exists $\delta(\epsilon) > 0$ such that if $|b_{ni}(\omega) - c_{ni}| < \delta(\epsilon)$, $i = 1, \ldots, k$, then $|g_j(b_n(\omega)) - g_j(c_n)| < \epsilon$. Since $|b_{ni}(\omega) - c_{ni}| < \delta(\epsilon)$ for all n sufficiently large and almost every ω, then $|g_j(b_n(\omega)) - g_j(c_n)| < \epsilon$ for all n sufficiently large and almost every ω. Hence $g(b_n(\omega)) - g(c_n) \xrightarrow{\text{a.s.}} 0$.

To state the results for the OLS and IV estimators below concisely, we define the following concepts, as given by White [1982, pp 484–485].

DEFINITION 2.17: A sequence of $k \times k$ matrices $\{\mathbf{A}_n\}$ is said to be *uniformly nonsingular* if and only if for some $\delta > 0$ and all n sufficiently large $|\det \mathbf{A}_n| > \delta$. If $\{\mathbf{A}_n\}$ is a sequence of positive semidefinite matrices, then $\{\mathbf{A}_n\}$ is *uniformly positive definite* if and only if $\{\mathbf{A}_n\}$ is uniformly nonsingular. If $\{\mathbf{A}_n\}$ is a sequence of $l \times k$ matrices, then $\{\mathbf{A}_n\}$ has *uniformly full column rank* if and only if there exists a sequence of $k \times k$ submatrices $\{\mathbf{A}_n^*\}$ which is uniformly nonsingular.

If a sequence of matrices is uniformly nonsingular, the elements of the sequence are prevented from getting "too close" to singularity. Similarity, if a sequence of matrices has uniformly full column rank, the elements of the sequence are prevented from getting "too close" to a matrix with less than full column rank.

Next we state the desired extensions of Theorem 2.12 and Exercise 2.13.

THEOREM 2.18: Suppose

(i) $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}_o + \boldsymbol{\epsilon}$;
(ii) $\mathbf{X}'\boldsymbol{\epsilon}/n \xrightarrow{\text{a.s.}} \mathbf{0}$;
(iii) $\mathbf{X}'\mathbf{X}/n - \mathbf{M}_n \xrightarrow{\text{a.s.}} \mathbf{0}$, where $\{\mathbf{M}_n\}$ is O(1) and uniformly positive definite.

Then $\hat{\boldsymbol{\beta}}_n$ exists a.s. for all n sufficient by large and $\hat{\boldsymbol{\beta}}_n \xrightarrow{\text{a.s.}} \boldsymbol{\beta}_0$.

Proof: Because $\mathbf{M}_n$ is O(1) for all n sufficiently large, it follows from Proposition 2.16 that $\det(\mathbf{X}'\mathbf{X}/n) - \det \mathbf{M}_n \xrightarrow{\text{a.s.}} \mathbf{0}$. Since $\det \mathbf{M}_n > \delta > 0$ for all n sufficiently large by Definition 2.17, it follows that $\det(\mathbf{X}'\mathbf{X}/n) > \delta/2 > 0$ for all n sufficiently large almost surely, so that $(\mathbf{X}'\mathbf{X}/n)^{-1}$ exists a.s. for all n sufficiently large. Hence $\hat{\boldsymbol{\beta}}_n \equiv (\mathbf{X}'\mathbf{X}/n)^{-1}\mathbf{X}'\mathbf{y}/n$ exists a.s. for all n sufficiently large.

Now $\hat{\boldsymbol{\beta}}_n = \boldsymbol{\beta}_o + (\mathbf{X}'\mathbf{X}/n)^{-1}\mathbf{X}'\boldsymbol{\epsilon}/n$ by (i). It follows from Proposition 2.16 that $\hat{\boldsymbol{\beta}}_n - (\boldsymbol{\beta}_o + \mathbf{M}_n^{-1} \cdot \mathbf{0}) \xrightarrow{\text{a.s.}} \mathbf{0}$ or $\hat{\boldsymbol{\beta}}_n \xrightarrow{\text{a.s.}} \boldsymbol{\beta}_o$, given (ii) and (iii).

