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April 8, 1997

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Federal Communications Commission
Office of Secretary

Ex Parte

Mr. William F. Caton
Acting Secretary
Federal Communications Commission
1919 M Street, N.W. Rm. 222
Washington, D.C. 20554

Re: CC Docket No. 94-1

Today, on behalf of Bell Atlantic, Dr. Melvyn Fuss of the University of Toronto, Maureen Keenan, Ed Shakin, and I met with Jay Atkinson, Alex Belinfante, Raj Kanan, Steve Spaeth, Mark Uretsky, and Brad Wimmer to discuss the above captioned proceeding. The attached handouts were used during the meeting.

Please enter this letter and material into the record as appropriate. Please do not hesitate to contact me if there are any questions.

Sincerely,

Joseph Mulieri

Attachment

cc: J. Atkinson
A. Belinfante
R. Kanan
S. Spaeth
M. Uretsky
B. Wimmer

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List ABCDE

Bell Atlantic

Ex Parte Presentation
of
Dr. Melvyn Fuss

April 8, 1997

An Input Price Growth Rate Differential Term Should Not Be Included in the Calculation of X

- The best prediction for the future value of the input price differential is zero.

- Correct statistical analysis of the input price data demonstrates that the shift in the relationship between the LEC and U.S. input price growth rates observed after Divestiture was temporary in nature.

- The period 1984-89 was a temporary departure from the long term relationship between the LEC input price growth and the U.S. economy's input price growth ; and this long term relationship was resumed in the 1990's.

- The above statements are confirmed by an analysis of the data set supplied recently by USTA to Dr Anthony Bush, which covers the period 1949-1995 and contains the updated simplified Christensen data. The analysis is presented in the enclosed tables. The average annual input price growth rate differential (LEC-U.S.) was:

1949-95 (excluding 1984-89)	+0.4 %
1990-95	+0.6 %
1984-89	-4.1 %

- The enclosed tables demonstrate that the criticisms of AT&T and Ad Hoc regarding the statistical procedures I employed are without merit with respect to the 1949-95 data set.

Total Factor Productivity Growth Should Be Calculated on a Total Company Basis

- There is no economically valid method for measuring interstate-only TFP, since significant amounts of inputs are used jointly with intrastate services to create joint and common costs.

- It is impossible to write down a formula to calculate interstate TFP.

- Joint and common costs cannot be allocated to separate services in an economically meaningful manner.

- AT&T's assumption that inputs used by all services grow at the same rate is a particularly simplistic form of fully distributed cost allocation. It has not been justified in AT&T's various submissions, either analytically or empirically.

AT&T's Last Round of Criticisms of My Methodology Are Without Merit (Ex Parte filed in CC Docket 94-1, June 28, 1997)

Input Price Differential

- Norsworthy continues to be wrong regarding his use of cointegration testing. His reference (Hamilton (1994)) contradicts his methodology and confirms mine (evidence in the form of copies of the relevant pages are attached). All of Hamilton's tests are tests of the null hypothesis that variables are not cointegrated. The Engle-Granger test statistic cannot test the reverse hypothesis. Norsworthy's interpretation of the word "Usually" in the TSP User's Guide is nonsensical, as the enclosed page from the Guide makes clear.

- The Engle-Granger test statistic cannot be used to test whether the residuals from a regression are non-stationary when one of the potentially cointegrated variables is stationary. This is clear from the references supplied by Norsworthy and myself.

- I have always addressed all available evidence, contrary to Norsworthy's complaint. I continue to do so by presenting test results for the newly available 1949-95 data set.

- In the 1949-95 data set, the U.S. input price series is taken from a common source (the Bureau of Labor Statistics). There is no need to consider adding dummy variables reflecting the time periods corresponding to each of the separate sources since there are no separate sources. The test results using this updated data set confirm my previous results.

- Structural breaks in the data are not <u>sources</u> of unit roots, as claimed by Norsworthy. His reference (Enders(1995)) states the finding, commonly noted in the cointegration literature, that in the presence of structural breaks in the data unit root tests are biased toward a finding of unit roots when they are not present (see enclosed page from Enders).

- The point made by Norsworthy in the section "Economics and the Input Price Differential" regarding different input cost shares, to the extent it has relevance, is equally applicable for <u>all</u> time periods. Yet the different cost shares did not lead to an average input price differential over the 1949-95 (excluding 1984-89) and 1990-95 periods. Norsworthy needs to explain why this "economic" effect is quantitatively important for only a short period of time (1984-89) and not for the other time periods.

- The mixup over the bond yield data series used by Bush and Uretsky (BU) was communicated to AT&T before the Norsworthy- Berndt (NB) Reply statement was filed. Christensen used the Moody public utility bond yield data in the Feb. ex parte, not the corporate yield series used by BU.

Interstate Only TFP

- Norsworthy admits that the NB algebraic equation for interstate "TFP" was incorrect (he says it needs adjustment). The equation in his Supplemental Statement adds no light on the argument, since it depends on the unsustainable assumption that input growth rates are equal for all outputs.

- Contrary to Norsworthy's assertion, Christensen includes the inputs and outputs from all non-regulated services which use inputs jointly with the regulated services.

- Norsworthy claims I should offer an alternative method of calculating interstate access only TFP growth. But how can I, since I argue that it does not exist conceptually.

- Norsworthy's claim that a LEC's choice of a high X factor demonstrates high interstate TFP growth is not correct logically. When the accounting rate of return starting point is in excess of the sharing rate of return and the selection is only for a limited period, it can be in a LEC's interest to choose a high X factor even when it expects to have a lower productivity growth rate.

- On p. 15 of his Supplemental Statement Norsworthy repeats Nadiri's mixup of levels and rates of growth. The discussion of variable cost elasticities is not relevant since variable costs do not include capital switching costs. In any case, cost elasticities refer to the effects of output changes on total company cost, not service-specific cost.

- The discussion of the Bernstein paper on p.16 ignores my criticism of that paper. My criticism demonstrates that the results are meaningless because of the specification error in the model (as explained in my earlier ex parte).

