numerical integration. In dB, (6) agrees exactly with the numerical results to three decimal places for $B_\varepsilon \le 40°$, and the error is less than 0.1 dB up to 75°. B_α has little effect as long as it is less than 90°. As can be seen from (6), the excess gain ranges from about 4.4 dB for small B_ε down to about 3.8 dB for $B_\varepsilon = 60°$. The AirTouch antenna gain formula must be divided by (6) so that (2) is satisfied.

AirTouch uses (4) to evaluate the average gain, with $f_\varepsilon(\varepsilon) = \cos\varepsilon/(1-\sin\varepsilon)$, which is a valid pdf (but does not represent a uniform distribution of transmitters over the Earth's surface, as shown below). The AirTouch average gain formula is:

$$G_{ATav} = \frac{1}{2\pi(1-\sin\varepsilon)}\int_{-\pi}^{\pi}\int_{\varepsilon_0}^{\pi/2} G(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon d\alpha \tag{7}$$

Using this formula, the average gain for any valid antenna pattern with elevation gain that is maximum at $\varepsilon = 0$, is symmetric about $\varepsilon = 0$, and decreases monotonically for $|\varepsilon| > 0$, will always be less than 0 dB (which is the result for an isotropic antenna). This can be seen by observing that the total power radiated through the portion of the upper hemisphere for which $\varepsilon \geq 0$ is:

$$P(\varepsilon) = \frac{P}{4\pi} \int_{-\pi}^{\pi} \int_{\varepsilon_0}^{\pi/2} G(\alpha, \varepsilon) \cos\varepsilon \, d\varepsilon d\alpha \ . \tag{8}$$

With the gain antenna, more power is directed at the elevation range excluded by the integral ($\varepsilon < \varepsilon_0$) than with the isotropic antenna, so there is less total power directed toward the upper portion of the hemisphere ($\varepsilon \geq \varepsilon_0$). The integral in (7) is therefore less than in the isotropic case for antennas with horizonal gain.

Evaluation of (7) using the AirTouch antenna pattern of (5), corrected to satisfy (2), bears this out, giving the results shown in Table 1.

Table 1: Corrected AirTouch Results

B_ε(degrees)	G_{av}(dBi)
10	-3.39
20	-1.55
30	-0.85
40	-0.53
50	-0.35
60	-0.24
70	-0.18

THE PDF OF THE ELEVATION ANGLE

The portion of the Earth's surface seen by the satellite subtends a polar angle β_{max} (see Fig. 1), where the polar axis extends from the satellite to the center of the Earth. An area element can be expressed as $dA = R d\beta \cdot R \sin\beta \, d\alpha$. Assuming that terrestrial transmitters are uniformly-distributed over area, the probability that a given transmitter is located within an angular increment $d\beta$ at an angle β from the axis is proportional to $\sin\beta$. Applying the requirement that the pdf must integrate to 1 gives:

$$f_\beta(\beta) = \frac{\sin\beta}{1 - \cos\beta_{max}}, \quad 0 \leq \beta \leq \beta_{max} \ . \tag{9}$$

The elevation angle ε can be related to β using Figure 3. The law of cosines gives:

$$d = \sqrt{R^2 \sin^2 \varepsilon + 2Rh + h^2} - R \sin \varepsilon . \tag{10}$$

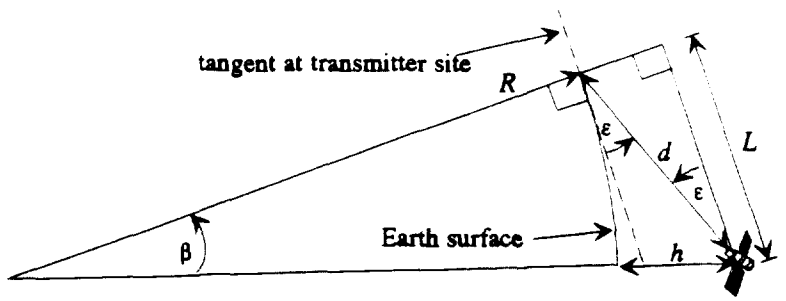

Figure 3: *Geometry for relating θ to ε*

From Fig. 3, $L = (R+h)\sin\beta = d\cos\varepsilon$. Hence, $\sin\beta = d\cos\varepsilon/(R+h)$, which with (10) gives β as a function of ε. Defining $u \equiv \sin\beta$, transforming (9) gives the pdf of u as:

$$f_u(u) = \frac{u}{(1-\cos\beta_{\max})\sqrt{1-u^2}}, \quad 0 \le u \le \sin\beta_{\max} . \tag{11}$$

The pdf of ε is then $f_\varepsilon(\varepsilon) = -f_u\left(\frac{d\cos\varepsilon}{R+h}\right)\frac{du}{d\varepsilon}$, with

$$\frac{du}{d\varepsilon} = -\frac{d}{R+h}\left[\frac{R\cos^2\varepsilon}{d+R\sin\varepsilon} + \sin\varepsilon\right]. \tag{12}$$

Figure 4 shows $f_\varepsilon(\varepsilon)$ and $\cos\varepsilon/(1-\sin\varepsilon_0)$ for $\varepsilon_0 = 10°$. Notice that $f_\varepsilon(\varepsilon)$ weights low elevation angles much more heavily than does $\cos\varepsilon/(1-\sin\varepsilon_0)$, so it can be expected to yield higher average gain for antennas with elevation gain.