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$\{f_n(\lambda)\}$ of characteristic
n $f(\lambda)$, then the conver-
$\mathcal{R}^k$.

of Lukacs [1970, p. 50].

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dom $k \times 1$ vectors with
$\lambda) \to f(\lambda)$. If $\{A_n\}$ is any
h that $\{A_n\}$ is O(1), then
ions $\{f_n^*(\theta)\}$, where θ is
$\to 0$.

). For fixed θ, $\lambda_n \equiv A_n\theta$
$\mathcal{N}_\theta$, for all n sufficiently

large because $\{A_n\}$ is O(1). Because $f_n(\lambda) \to f(\lambda)$, we have $f_n(\lambda_n) - f(\lambda_n) \to 0$ uniformly for all λ_n in $\mathcal{N}_\theta$, by Corollary 4.20. Hence for fixed θ, $f_n(A_n\theta) - f(A_n\theta) = f_n^*(\theta) - f(A_n\theta) \to 0$ for any O(1) sequence $\{A_n\}$. Because θ is arbitrary, the result follows.

The following consequence of this result is used many times below.

COROLLARY 4.24: Let $\{b_n\}$ be a sequence of random $k \times 1$ vectors such that $V_n^{-1/2}b_n \overset{A}{\sim} N(\mathbf{0}, \mathbf{I})$, where $\{V_n\}$ and $\{V_n^{-1}\}$ are O(1). Let $\{A_n\}$ be a O(1) sequence of (nonstochastic) $k \times q$ matrices with full column rank q for all n sufficiently large, uniformly in n. Then the sequence $\{A_n'b_n\}$ is such that $\Gamma_n^{-1/2}A_n'b_n \overset{A}{\sim} N(\mathbf{0}, \mathbf{I})$, where $\Gamma_n \equiv A_n'V_nA_n$ and $\{\Gamma_n\}$ and $\{\Gamma_n^{-1}\}$ are O(1).

Proof: $\{\Gamma_n\}$ is O(1) by Proposition 2.30. $\{\Gamma_n^{-1}\}$ is O(1) because $\{\Gamma_n\}$ is O(1) and $\det \Gamma_n > \delta > 0$ for all n sufficiently large, given the conditions on $\{A_n\}$ and $\{V_n\}$. Let $f_n^*(\theta)$ be the characteristic function of $\Gamma_n^{-1/2}A_n'b_n = \Gamma_n^{-1/2}A_n'V_n^{1/2}V_n^{-1/2}b_n$. Because $\{\Gamma_n^{-1/2}A_n'V_n^{1/2}\}$ is O(1), Lemma 4.23 applies, implying $f_n^*(\theta) - f(V_n^{1/2}A_n\Gamma_n^{-1/2}\theta) \to 0$, where $f(\lambda) = \exp(-\lambda'\lambda/2)$, the limiting characteristic function of $V_n^{-1/2}b_n$. Now $f(V_n^{1/2}A_n\Gamma_n^{-1/2}\theta) = \exp(-\theta'\Gamma_n^{-1/2}A_n'V_nA_n\Gamma_n^{-1/2}\theta/2) = \exp(-\theta'\theta/2)$ by definition of $\Gamma_n^{-1/2}$. Hence $f_n^*(\lambda) - \exp(-\theta'\theta/2) \to 0$, so $\Gamma_n^{-1/2}A_n'b_n \overset{A}{\sim} N(\mathbf{0}, \mathbf{I})$ by the continuity theorem 4.17.

This result allows us to complete the proof of the following very general asymptotic normality result for the least squares estimator.

THEOREM 4.25: Given

(i) $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}_0 + \boldsymbol{\epsilon}$;
(ii) $\mathbf{V}_n^{-1/2}n^{-1/2}\mathbf{X}'\boldsymbol{\epsilon} \overset{A}{\sim} N(\mathbf{0}, \mathbf{I})$, where $\mathbf{V}_n \equiv \text{var}(n^{-1/2}\mathbf{X}'\boldsymbol{\epsilon})$ is O(1) and uniformly positive definite;
(iii) $\mathbf{X}'\mathbf{X}/n - \mathbf{M}_n \xrightarrow{p} \mathbf{0}$, where $\mathbf{M}_n \equiv E(\mathbf{X}'\mathbf{X}/n)$ is O(1) and uniformly positive definite.