- Local verus global cost separability: My comments regarding cost complementarities in my previous ex parte are applicable to both the global and local forms. Only the illustrative example used the global form of cost complementarity for simplicity. Since it is the prevailing opinion (including that of Norsworthy - see first paragraph on page 12 of his Supplemental Statement) that telecommunications costs are not separable, it is Norsworthy who should provide empirical evidence that separability (whether global or local over all relevant output combinations) is a reasonable working hypothesis. I know of no such empirical evidence.

Guide To Enclosed Tables

The enclosed tables are based on the underlying LEC input [price] inflation data for the period 1949-95 submitted by USTA, Letter to Dr. Anthony Bush, March 24, 1997.

Tables 1-3 are new tables.

Tables A2(revised), A4(revised), A6(revised), D1(revised) and D2(revised) are revised tables from my previous Declarations. The revisions utilize the 1949-95 data*.

Tables A2, A4 and A6 appeared in my initial Declaration (December 15, 1995)

Tables D1 and D2 appeared in my third Declaration (May 31, 1996)

* The U.S. input price growth rate for 1995 is not yet available. The enclosed results use the 1989-94 average (3.2%) as an estimate of the 1995 data point.

Table 1

LEC-U.S. Input Price Differential

Time Period	Annual % Change in LEC Input Prices	Annual % Change in U.S. Input Prices	LEC-U.S. Input Price Differential
1949-95	4.5	4.7	-0.2 (t=0.4)
1949-95 (excluding 1984-89)	5.2	4.8	+0.4 (t=0.7)
1990-95	3.7	3.1	+0.6 (t=1.1)
1984-89	-0.2	3.9	-4.1 (t=4.1)

Table 2

Tests of the Unit Root Hypothesis: 1949-1995

Variable	Symmetric Mean Test Statistic*	P-Value	Dickey- Fuller Test Statistic	P-Value
% Change in LEC Input Prices (CPT)	-2.48	0.31	-2.25	0.46
% Change in U.S. Input Prices (CPE)	-2.22	0.49	-0.99	0.95
LEC-U.S. Input Price Differential (CPDIFF = CPT-CPE)	-3.39**	0.028	-3.19***	0.085
Moody's Corporate Aaa Bond Yield (MOODY)	-1.77	0.79	-1.37	0.87
Divestiture Dummy (DIVEST)	-1.78	0.78	-1.78	0.71
D90	-2.05	0.62	-1.64	0.78

* Preferred to the Dickey-Fuller test because it is a more powerful test (see the TSP 4.3 User's Guide, page 94.)
** Unit root rejected at 5% significance level
*** Unit root rejected at 10% significance level

Table 3

Tests of Cointegration

Equation	Engle-Granger Test Statistic	P-Value
Permanent Change Hypothesis (%change in LEC Input Prices as dependent variable)	-3.01	0.59
Temporary Change Hypothesis (%change in LEC Input Prices as dependent variable)	-3.33	0.42
Permanent Change Hypothesis (LEC-U.S. Input Price Differential as dependent variable)	not applicable since unit root test on dependent variable rejected	not applicable since unit root test on dependent variable rejected
Temporary Change Hypothesis (LEC-U.S. Input Price Differential as dependent variable)	not applicable since unit root test on dependent variable rejected	not applicable since unit root test on dependent variable rejected

Table A2 (Revised)

Values of the Standard Errors of Regression
Data to 1995

DX	Christensen Data Equation (4) *	Christensen Data Equation (5)
D85	3.552	3.579
D86	3.460	3.518
D87	3.446	3.528
D88	3.428	3.533
D89	3.181	3.351
D90	2.888	3.151
D91	2.933	3.214
D92	2.995	3.318
D93	3.115	3.413
D94	3.211	3.477
D95	3.190	3.474

* Failed cointegration test and hence may be a spurious regression.

Table A.4 (Revised)

Testing the Two Competing Hypotheses Using the J Test
Data to 1995

Data Set and Equation Nos.	Hypothesis	t - Statistic for α	Critical 5% Value of t	P-Value
Christensen Eqs (2)&(4) *	H1 versus HC	3.46	1.96	.0005
	H2 versus HC	1.81	1.96	.0700
Christensen Eqs (3)&(5)	H1 versus HC	3.05	1.96	.0023
	H2 versus HC	0.42	1.96	.6750

* Failed cointegration test and hence may be a spurious regression.

Table A.6 (Revised)

Testing the Two Competing Hypotheses Using the Cox Test
Data to 1995

Data Set and Equation Nos.	Hypothesis	Standard Normal (N) Statistic for α	Critical 5% Value of N	P-Value
Christensen Eqs (2)&(4) *	H1 is correct	-6.91	-1.96	.0000
	H2 is correct	-2.35	-1.96	.0189
Christensen Eqs (3)&(5)	H1 is correct	-6.45	-1.96	.0000
	H2 is correct	-0.43	-1.96	.6652

* Failed cointegration test and hence may be a spurious regression.

Time Series Analysis

James D. Hamilton

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and $\mathbf{W}(r)$ denotes n-dimensional standard Brownian motion while the integral sign indicates integration over r from 0 to 1. Similarly,

$$\Upsilon_T^{-1} \sum_{t=1}^{T} \mathbf{x}_t^* \varepsilon_{it} \xrightarrow{L} \begin{bmatrix} \mathbf{h}_1 \\ h_2 \\ \mathbf{h}_3 \\ h_4 \end{bmatrix}, \qquad [18.2.48]$$

where $\mathbf{h}_1 \sim N(\mathbf{0}, \sigma_{ii}\mathbf{V})$. The variables h_2 and h_4 are also Gaussian, though $\mathbf{h}_3$ is non-Gaussian. If we define $\boldsymbol{\omega}$ to be the vector of coefficients on lagged $\Delta\mathbf{y}$,

$$\boldsymbol{\omega} \equiv (\boldsymbol{\zeta}_{i1}', \boldsymbol{\zeta}_{i2}', \ldots, \boldsymbol{\zeta}_{i,n-1}')',$$