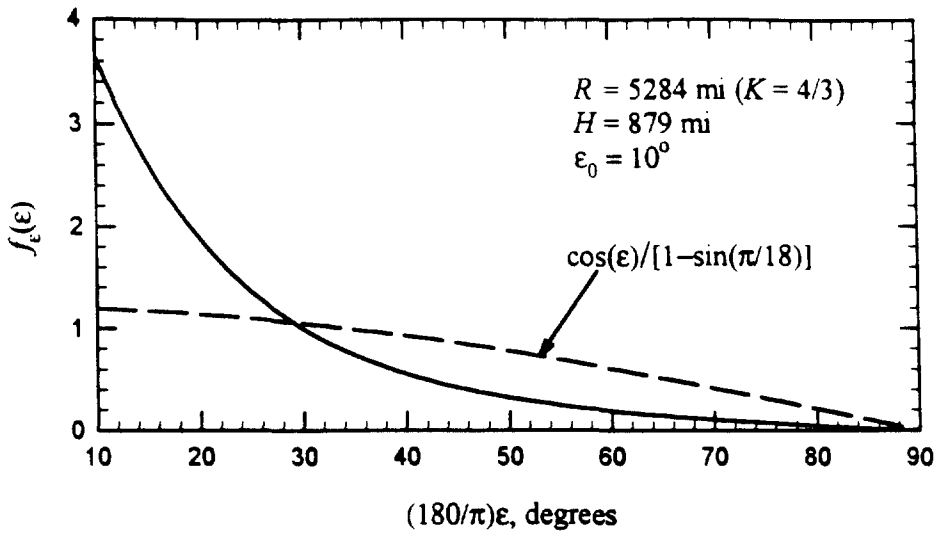

Figure 4: *Probability density function of the elevation angle, assuming terrestrial transmitters are uniformly-distributed over area.*

AVERAGE GAIN RESULTS FOR SEVERAL ANTENNAS

In this section, the average gain will be computed, using the "uniform density" pdf for the elevation angle developed above, for three different antenna types: (1) the adjusted "AirTouch" antenna discussed above; (2) parabolic dish antennas of various diameters; and (3) a half-wave dipole.

The AirTouch Antenna

Table 2 shows the average gain for the AirTouch antenna, computed using $f_\varepsilon(\varepsilon)$ instead of $\cos\varepsilon/(1-\sin\varepsilon_0)$. Note that in this case, there is an increase in average gain (albeit very small) over the isotropic case, due to the heavier weighting of lower elevation angles. The maximum is about 0.6 dB, occurring at elevation beamwidths of 30 to 40 degrees.

Table 2: *Average gain for the AirTouch antenna pattern using the "uniform density" probability density function for the elevation angle.*

B_ϵ (degrees)	G_{av}(dBi)
5.000	-4.381
10.000	-2.115
15.000	-0.433
20.000	0.253
25.000	0.519
30.000	0.604
35.000	0.606
40.000	0.571
45.000	0.521
50.000	0.467
55.000	0.415
60.000	0.368
65.000	0.325
70.000	0.288
75.000	0.255

Parabolic Dish Antennas

The circular parabolic reflector is an important antenna to consider, because near 5 GHz, considerable gain can be achieved with a small antenna. It is reasonable to expect that for NII/SUPERNet applications requiring antenna gain, the parabolic dish will often be the antenna of choice.

If ϕ is the polar angle between the field point and the antenna boresight (see Fig. 2), the normalized field pattern is:[1]

$$E(\phi) = \frac{2}{\pi D_\lambda} \frac{J_1(\pi D_\lambda \sin\phi)}{\sin\phi}, \tag{13}$$

where D_λ is the diameter of the aperture in wavelengths and $J_1(\cdot)$ is the first-order Bessel function of the first kind. Note that $E(\phi)_{max} = E(0) = 1$,[2] so the power gain pattern can be written as:

[1] John D. Krauss, *Antennas*, second edition, McGraw-Hill, 1988, section 12.7.

[2] $\lim_{x \to 0} J_1(x) = x/2$

$$G(\phi) = G_0 E^2(\phi) = G_0 \left[\frac{2}{\pi D_\lambda} \frac{J_1(\pi D_\lambda \sin\phi)}{\sin\phi} \right]^2 \tag{14}$$

Figure 5 shows power gain patterns for several values of D_λ = 2, 5, and 10.