Then $\mathbf{D}_n^{-1/2}\sqrt{n}(\hat{\boldsymbol{\beta}}_n - \boldsymbol{\beta}_0) \overset{A}{\sim} N(\mathbf{0}, \mathbf{I})$, where $\mathbf{D}_n \equiv \mathbf{M}_n^{-1}\mathbf{V}_n\mathbf{M}_n^{-1}$. Suppose in addition that

(iv) there exists $\hat{\mathbf{V}}_n$ positive semidefinite and symmetric such that $\hat{\mathbf{V}}_n - \mathbf{V}_n \xrightarrow{p} \mathbf{0}$. Then $\hat{\mathbf{D}}_n - \mathbf{D}_n \xrightarrow{p} \mathbf{0}$, where $\hat{\mathbf{D}}_n \equiv (\mathbf{X}'\mathbf{X}/n)^{-1}\hat{\mathbf{V}}_n(\mathbf{X}'\mathbf{X}/n)^{-1}$.

Proof: Because $\mathbf{X}'\mathbf{X}/n - \mathbf{M}_n \xrightarrow{p} \mathbf{0}$ and $\mathbf{M}_n$ is finite and nonsingular

Applied Econometric Time Series

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In practice, the choice of the most appropriate test can be difficult since you never know the true data-generating process. A safe choice is to use both types of unit roots tests. If they reinforce each other, you can have confidence in the results. Sometimes, economic theory will be helpful in that it suggests the most appropriate test. In the Corbae and Ouliaris example, excess returns should be positively correlated; hence, the Phillips–Perron test is a reasonable choice.

6. STRUCTURAL CHANGE

In performing unit root tests, special care must be taken if it is suspected that structural change has occurred. When there are structural breaks, the various Dickey–Fuller and Phillips–Perron test statistics are biased toward the nonrejection of a unit root. To explain, consider the situation in which there is a one-time change in the mean of an otherwise stationary sequence. In the top graph (a) of Figure 4.4, the $\{y_t\}$ sequence was constructed so as to be stationary around a mean of zero for $t = 0, \ldots, 50$ and then to fluctuate around a mean of 6 for $t = 51, \ldots, 100$. The sequence was formed by drawing 100 normally and independently distributed values for the $\{\epsilon_t\}$ sequence. By setting $y_0 = 0$, the next 100 values in the sequence were generated using the formula:

$$y_t = 0.5y_{t-1} + \epsilon_t + D_L \tag{4.29}$$

where D_L is a dummy variable such that $D_L = 0$ for $t = 1, \ldots, 50$ and $D_L = 3$ for $t = 51, \ldots, 100$. The subscript L is designed to indicate that the *level* of the dummy changes. At times, it will be convenient to refer to the value of the dummy variable in period t as $D_L(t)$; in the example at hand, $D_L(50) = 0$ and $D_L(51) = 3$.

In practice, the structural change may not be as apparent as the break shown in the figure. However, the large simulated break is useful for illustrating the problem of using a Dickey–Fuller test in such circumstances. The straight line shown in the figure highlights the fact that the series appears to have a deterministic trend. In fact, the straight line is the best-fitting OLS equation:

$$y_t = a_0 + a_2 t + e_t$$

In the figure, you can see that the fitted value of a_0 is negative and the fitted value of a_2 is positive. The proper way to estimate (4.29) is to fit a simple AR(1) model and allow the intercept to change by including the dummy variable D_L. However, suppose that we unsuspectingly fit the regression equation:

$$y_t = a_0 + a_1 y_{t-1} + e_t \tag{4.30}$$

As you can infer from Figure 4.4, the estimated value of a_1 is necessarily biased toward unity. The reason for this upward bias is that the estimated value of a_1 cap-