then the preceding results imply that

$$\Upsilon_T(\mathbf{b}_T^* - \boldsymbol{\beta}^*) = \begin{bmatrix} T^{1/2}(\hat{\boldsymbol{\omega}}_T - \boldsymbol{\omega}) \\ T^{1/2}(\hat{\boldsymbol{\alpha}}_{i,T}^* - \boldsymbol{\alpha}_i^*) \\ T(\hat{\boldsymbol{\rho}}_{i,T}^* - \boldsymbol{\rho}_i^*) \\ T^{3/2}(\hat{\gamma}_{i,T} - \gamma_i) \end{bmatrix} \xrightarrow{L} \begin{bmatrix} \mathbf{V}^{-1}\mathbf{h}_1 \\ \mathbf{Q}^{-1}\boldsymbol{\eta} \end{bmatrix}, \qquad [18.2.49]$$

where $\boldsymbol{\eta} \equiv (h_2, \mathbf{h}_3', h_4)'$ and $\mathbf{Q}$ is the $[(n+1) \times (n+1)]$ lower right block of the matrix in [18.2.46]. Thus, as usual, the coefficients on $\mathbf{u}_{t-s}$ in [18.2.43] are asymptotically Gaussian:

$$\sqrt{T}(\hat{\boldsymbol{\omega}}_{i,T} - \boldsymbol{\omega}_i) \xrightarrow{L} N(\mathbf{0}, \sigma_{ii}\mathbf{V}^{-1}).$$

These coefficients are, of course, numerically identical to the coefficients on $\Delta\mathbf{y}_{t-s}$ in [18.2.38]. Any F tests involving just these coefficients are also identical for the two parameterizations. Hence, an F test about $\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_{p-1}$ in [18.2.38] has the usual limiting χ^2 distribution. This is the same asymptotic distribution as if [18.2.38] were estimated with $\boldsymbol{\rho} = \mathbf{I}_n$ imposed; that is, it is the same asymptotic distribution whether the regression is estimated in levels or in differences.

Since $\hat{\boldsymbol{\rho}}_T^*$ and $\hat{\boldsymbol{\gamma}}_T$ converge at a faster rate than $\hat{\boldsymbol{\omega}}_T$, the asymptotic distribution of a linear combination of $\hat{\boldsymbol{\omega}}_T$, $\hat{\boldsymbol{\rho}}_T^*$, and $\hat{\boldsymbol{\gamma}}_T$ that puts nonzero weight on $\hat{\boldsymbol{\omega}}_T$ has the same asymptotic distribution as a linear combination that uses the true values for $\boldsymbol{\rho}$ and $\boldsymbol{\gamma}$. This means, for example, that the original coefficients $\hat{\boldsymbol{\Phi}}_s$ of the *VAR* estimated in levels as in [18.2.1] are all individually Gaussian and can be interpreted using the usual t tests. A Wald test of the null hypothesis of $p_0 \geq 1$ lag against the alternative of $p > p_0$ lags again has the usual χ^2 distribution. However, Granger-causality tests typically have nonstandard distributions.

18.3. *Spurious Regressions*

Consider a regression of the form

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + u_t,$$

for which elements of y_t and $\mathbf{x}_t$ might be nonstationary. If there does not exist some population value for $\boldsymbol{\beta}$ for which the residual $u_t = y_t - \mathbf{x}_t'\boldsymbol{\beta}$ is $I(0)$, then *OLS* is quite likely to produce spurious results. This phenomenon was first discovered in Monte Carlo experimentation by Granger and Newbold (1974) and later explained theoretically by Phillips (1986).

A general statement of the spurious regression problem can be made as follows. Let $\mathbf{y}_t$ be an $(n \times 1)$ vector of $I(1)$ variables. Define $g \equiv (n - 1)$, and

19

This chapter discusses a particular class of vector unit root processes known as *cointegrated* processes. Such specifications were implicit in the "error-correction" models advocated by Davidson, Hendry, Srba, and Yeo (1978). However, a formal development of the key concepts did not come until the work of Granger (1983) and Engle and Granger (1987).

Section 19.1 introduces the concept of cointegration and develops several alternative representations of a cointegrated system. Section 19.2 discusses tests of whether a vector process is cointegrated. These tests are summarized in Table 19.1. Single-equation methods for estimating a cointegrating vector and testing a hypothesis about its value are presented in Section 19.3. Full-information maximum likelihood estimation is discussed in Chapter 20.

19.1. *Introduction*

Description of Cointegration

An $(n \times 1)$ vector time series $\mathbf{y}_t$ is said to be *cointegrated* if each of the series taken individually is $I(1)$, that is, nonstationary with a unit root, while some linear combination of the series $\mathbf{a}'\mathbf{y}_t$ is stationary, or $I(0)$, for some nonzero $(n \times 1)$ vector $\mathbf{a}$. A simple example of a cointegrated vector process is the following bivariate system:

$$y_{1t} = \gamma y_{2t} + u_{1t} \qquad [19.1.1]$$

$$y_{2t} = y_{2,t-1} + u_{2t}, \qquad [19.1.2]$$

with u_{1t} and u_{2t} uncorrelated white noise processes. The univariate representation for y_{2t} is a random walk,

$$\Delta y_{2t} = u_{2t}, \qquad [19.1.3]$$

while differencing [19.1.1] results in

$$\Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t} = \gamma u_{2t} + u_{1t} - u_{1,t-1}. \qquad [19.1.4]$$

Recall from Section 4.7 that the right side of [19.1.4] has an $MA(1)$ representation:

$$\Delta y_{1t} = v_t + \theta v_{t-1}, \qquad [19.1.5]$$

where v_t is a white noise process and $\theta \neq -1$ as long as $\gamma \neq 0$ and $E(u_{2t}^2) > 0$. Thus, both y_{1t} and y_{2t} are $I(1)$ processes, though the linear combination