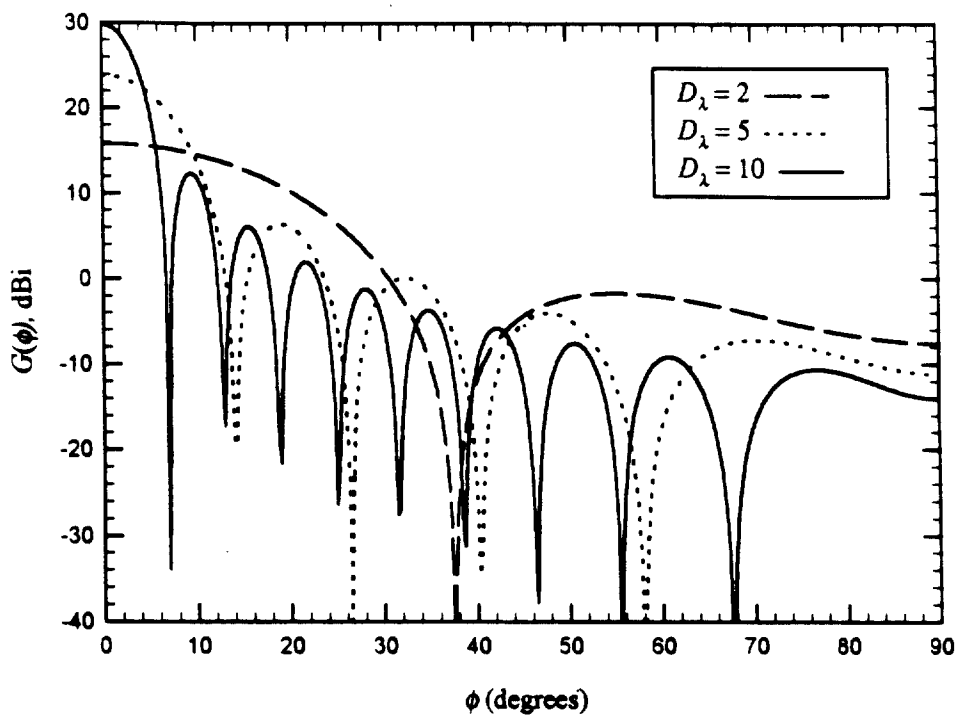

Figure 5: *Gain pattern for parabolic dish antenna; D_λ is the diameter in wavelengths.*

The average gain can be computed using (4) with (see Fig. 2) $\phi = \cos^{-1}(\cos\varepsilon \cdot \cos\alpha)$. Assuming that all of the power is directed in the hemisphere $-\pi/2 \le \alpha \le \pi/2$, the average gain can be calculated as:

$$G_{av} = \frac{1}{\pi} \int_0^{\pi/2} \int_{\varepsilon_0}^{\pi/2} G\left[\cos^{-1}(\cos\alpha \cos\varepsilon)\right] f_\varepsilon(\varepsilon) d\varepsilon d\alpha \ . \tag{15}$$

Table 3 shows the average gain and other parameters of interest for parabolic dish antennas with D_λ ranging from 1 to 10. The beamwidths are in degrees. The maximum gain was computed using:

$$G_0 = \frac{\pi}{\int_0^{\pi/2} \int_{\varepsilon_0}^{\pi/2} E^2\left[\cos^{-1}(\cos\alpha \cos\varepsilon)\right] \cos\varepsilon \, d\varepsilon d\alpha}, \tag{16}$$

and agrees well with the theoretical directive gain for a large, uniformly-illuminated aperture:[3] $G = 9.87 D_\lambda^2$; note that agreement is closer as the aperture size increases. As with the AirTouch antenna formula in (5), the average gain decreases as the maximum gain increases, since more power is directed below the minimum elevation angle seen by the satellite.

Table 3: *Beamwidths, maximum gain, and average gain for parabolic dish antennas using the "uniform density" pdf for the elevation angle.*

D_λ	half-power beamwidth	first null beamwidth	maximum gain (dBi)	average gain (dBi)
1.000	58.000	140.000	9.659	.747
2.000	29.000	70.000	15.858	.097
3.000	19.333	46.667	19.427	-1.836
4.000	14.500	35.000	21.946	-4.123
5.000	11.600	28.000	23.895	-5.806
6.000	9.667	23.333	25.485	-6.397
7.000	8.286	20.000	26.828	-6.542
8.000	7.250	17.500	27.991	-6.898
9.000	6.444	15.556	29.016	-7.577
10.000	5.800	14.000	29.933	-8.343

The Half-Wave Dipole

For the coordinate system used here, the pattern of a vertical (normal to the Earth's surface) half-wave dipole can be written as:

$$G_{\lambda/2}(\varepsilon) = G_0 \left[\frac{\cos\left(\frac{\pi}{2}\sin\varepsilon\right)}{\cos\varepsilon} \right]^2 \tag{17}$$

The pattern is omnidirectional in azimuth; hence:

$$\int_0^{\pi/2} G_{\lambda/2}(\varepsilon)\cos\varepsilon \, d\varepsilon = 1, \tag{18}$$

[3] Krauss, *op. cit.*

which allows G_0 to be computed numerically and compared to the well-known value of 1.64 (2.15 dBi) as a sanity check on the numerical computations. Table 4 shows the average gain computed using (4), as well as using the AirTouch formula (7).

Table 4: *Average gain for a half-wave dipole, using the "uniform density" pdf for the elevation angle, and also using the AirTouch formula*

minimum elevation angle (deg.)	average gain (uniforum density), dBi	average gain (AirTouch formula), dBi
0	1.448	0.000
2	1.345	-0.102
4	1.226	-0.213
6	1.091	-0.333
8	0.941	-0.463
10	**0.776**	**-0.603**
12	0.596	-0.753
14	0.401	-0.914
16	0.191	-1.085
18	-0.034	-1.267
20	-0.272	-1.461

As can be seen, for a minimum elevation angle greater than zero, the AirTouch average gain formula yields a result less than 0 dBi, because the dipole has gain in the elevation dimension. The computing the average gain using the "uniform density" pdf for ε gives a larger average gain (as would be expected), but with a 10° minimum elevation angle, the average is less than 0.8 dBi. However, this is larger than the average gain for either the parabolic dish or the AirTouch antenna pattern under any of the conditions investigated here.