Productivity growth in Canadian telecommunications

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Abstract. Canadian telecommunications firms do not price proportionately to marginal cost. The prices of toll services tend to be above marginal costs, whereas the prices of basic local services are typically set below marginal costs by regulators. In such circumstances, estimates of TFP growth using the conventional Törnqvist (Divisia) formula which weights outputs by revenue shares in determining the rate of growth of aggregate output is theoretically incorrect and needs to be replaced by a formula which uses cost elasticity weights. Empirically, the conventional Törnqvist index yields a very distorted picture of efficiency gains in the two largest Canadian telephone companies during the 1980s. For Bell Canada, I calculate the upward bias to be approximately 75 per cent over the period 1980–9 and 80 per cent over the period 1985-9. For B.C. Tel a similar calculation yields an upward bias of 37 per cent over the period 1980–9 and 48 per cent over the period 1985–9.

La croissance de la productivité dans les télécommunications au Canada. Les entreprises dans le monde des télécommunications au Canada ne pratiquent pas une tarification proportionnelle au coût marginal. Les prix des services interurbains tendent à s'établir au dessus des coûts marginaux tandis que les prix des services de base sont généralement fixés au dessous des coûts marginaux par les agences de réglementation. Dans ces circonstances, les évaluations de la croissance de la productivité totale des facteurs de production fondées sur la formule Törnquist (Divisia) – qui pondère les extrants selon la portion des revenus qu'ils engendrent dans la détermination du taux de croissance de la production agrégée – est incorrecte au plan théorique et doit être remplacée par une formule qui utilise des pondérations définies à partir des élasticités des coûts. Empiriquement, l'indice Törnquist donne une image distorsionnée des gains d'efficacité des deux grandes entreprises canadiennes de téléphone au cours des années 1980. Pour Bell Canada, l'auteur calcule un biais à la hausse d'à peu

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près 75 pour cent pour la période 1980–9 et de 80 pour cent pour la période 1985–9. Pour B.C. Tel, des calculs similaries montrent un biais à la hausse de 37 pour cent pour la période 1980–9 et de 48 pour cent pour la période 1985–9.

I. INTRODUCTION

Empirical estimates of productivity growth play an increasing role in the regulation of telecommunications, one of the most important service industries in industrialized economies. This is true whether rates (prices) are regulated through the utilization of traditional rate-of-return/rate-structure considerations or through the newly emerging 'price caps' form of regulation. In the recently completed Canadian Radio-Television and Telecommunications Commission (CRTC) Hearings (CRTC) 1992 regarding the possibility of competition in public long-distance telephone service (hereafter denoted Interconnect Hearings), productivity growth was a central area of deliberation as a part of an attempt to evaluate both the past and forecasted future performances of monopoly providers of toll service and the effect on industry performance of the introduction of competition.

The voluminous data submitted by Bell Canada and British Columbia Telephone (B.C. Tel) during the Interconnect Hearings provide us with an opportunity to update through 1989 previous published estimates of total factor productivity (TFP) growth for Canadian telecommunications, the most current of which stopped in 1980 (Denny, Fuss, and Waverman 1981; Denny, Fuss, Everson, and Waverman 1981; Denny, de Fontenay, and Werner 1983a; Kiss 1983; Bernstein 1989.

Both Bell Canada and B.C. Tel have continued to present estimates of TFP growth in various rate hearings during the 1980s, using the conventional Törnqvist (Divisia) formula which weights outputs by revenue shares in determining the rate of growth of aggregate output. One of the characteristics of the regulatory environment within which Canadian telecommunications firms operate is that firms do not price proportionately to marginal cost. The prices of toll services tend to be above marginal costs, whereas the prices of local services are typically set by the regulatory authorities below marginal costs.[1] In such circumstances, where prices are not set proportional to marginal costs and when the goal of TFP growth measurement is to calculate the growth in production efficiency (as it usually is),[2] the use of revenue share weights is theoretically incorrect and needs to be replaced,

1 This is a general statement for the broad service categories – local and toll. It will not be true for all subaggregates. For example, the enhanced services category of local service is probably priced above marginal cost.