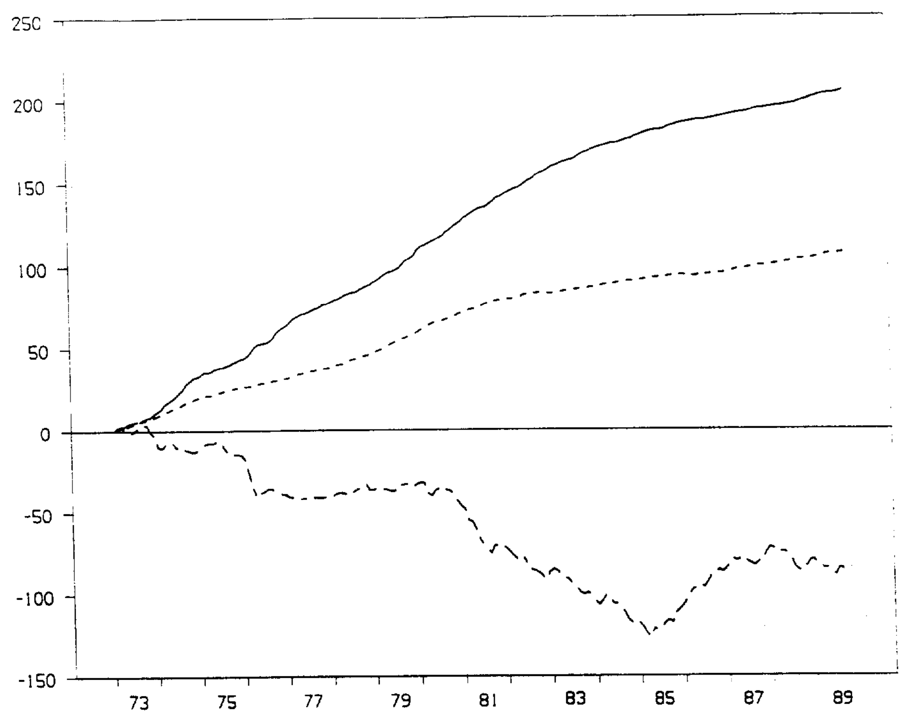

FIGURE 19.2 One hundred times the log of the price level in the United States (p_t), the dollar-lira exchange rate (s_t), and the price level in Italy (p_t^*), monthly, 1973–89. Key: ---- p_t; —–— s_t; ——— p_t^*.

exchange rate (s_t), where s_t is in terms of the number of U.S. dollars needed to purchase an Italian lira. Natural logs of the raw data were taken and multiplied by 100, and the initial value for 1973:1 was then subtracted, as in

$$p_t = 100\cdot[\log(P_t) - \log(P_{1973:1})].$$

The purpose of subtracting the constant $\log(P_{1973:1})$ from each observation is to normalize each series to be zero for 1973:1 so that the graph is easier to read. Multiplying the log by 100 means that p_t is approximately the percentage difference between P_t and its starting value $P_{1973:1}$. The graph shows that Italy experienced about twice the average inflation rate of the United States over this period and that the lira dropped in value relative to the dollar (that is, s_t fell) by roughly this same proportion.

Figure 19.3 plots the real exchange rate,

$$z_t \equiv p_t - s_t - p_t^*.$$

It appears that the trends are eliminated by this transformation, though deviations of the real exchange rate from its historical mean can persist for several years.

To test for cointegration, we first verify that p_t, p_t^*, and s_t are each individually $I(1)$. Certainly, we anticipate the average inflation rate to be positive ($E(\Delta p_t) > 0$), so that the natural null hypothesis is that p_t is a unit root process with positive drift, while the alternative is that p_t is stationary around a deterministic time trend. With monthly data it is a good idea to include at least twelve lags in the regression. Thus, the following model was estimated by OLS for the U.S. data for t = 1974:2

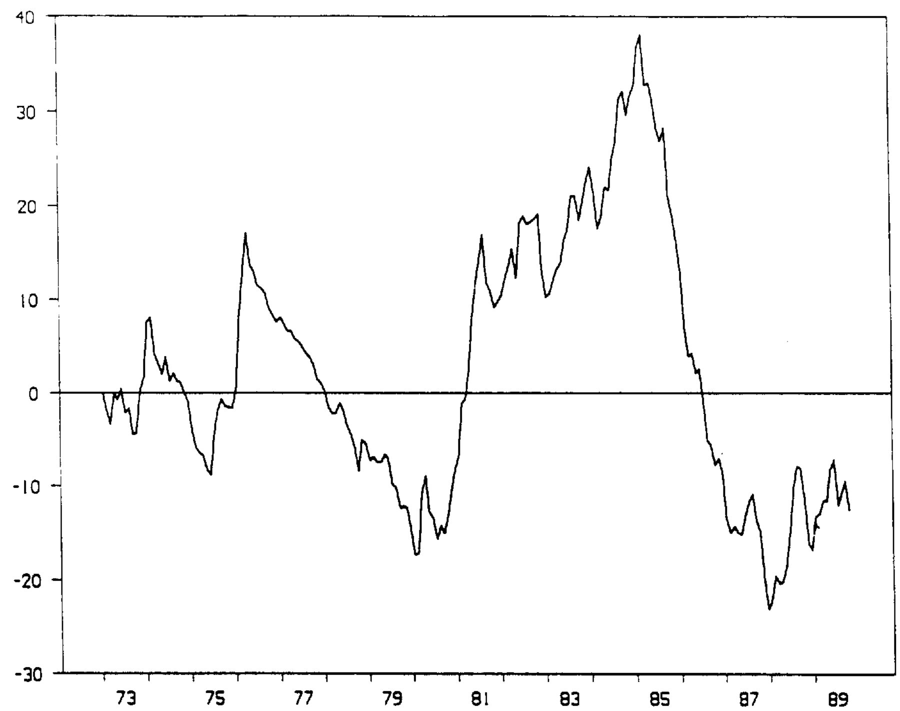

FIGURE 19.3 The real dollar-lira exchange rate, monthly, 1973–89.

through 1989:10 (standard errors in parentheses):

$$\begin{aligned} p_t = {} & \underset{(0.08)}{0.55}\,\Delta p_{t-1} - \underset{(0.09)}{0.06}\,\Delta p_{t-2} + \underset{(0.08)}{0.07}\,\Delta p_{t-3} + \underset{(0.08)}{0.06}\,\Delta p_{t-4} \\ & - \underset{(0.08)}{0.08}\,\Delta p_{t-5} - \underset{(0.07)}{0.05}\,\Delta p_{t-6} + \underset{(0.07)}{0.17}\,\Delta p_{t-7} - \underset{(0.07)}{0.07}\,\Delta p_{t-8} \\ & + \underset{(0.07)}{0.24}\,\Delta p_{t-9} - \underset{(0.07)}{0.11}\,\Delta p_{t-10} + \underset{(0.07)}{0.12}\,\Delta p_{t-11} + \underset{(0.07)}{0.05}\,\Delta p_{t-12} \\ & + \underset{(0.09)}{0.14} + \underset{(0.00307)}{0.99400}\,p_{t-1} + \underset{(0.0018)}{0.0029}\,t. \end{aligned} \tag{19.2.1}$$