CONCLUSIONS

It is clear from the derivations and examples given here that, consistent with intuition, the potential for interference from unlicensed devices to the MSS feeder uplink at 5150-5250 MHz will be determined not by the average EIRP of the unlicensed devices, but rather by the average transmit power. The maximum average gain observed in the calculations given here was for the half-wave dipole (less than 0.8 dBi), which has a maximum gain of about 2.15 dBi. The average gain of "high-gain" antennas (such as a parabolic dish) is less than 0 dBi, because much of the power is directed below the satellite's field of vision. For the average gain computations discussed here, all antennas were assumed to be of the same type. In reality, there will be a mix of antenna types. If these include high-gain antennas, then the results given here suggest that the average gain over all antennas will be less than 0 dBi.

It is recommended that the power output limit for unlicensed devices operating in the 5150-5250 MHz band be stated as a limit on power into the antenna terminals, rather than as an EIRP limit. This approach would obviously benefit the unlicensed devices, allowing design flexibility and simplifying compliance measurements. However, it would benefit the MSS feeder uplinks as well. A transmit power limit would encourage the designers of the unlicensed devices to use directional antennas when it would benefit the applications. As shown here, the average gain (seen by the satellite) for directional antennas is actually less than that for antennas that are more nearly isotropic. Thus, a transmitter output power limit, rather than an EIRP limit, may actually *reduce* interference to MSS feeder uplinks.

APPENDIX

EXCESS GAIN OF THE AIRTOUCH ANTENNA PATTERN

The AirTouch antenna pattern is:

$$G_{AT}(\alpha,\varepsilon)=\frac{27000}{B_\alpha B_\varepsilon}10^{-\frac{1}{2}\left[\left(\frac{\varepsilon}{B_\varepsilon}\right)^2+\left(\frac{\alpha}{B_\alpha}\right)^2\right]}+1, \tag{A-1}$$

which can be written as:

$$G_{AT}(\alpha,\varepsilon)=27000\frac{e^{-K_\alpha^2\alpha^2}}{B_\alpha}\cdot\frac{e^{-K_\varepsilon^2\varepsilon^2}}{B_\varepsilon}+1, \tag{A-2}$$

where $K_\alpha=\frac{\sqrt{\ln 10/2}}{B_\alpha}$ and $K_\varepsilon=\frac{\sqrt{\ln 10/2}}{B_\varepsilon}$.

The conservation-of-energy constraint on the gain pattern is:

$$\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2}G(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon d\alpha=4\pi \tag{A-3}$$

Since the AirTouch gain pattern is symmetric about $\alpha=0$ and about $\varepsilon=0$, the excess gain can be expressed as:

$$G_{excess}=\frac{1}{\pi}\int_0^{\pi}\int_0^{\pi/2}G_{AT}(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon d\alpha=1+\frac{27000}{\pi}\left(\frac{\pi}{180}\right)^2 I_\alpha I_\varepsilon, \tag{A-4}$$

where the factor $(\pi/180)^2$ accounts for the conversion from degrees to radians, and:

$$I_\alpha=\frac{1}{B_\alpha}\int_0^{\pi}e^{-K_\alpha^2\alpha^2}d\alpha \tag{A-5a}$$

$$I_\varepsilon=\frac{1}{B_\varepsilon}\int_0^{\pi/2}e^{-K_\varepsilon^2\varepsilon^2}\cos\varepsilon\, d\varepsilon \tag{A-5b}$$

Using the substitution $x = K_\alpha \alpha$, (A-5a) becomes:

$$I_\alpha = \sqrt{\frac{2}{\ln 10}} \int_0^{\pi K_\alpha} e^{-x^2} dx = \sqrt{\frac{\pi}{2\ln 10}} \cdot \mathrm{erf}\left(\frac{\pi}{B_\alpha} \sqrt{\frac{\ln 10}{2}} \right), \tag{A-6}$$

where $\mathrm{erf}(z) \equiv \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$ is the error function. Unless B_α is large (i.e., $\geq \pi/2$), $\mathrm{erf}\left(\pi / B_\alpha \sqrt{\ln 10/2}\right) \approx 1$, and

$$I_\alpha \approx \sqrt{\frac{\pi}{2\ln 10}} . \tag{A-7}$$

Similarly, substituting $x = K_\varepsilon \varepsilon$ in (A-5b) gives:

$$I_\varepsilon = \sqrt{\frac{2}{\ln 10}} \int_0^{\pi K_\varepsilon/2} e^{-x^2} \cos\left(x/K_\varepsilon\right) d\varepsilon \tag{A-8}$$

Using $\int_0^\infty e^{-x^2} \cos ax \, dx = \frac{\sqrt{\pi}}{2} e^{-a^2/4}$, and assuming that $B_\varepsilon < \pi/2$, I_ε can be approximated as:

$$I_\varepsilon \approx \sqrt{\frac{\pi}{2\ln 10}} e^{-B_\varepsilon^2/2\ln 10} \tag{A-9}$$