2 The growth rate of production efficiency in this context refers to the net effects of technical change and efficiency improvements due to output expansion in the presence of increasing returns to scale. Intertemporal improvements in X-inefficiency will be included in the empirical TFP growth numbers, even though the underlying theoretical model assumes production takes place on the efficient frontier. Changes in allocative efficiency due to movements of prices in general towards marginal costs will not be included, except indirectly to the extent that lower prices induce output expansion in the presence of increasing returns.

when possible, by cost elasticity weights (Caves and Christensen 1980; Denny, Fuss, and Waverman 1981).[3]

Denny, Fuss, and Waverman (1981, table 12) show that for Bell Canada over the period 1952–76, use the conventional measure leads to an upward bias of 19 per cent in the estimation of annual efficiency gains.[4] One of the main results of this paper is the fact that during the 1980s the upward bias increased substantially, so that the conventional TFP index yields a very distorted picture of efficiency gains in the two largest Canadian telephone companies. For Bell Canada, I calculate the upward bias to be approximately 75 per cent over the period 1980–9 and 80 per cent over the period of 1985–9. For B.C. Tel a similar calculation yields an upward bias of 37 per cent over the period 1980–9 and 48 per cent over the period 1985–9.

II. REVENUE WEIGHTS VERSUS COST WEIGHT IN THE MEASUREMENT OF TFP GROWTH

The majority of Canadian telephone companies calculate TFP growth employing what has been called the 'conventional' Törnqvist (Divisia) index of TFP (Denny, Fuss, and Waverman 1981). This conventional measure has been used extensively in the calculation of TFP in studies of the overall economy, major sectors such as manufacturing, and subsectors like the two-digit manufacturing industries (e.g., food and beverages). The appropriateness of its use in such settings is now well established, in both academic circles and government statistical agencies. But this conventional measure is not appropriate (from a conceptual perspective) for determining productivity growth in the case of telephone companies such as Bell Canada and B.C. Tel. The latter statement is true for any situation similar to that found in telecommunications – substantial departures from price/marginal cost proportionality and unequal growth rates of outputs.

The conventional Törnqvist index for measuring TFP growth between years $t-1$ and t is calculated from the log difference formula:

$$\Delta \log \mathrm{TFP}^R = \Delta \log Q^R - \Delta \log X, \tag{1}$$

where

$$\Delta \log Q^R = \Sigma(1/2)\cdot(R_{it} + R_{i,t-1})\cdot[\log Q_{it} - \log Q_{i,t-1}] \tag{2}$$

$$\Delta \log X = \Sigma(1/2)\cdot(S_{it} + S_{i,t-1})\cdot[\log X_{it} - \log X_{i,t-1}]. \tag{3}$$

3 The change in weighting procedures is the result of replacing output prices by marginal costs in the conventional Törnqvist (Divisia) formula. The analyses of Caves and Christensen (1980) and Denny, Fuss, and Waverman (1981) are specific to this functional form. For a theoretical justification of the statement in the text as applied to any exact index number aggregation formula see Diewert (1991, theorem 1). In CRTC (1992) the commission concluded that cost weights are superior to revenue weights for the reasons stated in this paper, and it appeared to direct the telephone companies to provide cost-weighted TFP growth estimates in future hearings.

4 The conventional (revenue weighted) measure was 3.35 per cent per annum, whereas the cost elasticity weighted measure was 2.81 per cent per annum, resulting in an upward bias of $(3.35 - 2.81)/2.81 = 0.19$.

Q_{it} is the amount of the ith output produced at time t;
X_{it} is the amount of the ith input utilized at time t;
R_{it} is the revenue share of the ith output in total revenue;
S_{it} is the cost share of the ith input in total cost.

The superscript R indicates that revenue weights are used in the calculation of $\Delta \log Q$. $\Delta \log Q$ and $\Delta \log X$ are often referred to as the rates of change of aggregate output and aggregate input, respectively, since they result from procedures that aggregate the rates of change of individual outputs and inputs.