The t statistic for testing the null hypothesis that ρ (the coefficient on p_{t-1}) is unity is

$$t = (0.99400 - 1.0)/(0.00307) = -1.95.$$

Comparing this with the 5% crtical value from the case 4 section of Table B.6 for a sample of size $T = 189$, we see that $-1.95 > -3.44$. Thus, the null hypothesis of a unit root is accepted. The F test of the joint null hypothesis that $\rho = 1$ and $\delta = 0$ (for ρ the coefficient on p_{t-1} and δ the coefficient on the time trend) is 2.41. Comparing this with the critical value of 6.40 from the case 4 section of Table B.7, the null hypothesis is again accepted, further confirming the impression that U.S. prices follow a unit root process with drift.

If p_t in [19.2.1] is replaced by p_t^*, the augmented Dickey-Fuller t and F tests are calculated to be -0.13 and 4.25, respectively, so that the null hypothesis that the Italian price level follows an $I(1)$ process is again accepted. When p_t in [19.2.1] is replaced by s_t, the t and F tests are -1.58 and 1.49, so that the exchange rate likewise admits an $ARIMA(12, 1, 0)$ representation. Thus, each of the three series individually could reasonably be described as a unit root process with drift.

Proposition 19.1: (*Gra* [redacted] *nsider an* $(n \times 1)$ *vector* $\mathbf{y}_t$ *where* $\Delta\mathbf{y}_t$ *satisfies* [*19.1.29*] *for* $\boldsymbol{\varepsilon}_t$ *white noise with positive definite variance-covariance matrix and* $\{s \cdot \boldsymbol{\Psi}_s\}_{s=0}^{\infty}$ *absolutely summable. Suppose that there are exactly* h *cointegrating relations among the elements of* $\mathbf{y}_t$. *Then there exists an* $(h \times n)$ *matrix* $\mathbf{A}'$ *whose rows are linearly independent such that the* $(h \times 1)$ *vector* $\mathbf{z}_t$ *defined by*

$$\mathbf{z}_t \equiv \mathbf{A}'\mathbf{y}_t$$

is stationary. The matrix $\mathbf{A}'$ *has the property that*

$$\mathbf{A}'\boldsymbol{\Psi}(1) = \mathbf{0}.$$

If, moreover, the process can be represented as the pth-order VAR in levels as in equation [*19.1.26*], *then there exists an* $(n \times h)$ *matrix* $\mathbf{B}$ *such that*

$$\boldsymbol{\Phi}(1) = \mathbf{B}\mathbf{A}',$$

and there further exist $(n \times n)$ *matrices* $\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_{p-1}$ *such that*

$$\Delta\mathbf{y}_t = \boldsymbol{\zeta}_1\Delta\mathbf{y}_{t-1} + \boldsymbol{\zeta}_2\Delta\mathbf{y}_{t-2} + \cdots + \boldsymbol{\zeta}_{p-1}\Delta\mathbf{y}_{t-p+1} + \boldsymbol{\alpha} - \mathbf{B}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t.$$

19.2. *Testing the Null Hypothesis of No Cointegration*

This section discusses tests for cointegration. The approach will be to test the null hypothesis that there is no cointegration among the elements of an $(n \times 1)$ vector $\mathbf{y}_t$; rejection of the null is then taken as evidence of cointegration.

Testing for Cointegration When the Cointegrating Vector Is Known

Often when theoretical considerations suggest that certain variables will be cointegrated, or that $\mathbf{a}'\mathbf{y}_t$ is stationary for some $(n \times 1)$ cointegrating vector $\mathbf{a}$, the theory is based on a particular known value for $\mathbf{a}$. In the purchasing power parity example [19.1.6], $\mathbf{a} = (1, -1, -1)'$. The Davidson, Hendry, Srba, and Yeo hypothesis (1978) that consumption is a stable fraction of income implies a cointegrating vector of $\mathbf{a} = (1, -1)'$, as did Kremers's assertion (1989) that government debt is a stable multiple of GNP.

If the interest in cointegration is motivated by the possibility of a particular known cointegrating vector $\mathbf{a}$, then by far the best method is to use this value directly to construct a test for cointegration. To implement this approach, we first test whether each of the elements of $\mathbf{y}_t$ is individually $I(1)$. This can be done using any of the tests discussed in Chapter 17. Assuming that the null hypothesis of a unit root in each series individually is accepted, we next construct the scalar $z_t = \mathbf{a}'\mathbf{y}_t$. Notice that if $\mathbf{a}$ is truly a cointegrating vector, then $\mathbf{a}'\mathbf{y}_t$ will be $I(0)$. If $\mathbf{a}$ is not a cointegrating vector, then $\mathbf{a}'\mathbf{y}_t$ will be $I(1)$. Thus, a test of the null hypothesis that z_t is $I(1)$ is equivalent to a test of the null hypothesis that $\mathbf{y}_t$ is not cointegrated. If the null hypothesis that z_t is $I(1)$ is rejected, we would conclude that $z_t = \mathbf{a}'\mathbf{y}_t$ is stationary, or that $\mathbf{y}_t$ is cointegrated with cointegrating vector $\mathbf{a}$. The null hypothesis that z_t is $I(1)$ can also be tested using any of the approaches in Chapter 17.

For example, Figure 19.2 plots monthly data from 1973:1 to 1989:10 for the consumer price indexes for the United States (p_t) and Italy (p_t^*), along with the

Of course, we could instead imagine including a time trend directly in the regression, as in

$$y_{1t} = \alpha + \gamma_2 y_{2t} + \gamma_3 y_{3t} + \cdots + \gamma_n y_{nt} + \delta t + u_t. \qquad [19.2.48]$$

Since [19.2.48] is in the same form as the regression of [19.2.47], critical values for such a regression could be found by treating this as if it were a regression involving $(n + 1)$ variables and looking in the case 3 section of Table B.8 or B.9 for the critical values that would be appropriate if we actually had $(n + 1)$ rather than n total variables. Clearly, the specification in [19.2.42] has more power to reject a false null hypothesis than [19.2.48], since we would use the same table of critical values for [19.2.42] or [19.2.48] with one more degree of freedom used up by [19.2.48]. Conceivably, we might still want to estimate the regression in the form of [19.2.48] to cover the case when we are not sure whether any of the elements of $\mathbf{y}_t$ have a nonzero trend or not.