Combining (A-4), (A-7), and (A-9), the excess gain can be approximated as:

$$G_{excess} \approx 1 + \left(\frac{\pi}{180}\right)^2 \frac{27000}{2\ln 10} e^{-B_\varepsilon^2/2\ln 10} = 1 + 1.786 e^{-B_\varepsilon^2/2\ln 10} , \tag{A-10}$$

where B_ε is in radians. The approximation of (A-10) was checked against numerical results, and found to be accurate to within less than 0.1 dB for $B_\varepsilon \leq 75°$ and $B_\alpha \leq 120°$. For $B_\varepsilon \leq 50°$ and $B_\alpha \leq 80°$, it agrees with the numerical results to within 0.01 dB. The largest error observed for any combination of B_ε and B_α was less than 0.4 dB for $0 < B_\alpha \leq 180°$ and $0 < B_\varepsilon \leq 90°$.

ATTACHMENT C

ATTACHMENT 1

EFFECT OF NII/SUPERNET DEVICE DEPLOYMENT ON GLOBALSTAR™ CAPACITY

Introduction and Abstract

In ET Docket 96-102, the Federal Communications Commission has proposed to allow the operation of unlicensed "NII/SUPERNet" devices in the bands 5150-5350 MHz and 5725-5875 MHz, under Part 15 of its Rules. The band 5150-5250 MHz is also used for the feeder uplink in the Mobile Satellite Service (MSS). The feeder uplink from the terrestrial gateway plus the associated downlink to the subscriber unit (i.e., a portable handset) constitute the forward link. Concerns have been raised by MSS interests about the potential for interference from the NII/SUPERNet devices to the MSS forward link. Specifically, AirTouch Communications has analyzed the capacity reduction to the Globalstar system that might occur due to transmissions from the NII/SUPERNet devices.

On November 1, 1996, Apple Computer and WINForum jointly proposed a set of Part 15 rules to regulate operation of NII/SUPERNet devices in the 5150-5250 MHz band which would prevent the NII/SUPERNet devices from causing harmful interference to the MSS. In this paper, the AirTouch capacity reduction formula is used to calculate the effect on Globalstar of the NII/SUPERNet devices, assuming that the Apple/WINForum proposal is adopted. The AirTouch assumptions on outdoor deployment and building attenuation are used in the calculations. From these calculations, it is clear that the effect of the NII/SUPERNet devices on Globalstar would be insignificant, even with unrealistically aggressive assumptions about the transmit duty cycle of NII/SUPERNet devices. For example, 30 million NII/SUPERNet devices, each operating at a duty cycle of 50% (i.e., every unit is always either transmitting or receiving), would cause a reduction in the baseband signal-to-noise ratio of only about 0.003 dB, and a capacity reduction (according to the AirTouch formula) of less than one-tenth of one percent. With more realistic assumptions about NII/SUPERNet operation (e.g., a 1% average duty cycle), the impact would be vanishingly small. Considering that the Globalstar forward link includes more than 13 dB of margin, it is clear that the NII/SUPERNet devices will not have any significant impact on the operation of Globalstar.

It is concluded that it would be in the public interest for the Commission to adopt the Apple/WINForum proposal, which would allow the NII/SUPERNet devices adequate flexibility, while protecting MSS from the possibility of harmful interference. Restrictions more severe than those proposed by Apple and WINForum are unnecessary to protect MSS, and would undermine the viability of NII/SUPERNet devices.

The AirTouch Analysis

On November 27, 1996, AirTouch Communications filed an *ex parte* document (Attachment 2) which included two figures showing the capacity degradation to the Globalstar mobile satellite system that would reportedly result from various numbers of NII/SUPERNet devices in the 5150-5250 MHz band, which is used for the Globalstar

feeder uplink. Although the basis for these curves was not provided, they seem to have been derived using the formula on page 4 of the Appendix to the AirTouch Reply Comments in ET Docket 96-102.[1] That formula is:

$$\Delta C = \frac{10^{I_t/10}}{10^{I_{p15}/10} + 10^{I_t/10}}, \tag{1}$$

where ΔC is the ratio of the Globalstar capacity with NII/SUPERNet devices to that without them, $I_t = -202.86\,\mathrm{dBW/Hz}$ is the total interference plus noise at the Globalstar subscriber unit without the NII/SUPERNet devices, and I_{p15} is the aggregate power density from the Part 15 NII/SUPERNet devices, in dBW/Hz.