It has been recognized since the late 1970s that a crucial assumption used in establishing the linkage between $\Delta \log \text{TFP}^R$ and the annual change in production efficiency is that output prices are in the *same* proportion to marginal costs for all outputs at any point in time. This is an inappropriate assumption in the case of many regulated firms, including Canadian telecommunications firms, since for these firms the price of toll output (as a broad service category) exceeds the marginal cost of toll production, and the local service price (including the access price[5]) is less than the relevant marginal cost. Empirical support for these assertions can be found in Fuss and Waverman (1981a,b), where Bell Canada prices are compared with econometric estimates of marginal costs. This pattern of cross-subsidization eliminates any possibility that the proportionality assumption could be satisfied in the historical data.

In an appendix I formally demonstrate the fact that equation (1) measures production efficiency growth only when the price/marginal-cost proportionality assumption holds. I also demonstrate that, when this assumption does not hold, the correct form of the TFP index for a cost-minimizing firm is

$$\Delta \log \text{TFP}^C = \Delta \log Q^C - \Delta \log X, \tag{4}$$

where

$$\Delta \log Q^C = \Sigma(1/2) \cdot (M_{it} + M_{i,t-1}) \cdot [\log Q_{it} - \log Q_{i,t-1}], \tag{5}$$

and M_{it} is the cost elasticity of the ith output divided by the sum of the cost elasticities, summed over all outputs. M_{it} is denoted a 'cost elasticity share' to distinguish it from the cost elasticity itself.

The above definition of $\Delta \log Q^c$ differs from that found in Caves and Christensen (1980), who replace the revenue shares with cost elasticities rather than cost elasticity shares. Whether the average cost elasticities or average cost elasticity shares are the correct weights for weighting the rates of growth of the individual outputs depends on the definition of TFP utilized. If productivity growth associated with scale economies is excluded from the definition, the correct weight is the cost elasticity. This will be the case when TFP growth and technical change are by definition

5 Access is included as one of the outputs in the subaggregate 'local' in the productivity accounts of most Canadian telephone companies, including Bell Canada and B.C. Tel.

synonymous. If scale economies are included as a potential source of TFP growth, the correct weight is the cost elasticity share.[6] Both definitions of TFP growth can be found in the productivity literature. However, Canadian telecommunications TFP growth rate estimates produced by both academics and the telephone companies are clearly intended to include any productivity change due to non-constant returns to scale in their measures (see Kiss 1983). Therefore the aggregation weights that are appropriate for providing estimates of TFP growth rates for Bell Canada and B.C. Tel involve the use of cost elasticity shares.

Which cost elasticity shares to use depends on whether one believes a short-run model (with capital quasi-fixed) or a long-run model (with capital variable) is appropriate. From the point of view of TFP growth measurement, the difference in model implies a difference in the valuation of the capital input, both in the calculation of the output cost elasticity aggregation weights and the input aggregation weights. For the long-run model, the capital input is valued at its user cost. For the short-run model, the price of capital services to be used is its shadow value at the point of temporary equilibrium.[7] In this paper, I present estimates for Bell Canada based on both models. For B.C. Tel, only estimates based on the short-run model are possible, since no user cost data are available.[8]

One of the main objections to the use of cost weights that has been raised is the fact that unlike revenue shares, cost elasticity shares, whether based on user cost or shadow-price valuation of the capital input, are not directly measured in the basic productivity data provided by the telephone companies. In section IV I discuss the measurement of these shares. Before turning to that issue, however, I provide an outline of the data used in this study.

III. DATA AND DATA SOURCES

The basic data on the prices and quantities of outputs and inputs are taken from

6 When production is subject to constant returns to scale, scale economies do not contribute to TFP growth and the two possible definitions of TFP growth coincide. Also, this is the case where the cost elasticity share and the cost elasticity are identical, since with constant returns to scale the cost elasticities sum to unity. The above statements are directly applicable to a long-run equilibrium analysis. With respect to a short-run equilibrium model, these statements remain valid as long as the output cost elasticities are based on shadow-price valuation of the quasi-fixed inputs. (See fn7 for further elaboration on the different implications for TFP measurement of long-run versus short-run equilibrium models.)