Summary of Residual-Based Tests for Cointegration

The Phillips-Ouliaris-Hansen procedure for testing for cointegration is summarized in Table 19.1.

To illustrate this approach, consider again the purchasing power parity example where p_t is the log of the U.S. price level, s_t is the log of the dollar-lira exchange rate, and p_t^* is the log of the Italian price level. We have already seen that the vector $\mathbf{a} = (1, -1, -1)'$ does not appear to be a cointegrating vector for $\mathbf{y}_t = (p_t, s_t, p_t^*)'$. Let us now ask whether there is any cointegrating relation among these variables.

The following regression was estimated by *OLS* for $t = 1973{:}1$ to $1989{:}10$ (standard errors in parentheses):

$$p_t = \underset{(0.37)}{2.71} + \underset{(0.012)}{0.051}\, s_t + \underset{(0.0067)}{0.5300}\, p_t^* + \hat{u}_t. \qquad [19.2.49]$$

The number of observations used to estimate [19.2.49] is $T = 202$. When the sample residuals $\hat{u}_t$ are regressed on their own lagged values, the result is

$$\hat{u}_t = \underset{(0.01172)}{0.98331}\, \hat{u}_{t-1} + \hat{e}_t$$

$$s^2 = (T-2)^{-1} \sum_{t=2}^{T} \hat{e}_t^2 = (0.40374)^2$$

$$\hat{c}_0 = 0.1622$$

$$\hat{c}_j = (T-1)^{-1} \sum_{t=j+2}^{T} \hat{e}_t \hat{e}_{t-j}$$

$$\hat{\lambda}^2 = \hat{c}_0 + 2\cdot\sum_{j=1}^{12} [1 - (j/13)]\hat{c}_j = 0.4082.$$

The Phillips-Ouliaris Z_ρ test is

$$\begin{aligned} Z_\rho &= (T-1)(\hat{\rho} - 1) - (1/2)\{(T-1)\cdot\hat{\sigma}_{\hat{\rho}} \div s\}^2(\hat{\lambda}^2 - \hat{c}_0) \\ &= (201)(0.98331 - 1) \\ &\quad - \tfrac{1}{2}\{(201)(0.01172) \div (0.40374)\}^2(0.4082 - 0.1622) \\ &= -7.54. \end{aligned}$$

Given the evidence of nonzero drift in the explanatory variables, this is to be compared with the case 3 section of Table B.8. For $(n - 1) = 2$, the 5% critical

TABLE 19.1
Summary of Phillips-Ouliaris-Hansen Tests for Cointegration

Case 1:

Estimated cointegrating regression:

$$y_{1t} = \gamma_2 y_{2t} + \gamma_3 y_{3t} + \cdots + \gamma_n y_{nt} + u_t$$

True process for $\mathbf{y}_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$:

$$\Delta \mathbf{y}_t = \sum_{s=0}^{\infty} \mathbf{\Psi}_s \boldsymbol{\varepsilon}_{t-s}$$

Z_ρ has the same asymptotic distribution as the variable described under the heading Case 1 in Table B.8.

Z_t and the augmented Dickey-Fuller t test have the same asymptotic distribution as the variable described under Case 1 in Table B.9.

Case 2:

Estimated cointegrating regression:

$$y_{1t} = \alpha + \gamma_2 y_{2t} + \gamma_3 y_{3t} + \cdots + \gamma_n y_{nt} + u_t$$

True process for $\mathbf{y}_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$:

$$\Delta \mathbf{y}_t = \sum_{s=0}^{\infty} \mathbf{\Psi}_s \boldsymbol{\varepsilon}_{t-s}$$

Z_ρ has the same asymptotic distribution as the variable described under Case 2 in Table B.8.

Z_t and the augmented Dickey-Fuller t test have the same asymptotic distribution as the variable described under Case 2 in Table B.9.

Case 3:

Estimated cointegrating regression:

$$y_{1t} = \alpha + \gamma_2 y_{2t} + \gamma_3 y_{3t} + \cdots + \gamma_n y_{nt} + u_t$$

True process for $\mathbf{y}_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$:

$$\Delta \mathbf{y}_t = \boldsymbol{\delta} + \sum_{s=0}^{\infty} \mathbf{\Psi}_s \boldsymbol{\varepsilon}_{t-s}$$

with at least one element of $\delta_2, \delta_3, \ldots, \delta_n$ nonzero.

Z_ρ has the same asymptotic distribution as the variable described under Case 3 in Table B.8.

Z_t and the augmented Dickey-Fuller t test have the same asymptotic distribution as the variable described under Case 3 in Table B.9.

Notes to Table 19.1

Estimated cointegrating regression indicates the form in which the regression that could describe the cointegrating relation is estimated, using observations $t = 1, 2, \ldots, T$.

True process describes the null hypothesis under which the distribution is calculated. In each case, $\boldsymbol{\varepsilon}_t$ is assumed to be i.i.d. with mean zero, positive definite variance-covariance matrix, and finite fourth moments, and the sequence $\{s \cdot \mathbf{\Psi}_s\}_{s=0}^{\infty}$ is absolutely summable. The matrix $\mathbf{\Psi}(1)$ is assumed to be nonsingular, meaning that the vector $\mathbf{y}_t$ is not cointegrated under the null hypothesis. If the test statistic is below the indicated critical value (that is, if Z_ρ, Z_t, or t is negative and sufficiently large in absolute value), then the null hypothesis of no cointegration is rejected.