AirTouch calculates the NII/SUPERNet interference by assuming a total round-trip path loss of 193.9 dB, a transmitted power density per device of −80 dBW/Hz, a net building attenuation of 17 dB for indoor devices, and a bandwidth of 20 MHz for each NII/SUPERNet device. Therefore, 20% of the total number of devices in the 5150-5250 MHz band affect a given Globalstar subscriber unit. With those assumptions, if all devices are indoors (as assumed in the first AirTouch figure in Attachment 2), $i_{p15} = 3.2 \times 10^{-4} N_M d\, i_t$, where $i_{p15} = 10^{I_{p15}/10}$ W/Hz, $i_t = 10^{I_t/10}$ W/Hz, N_M is the total number of NII/SUPERNet devices (millions) in the 5150-5250 MHz band, and d is the average duty cycle per device. Substituting into (1), ΔC then becomes:

$$\Delta C = \frac{i_t}{i_{p15} + i_t} = \frac{1}{1 + 3.2 \times 10^{-4} N_M d}, \tag{2}$$

The percentage capacity decrease as shown on the figures in Attachment 2 is:

$$\Delta_{\%} = 100(1 - \Delta C) = 100\left(\frac{3.2 \times 10^{-4} N_M d}{1 + 3.2 \times 10^{-4} N_M d}\right) < 0.032 N_M d\,. \tag{3}$$

The bound is tight for small $\Delta_{\%}$, and it can be seen that it agrees with the curves in the first figure in Attachment 2.

In the second figure of Attachment 2, AirTouch assumes that 5% of the devices are operating outdoors. If G is the effective average antenna gain of the outdoor devices, then the effective number of indoor devices is:

$$N_{M,eff} = N_M\left(0.05 \cdot 10^{(G+17)/10} + 0.95\right). \tag{4}$$

[1]That Appendix is entitled "Technical Analysis Regarding Interference to MSS Links by Part 15 Devices Using 5.15-5.25 GHz Frequency Band" [*sic*].

If $G = 2$ dBi as AirTouch assumes, then $N_{M,eff} = 4.9 N_M$ and $\Delta_{\%} < 0.16 N_M d$. The upper bound is slightly above the upper curve for the higher values of $\Delta_{\%}$, as would be expected.

Globalstar Capacity Degradation under the Apple/WINForum Proposal

On November 1, 1996 Apple Computer and WINForum jointly filed a proposal for rules governing NII/SUPERNet devices operating in the 5150-5250 MHz band, which would impose the following restrictions:

1) For devices operated indoors and in the 5.15-5.25 GHz band, a maximum burst-average transmit power of 10 dBm + 10 log B or 24 dBm (250 mW), whichever is less, where B is the 20-dB emission bandwidth in MHz.
2) A provision for outdoor operation in the 5.15-5.25 GHz band with a maximum 60-second average transmit power of 0 dBm + 10 log B or 14 dBm (25 mW), whichever is less.
3) In both cases, reduction of these maximum power limits by the amount by which the antenna gain exceeds 6 dBi (as in 47 CFR 15.247).

The AirTouch formula will be used here to calculate the impact on Globalstar's capacity, with several minor changes in parameters:

- 3 dB will be added to the path loss, to account for the fact that the Globalstar feeder link uses circular polarization, and the polarization of the signals from the NII/SUPERNet devices will be random.
- The average antenna gain for outdoor NII/SUPERNet devices will be assumed to be 0 dBi, regardless of the gains of individual antennas, based on extensive analysis by members of WINForum (see WINForum's *ex parte* letter of December 6, 1996). During the course of that analysis, an error of roughly 4 dB in the AirTouch antenna gain analysis was discovered. This error was partially offset by an error in AirTouch's assumed distribution of NII/SUPERNet devices over the Earth's surface.
- The total number of NII/SUPERNet devices will be assumed to be evenly-distributed over the entire 350 MHz of the proposed spectrum allocation: 5.15-5.35 GHz and 5.725-5.875 GHz. The bandwidth per device will be assumed to be 20 MHz, consistent with the AirTouch analysis.
- The value of I_t used by AirTouch will be increased to reflect a realistic receiver noise figure. The value used by AirTouch was based on a receiver noise temperature of 293° K (room temperature) for the subscriber unit.[2] Even a low-noise receiver front-end will have a noise figure greater than 0 dB. A 2-dB noise figure will be used, which equates to a noise temperature of about 460° K. With that change, $I_t = -201.2$ dBW/Hz (the full 2 dB increase is not reflected here because the other components of I_t are assumed unchanged).

[2]This was taken from the February 29, 1996 Globalstar "Applications for Modification to Order and Authorization for GLOBALSTAR™ 19-DSS-P-91(48) and CSS-91-014, tables 3 and 4.

The 17-dB building attenuation and the 5% outdoor usage assumed by AirTouch will remain the same, as will the AirTouch expression for capacity degradation in (1).

With these parameters, $i_{p15} = 3.1 \times 10^{-5} N_M d i_t$ and the percentage capacity decrease, assuming all devices operate indoors, becomes:

$$\Delta_{\%} = 100\left(\frac{3.1 \times 10^{-5} N_M d}{1 + 3.1 \times 10^{-5} N_M d}\right) < 0.0031 N_M d. \tag{5}$$

Under the Apple/WINForum proposal, outdoor devices would be limited to a 60-second average transmit power that is 10 dB less than the "instantaneous" (burst-average) limit for indoor devices. Hence, if an outdoor device is transmitting with duty cycle d, the power limit is:

$$P_{out} = \begin{cases} P_{in}, & d \le 0.1 \\ 0.1 P_{in}/d, & d > 0.1 \end{cases} \tag{6}$$

where P_{in} and P_{out} are the transmit power limits for indoor and outdoor devices, respectively. It will be assumed here that all devices are transmitting the maximum allowed power, so outdoor devices transmit at the highest level allowed by the duty cycle. This clearly is a worst-case assumption.

From (6), the effective number of indoor devices $N_{M,eff}$ is related to the total number of devices by:

$$N_{M,eff} = \begin{cases} N_M\left(0.05 \cdot 10^{17/10} + 0.95\right) & d \le 0.1 \\ N_M\left(0.05 \cdot 10^{17/10} \cdot 0.1/d + 0.95\right) & d > 0.1 \end{cases} \tag{7}$$

Therefore,

$$N_{M,eff} d = \begin{cases} 3.45 N_M d & d \le 0.1 \\ (0.25 + 0.95d) N_M & d > 0.1 \end{cases} \tag{8}$$

Substituting $N_{M,eff} d$ per (8) for $N_M d$ in (5) gives $\Delta_{\%}$ for the mix of indoor and outdoor devices. Figure 1 shows the resulting Globalstar capacity degradation vs. the number of NII/SUPERNet devices for various duty cycles. The upper bound is:

$$\Delta_{\%} < \begin{cases} 0.01 N_M d & d \le 0.1 \\ \left(7.75 \times 10^{-4} + 2.94 \times 10^{-3} d\right) N_M & d > 0.1 \end{cases} \tag{9}$$

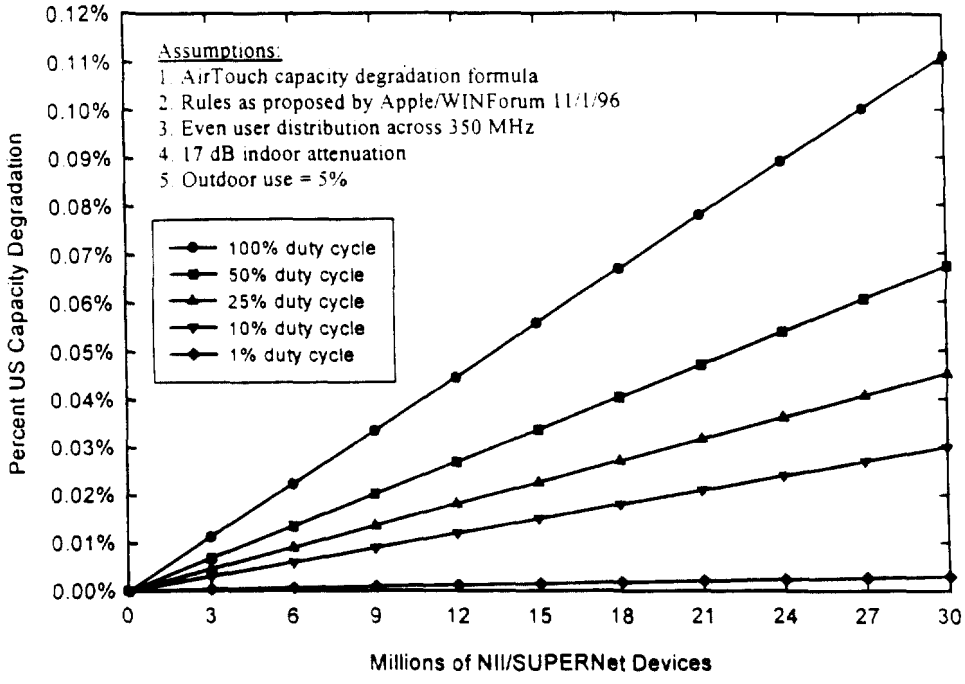

Figure 1: *Globalstar US capacity degradation from NII/SUPERNet devices using the AirTouch formula.*

The average NII/SUPERNet duty cycle is projected to be 1% or less.[3] In that case, the Globalstar capacity degradation with 30 million NII/SUPERNet devices is less than 0.003%. However, other duty cycles are shown for reference and comparison with the AirTouch results. Note that in the pathological case in which all devices are always either transmitting or receiving, the duty cycle is 50% and even then, the capacity reduction is about 0.07%. It is clear from this that even if NII/SUPERNet device deployment approaches that of cellular service in the U. S., and all devices are always active, the impact on Globalstar is negligible. With more realistic deployment and duty cycle assumptions, the impact is essentially nonexistent.

Another useful way of assessing the impact of the NII/SUPERNet devices is to calculate the reduction in E_b/N_0, the baseband signal-to-noise ratio, at the subscriber unit, given (in dB) by:

[3]See Exhibit A of WINForum's September 12, 1996 letter to Warren Richards, Chair, U.S. National Committee, International Communications and Information Policy, U.S. Department of State, filed as *ex parte* in ET Docket 96-102 on October 21, 1996.

$$\Delta(E_b/N_0) = 10\log\left(\frac{E_b/(i_{p15}+i_t)}{E_b/i_t}\right) = 10\log\left(\frac{i_t}{i_{p15}+i_t}\right) \approx -\frac{10}{\ln 10}\left(\frac{i_{p15}}{i_t}\right)\text{dB}, \tag{10}$$

with

$$\frac{i_{p15}}{i_t} = \begin{cases} 10^{-4} N_M d & d \le 0.1 \\ \left(7.75\times10^{-6} + 2.94\times10^{-5} d\right) N_M & d > 0.1 \end{cases} \tag{11}$$

The approximation in (10) uses the first term in the Taylor series for $\ln x$. Figure 2 shows $\Delta(E_b/N_0)$ vs. N_M for various values of d (computed using the exact expression, not the approximation).