Z_ρ is the following statistic,

$$Z_\rho \equiv (T-1)(\hat{\rho}_T - 1) - (1/2)\{(T-1)^2 \cdot \hat{\sigma}^2_{\hat{\rho}_T} \div s_T^2\}(\hat{\lambda}_T^2 - \hat{c}_{0,T}),$$

where $\hat{\rho}_T$ is the estimate of ρ based on OLS estimation of $\hat{u}_t = \rho \hat{u}_{t-1} + e_t$ for $\hat{u}_t$ the OLS sample residual value for Z_p is -27.1. Since $-7.54 > -27.1$, the null hypothesis of no cointegration is accepted. Similarly, the Phillips-Ouliaris Z_t statistic is

$$\begin{aligned} Z_t &= (\hat{c}_0/\hat{\lambda}^2)^{1/2}(\hat{\rho} - 1)/\hat{\sigma}_{\hat{\rho}} - (1/2)\{(T - 1)\cdot\hat{\sigma}_{\hat{\rho}} \div s\}(\hat{\lambda}^2 - \hat{c}_0)/\hat{\lambda} \\ &= \{(0.1622)/(0.4082)\}^{1/2}(0.98331 - 1)/(0.01172) \\ &\quad - \tfrac{1}{2}\{(201)(0.01172) \div (0.40374)\}(0.4082 - 0.1622)/(0.4082)^{1/2} \\ &= -2.02. \end{aligned}$$

Comparing this with the case 3 section of Table B.9, we see that $-2.02 > -3.80$, so that the null hypothesis of no cointegration is also accepted by this test. An *OLS* regression of $\hat{u}_t$ on $\hat{u}_{t-1}$ and twelve lags of $\Delta\hat{u}_{t-j}$ produces an *OLS* t test of $\rho = 1$ of -2.73, which is again above -3.80. We thus find little evidence that p_t, s_t, and p_t^* are cointegrated. Indeed, the regression [19.2.49] displays the classic symptoms of a spurious regression—the estimated standard errors are small relative to the coefficient estimates, and the estimated first-order autocorrelation of the residuals is near unity.

As a second example, Figure 19.5 plots 100 times the logs of real quarterly aggregate personal disposable income (y_t) and personal consumption expenditures (c_t) for the United States over 1947:I to 1989:III. In a regression of y_t on a constant, a time trend, y_{t-1}, and Δy_{t-j} for $j = 1, 2, \ldots, 6$, the *OLS* t test that the coefficient on y_{t-1} is unity is -1.28. Similarly, in a regression of c_t on a constant, a time trend, c_{t-1}, and Δc_{t-j} for $j = 1, 2, \ldots, 6$, the *OLS* t test that the coefficient on c_{t-1} is unity is -1.88. Thus, both processes might well be described as $I(1)$ with positive drift.

The *OLS* estimate of the cointegrating relation is

$$c_t = \underset{(2.35)}{0.67} + \underset{(0.0032)}{0.9865}\, y_t + u_t. \qquad [19.2.50]$$

A first-order autoregression fitted to the residuals produces

$$\hat{u}_t = \underset{(0.048)}{0.782}\, \hat{u}_{t-1} + \hat{e}_t,$$

Notes to Table 19.1 (continued).

from the estimated regression. Here,

$$s_T^2 = (T - 2)^{-1} \sum_{t=2}^{T} \hat{e}_t^2,$$

where $\hat{e}_t = \hat{u}_t - \hat{\rho}_T\hat{u}_{t-1}$ is the sample residual from the autoregression describing $\hat{u}_t$ and $\hat{\sigma}_{\hat{\rho}_T}$ is the standard error for $\hat{\rho}_T$ as calculated by the usual *OLS* formula:

$$\hat{\sigma}_{\hat{\rho}_T}^2 = s_T^2 \div \sum_{t=2}^{T} \hat{u}_{t-1}^2.$$

Also,

$$\hat{c}_{j,T} = (T - 1)^{-1} \sum_{t=j+2}^{T} \hat{e}_t\hat{e}_{t-j}$$

$$\hat{\lambda}_T^2 = \hat{c}_{0,T} + 2\cdot\sum_{j=1}^{q} [1 - j/(q + 1)]\hat{c}_{j,T}.$$

Z_t is the following statistic:

$$Z_t = (\hat{c}_{0,T}/\hat{\lambda}_T^2)^{1/2}\cdot(\hat{\rho}_T - 1)/\hat{\sigma}_{\hat{\rho}_T} - (1/2)(\hat{\lambda}_T^2 - \hat{c}_{0,T})(1/\hat{\lambda}_T)\{(T - 1)\cdot\hat{\sigma}_{\hat{\rho}_T} \div s_T\}.$$

Augmented Dickey-Fuller t statistic is the *OLS* t test of the null hypothesis that $\rho = 1$ in the regression

$$\hat{u}_t = \zeta_1\Delta\hat{u}_{t-1} + \zeta_2\Delta\hat{u}_{t-2} + \cdots + \zeta_{p-1}\Delta\hat{u}_{t-p+1} + \rho\hat{u}_{t-1} + e_t.$$