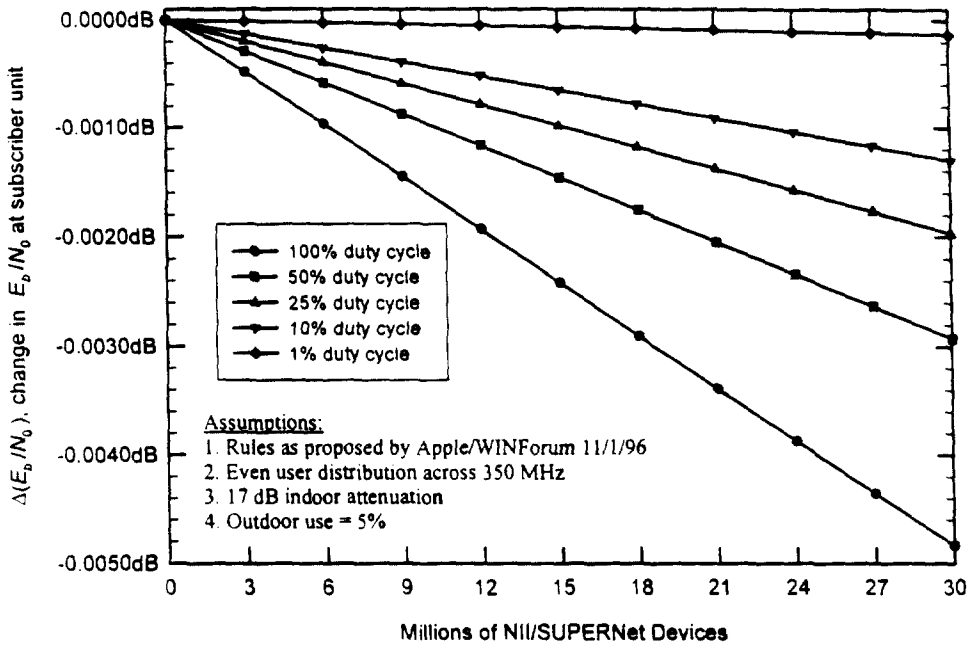

Figure 2: *Reduction in E_b/N_0 at the Globalstar subscriber unit due to NII/SUPERNet devices.*

Substituting (11) into the approximation of (10) gives:

$$\Delta(E_b/N_0) \approx \begin{cases} -4.65\times10^{-4} N_M d\,\text{dB} & d \le 0.1 \\ -\left(3.37\times10^{-5} + 1.28\times10^{-4} d\right) N_M\,\text{dB} & d > 0.1 \end{cases} \tag{12}$$

For 30 million NII/SUPERNet devices and a 1% duty cycle, E_b/N_0 at the Globalstar subscriber unit is reduced only 0.00014 dB. Even with the pathological 50% duty cycle, the reduction is only about 0.003 dB. Considering that the available link margin is 16 dB

with diversity and both paths clear, and 13.5 dB with one path fully blocked,[4] the negligible amount of degradation that could be caused by NII/SUPERNet devices would not impact service.

Conclusion

The Globalstar capacity reduction that would result from NII/SUPERNet devices has been analyzed, using the AirTouch formula and assumptions regarding building attenuation and percentage of outdoor operation, and assuming that the Apple/WINForum proposal of November 1, 1996 is adopted. In addition, the impact on the baseband signal-to-noise ratio E_b/N_0 has been analyzed. It is clear that if the Apple/WINForum proposal is adopted, the effect of NII/SUPERNet devices on the operation of the Globalstar system will be insignificant. The Part 15 rules proposed by Apple and WINForum will allow NII/SUPERNet devices adequate flexibility, while protecting the services to be provided by Globalstar. Restrictions on NII/SUPERNet devices operation more severe than those proposed by Apple and WINForum are unnecessary, and would only serve to undermine the usefulness of NII/SUPERNet devices.

[4]See Tables 3 and 4 of Globalstar, Feb. 29, 1996, *op. cit.*

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EX PARTE OR LATE FILED

December 2, 1996

EX PARTE

Mr. William F. Caton
Acting Secretary
Federal Communications Commission
1919 M Street, NW, Room 222
Washington, DC 20554

RECEIVED
DEC 2 - 1996
Federal Communications Commission
Office of Secretary

RE: Amendment of the Commission's Rules to Provide for Unlicensed NII/SUPERNet Operations in the 5 GHz Frequency Range (ET Docket No. 96-102)

Dear Mr. Caton:

On Wednesday, November 27, 1996, I, on behalf of AirTouch Communications, spoke with Harry Ng to discuss issues relating to this proceeding and he requested the attached. Please associate the attached material with the above-referenced proceeding.

Two copies of this notice are being submitted to the Secretary of the FCC in accordance with Section 1.1206(a)(1) of the Commission's Rules.

Please stamp and return the provided copy to confirm your receipt. Please contact me at 202-293-4957 should you have any questions or require additional information concerning this matter.

Sincerely,

Donna L. Bethea

Attachment

cc: Harry Ng

No. of Copies rec'd 0+1
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