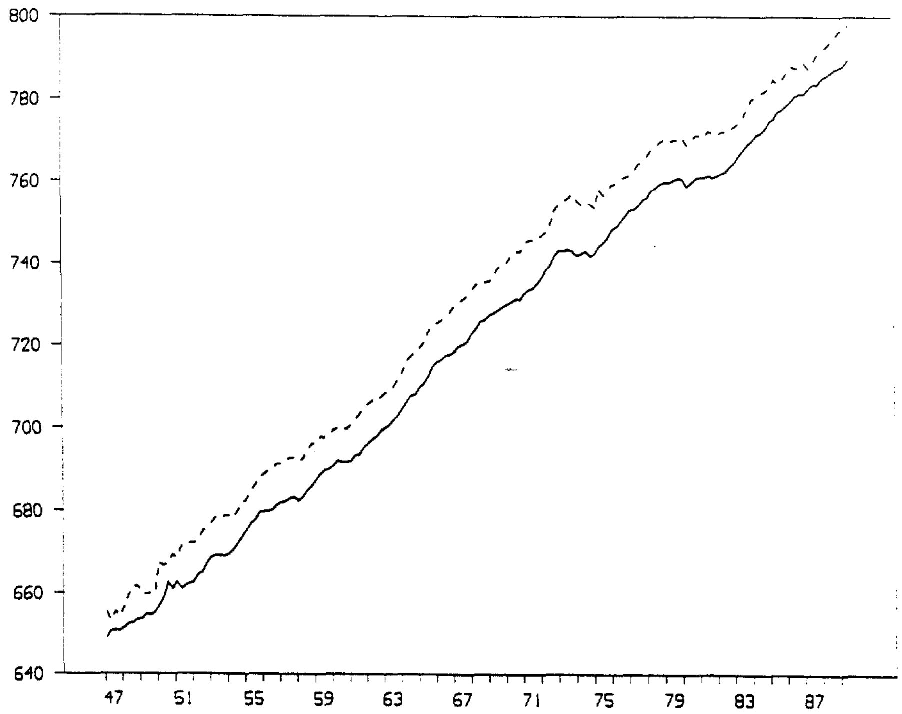

FIGURE 19.5 One hundred times the log of personal consumption expenditures (c_t) and personal disposable income (y_t) for the United States in billions of 1982 dollars, quarterly, 1947–89. Key: ——— c_t; ---- y_t.

for which the corresponding Z_ρ and Z_t statistics for $q = 6$ are -32.0 and -4.28. Since there is again ample evidence that y_t has positive drift, these are to be compared with the case 3 sections of Tables B.8 and B.9, respectively. Since $-32.0 < -21.5$ and $-4.28 < -3.42$, in each case the null hypothesis of no cointegration is rejected at the 5% level. Thus consumption and income appear to be cointegrated.

Other Tests for Cointegration

The tests that have been discussed in this section are based on the residuals from an *OLS* regression of y_{1t} on $(y_{2t}, y_{3t}, \ldots, y_{nt})$. Since these are not the same as the residuals from a regression of y_{2t} on $(y_{1t}, y_{3t}, \ldots, y_{nt})$, the tests can give different answers depending on which variable is labeled y_1. Important tests for cointegration that are invariant to the ordering of variables are the full-information maximum likelihood test of Johansen (1988, 1991) and the related tests of Stock and Watson (1988) and Ahn and Reinsel (1990). These will be discussed in Chapter 20. Other useful tests for cointegration have been proposed by Phillips and Ouliaris (1990), Park, Ouliaris, and Choi (1988), Stock (1990), and Hansen (1990).

19.3. *Testing Hypotheses About the Cointegrating Vector*

The previous section described some ways to test whether a vector $\mathbf{y}_t$ is cointegrated. It was noted that if $\mathbf{y}_t$ is cointegrated, then a consistent estimate of the cointegrating

```
OLSQ DCONS  LCONS(-1) C TIME;
CDF(DICKEYF) @T(1) ;
```

The resulting statistic for this example was -1.84 with a corresponding asymptotic P-value of .69, so the null of a unit root is accepted at the .05 level. Note that if we had used the conventional t-table to evaluate this P-value, we might have rejected this hypothesis. Options for CDF exist which allow you to compute the P-values without assuming the presence of a trend or constant. See the references and the *Reference Manual* for further details. Also be aware that the residuals from the Dickey-Fuller regression should be serially uncorrelated for the test to be valid, although they do not generally need to be homoskedastic (Phillips 1987). The Weighted Symmetric test is recommended over the Dickey-Fuller test, because it has (sometimes only slightly) higher power. That is, the WS test is more likely to reject the unit root (null hypothesis) when it is in fact false. The Phillips-Perron test is a variant of the Dickey-Fuller which tackles the problem of additional serial correlation in the residuals by using a GMM-type method to compute a residual variance that is "robust" to autocorrelation.

The cointegration of time series is a methodology for the analysis of time series pioneered by Engle and Granger (1987). Two or more time series are said to be *cointegrated* if a linear combination of them is I(0) (is stationary, or has all roots outside the unit circle) even though individually they are each I(1). Thus the hypothesis of cointegration consists of two parts: tests for I(1) of the individual series and I(0) of a linear combination. Usually the term cointegration testing refers only to the second part of the hypothesis; the test is performed *conditional* on the fact that each component series is I(1). Although this hypothesis sounds quite different from the hypothesis of a unit root, the practice of testing for cointegration is quite similar, and TSP provides the P-values for the Engle-Granger versions of these tests in the CDF procedure under the DICKEYF option.

As an example, consider testing that real consumption and real GNP from the Illustrative Model are cointegrated. It is easy to establish that each is I(1) separately (with asymptotic P-values of .69 and .66). The TSP commands to evaluate the second part of the hypothesis are the following:

```
SMPL 49 75 ;
COINT(ALLORD) LCONS LGNP;   ? this also performs the individual unit root tests at the same time
```

When LCONS is the dependent variable of the cointegrating regression, COINT chooses 2 augmenting lags, and obtains a test statistic of -1.65, which has a P-value of .89 . When LGNP is the dependent variable, 10 augmenting lags are chosen, and the test statistic and P-value are -1.29 and .95 respectively. So the null hypothesis of a unit root in the cointegrating regression cannot be rejected at the .05 level in either test. We can conclude that the linear combination of LCONS and LGNP is not I(0), so they are not cointegrated (at this significance level).

If done manually (without augmentation, and only shown for LCONS as the dependent variable):

```
SMPL 49 75 ;
OLSQ LCONS LGNP C TIME ;    ? the cointegrating regression
SMPL 50 75 ;
DRES = @RES-@RES(-1) ;
OLSQ DRES @RES(-1) ;    ? Engle-Granger test (Dickey-Fuller on residuals from cointegrating regression)
CDF(DICKEYF,NVAR=2) @T ;
```

We regress consumption on a constant, time, and GNP to obtain the cointegrating vector, construct the residuals from this vector, and then regress the first-differenced residuals on the lagged residual. Under the hypothesis of stationarity, the coefficient on this variable should be -1; the t-statistic for this hypothesis is the Engle-Granger statistic. One complication is that the actual value of the Engle-Granger statistic (although not its distribution) will be affected by the choice of left-hand variable in the first regression (consumption or GNP); the COINT(ALLORD) will try both.

To compute the asymptotic P-value manually for the Engle-Granger statistic, use the DICKEYF option of the CDF procedure with the NVAR option to specify the number of cointegrating variables used in computing the test statistic. TSP provides P-values for cointegrating regressions with up to 6 variables, using the response surface estimates given

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