7 Most TFP growth calculations for telecommunications are based implicitly on a long-run model, since the user cost of capital services is used for the capital input price. But, as Bernstein (1988, 1989) has emphasized, given the capital-intensive nature of the production process, it may be that the short-run model is more appropriate. In the single output case Berndt and Fuss (1986) have shown that the short-run model can be implemented through the replacement of the user cost of capital by a shadow value in the calculation of the input aggregation weights. Berndt and Fuss (1989) extend the analysis to the multi-output case and demonstrate that the appropriate procedure is, in addition, to replace the user cost with the shadow price in the calculation of the cost-elasticity shares.

8 It may seem strange that user costs are unavailable but shadow values can be obtained. As discussed below, I shall be approximating shadow values by using residual rates of return, and these data can be obtained from the productivity data bases.

Bell Canada and B.C. Tel's productivity studies, which were submitted in evidence at the Interconnection Hearings.[9] The Bell Canada data cover the period 1952–89 and represent a revision and update for the 1980s of data used by Denny, Fuss, and Waverman (1981), Kiss (1983), and Bernstein (1988, 1989). The reader is referred to these articles for an extensive discussion of the data. Prices and quantities (constant dollar outputs) are available for three categories of output: local services (monopoly local, access, competitive terminal), toll service (monopoly toll, competitive network), and miscellaneous service (all other) and three categories of input: labour, capital, and materials.[10] The B.C. Tel data cover the period 1981–9, and are constructed in a manner very similar to that of the Bell data, resulting in the same three output and same three input categories.[11]

The data used to calculate cost elasticity shares are the annual cost allocation studies,[12] which have been filed annually with the CRTC since 1979 (except 1985) by Bell Canada and intermittently since 1980 by B.C. Tel. A more extensive discussion of these data is contained in the next section of the paper.

IV. ESTIMATION OF COST ELASTICITY SHARES

What we require, at the most basic level, for each output to be aggregated is a datum that is proportional to the marginal cost of the output. As noted earlier, in the case of significant cross-subsidization such as that we find in Canadian telecommunications, the price of the output is a poor approximation of the required datum. This is why the revenue share is a poor choice as an aggregation weight and an attempt must be made to estimate the cost elasticity share M_{it} directly.

There are two procedures that have been used to estimate marginal cost (or equivalently the cost elasticity share). The first procedure is to estimate an econometric multiple output cost function. This procedure has been used by Denny, Fuss, and Waverman (1981), Caves and Christensen (1980), Caves, Christensen, and Swanson (1980), and Kim and Weiss (1989), among others. The second procedure is to utilize the results of cost allocation studies to approximate the cost elasticity weights. This procedure has not been employed previously in telecommunications, but was used by Christensen et al. (1985, 1990) in their analysis of the United States Postal Service TFP.[13]

Estimation of a multiple output cost function would appear to be an obvious way to obtain the needed cost weight information. The logarithmic derivatives of the cost function with respect to the individual outputs provide the cost elasticities needed to construct the cost elasticity weights. However, this method is not without its problems. For both the United States and Canada it has proved difficult to obtain well-behaved multiple output telecommunications cost function estimates with positive cost elasticities. This appears to be especially true when data for the 1980s are added to the sample.[14] Even when cost functions with satisfactory theoretical properties have been obtained, the economic characteristics estimated have remained a subject of controversy (see Kiss and Lefebvre 1987; Fuss 1992). Finally, with econometric cost functions, at most three aggregate cost elasticities can be obtained, whereas for the cost allocation data used in this study, seven cost elasticities can be obtained, permitting a more disaggregated analysis.[15]

In a similar kind of setting, Christensen et al. (1985) proposed as a practical empirical approximation to the required marginal cost datum the average (unit) allocated cost obtained from cost allocation studies. The use of cost allocation studies relies on the fact that the methodology of cost allocation leads to careful attempts to allocate costs to service categories that are causally related to the production of those services. In Canadian telecommunications cost studies (unlike those undertaken in the United States), not all costs are allocated, since it is recognized that some costs, the 'common costs,' cannot be allocated on a conceptually sound basis. The procedures adopted by the CRTC use peak traffic in the allocation of usage sensitive costs and hence the costs allocated are more closely related to incremental costs than is the case in the United States.

It is well known that the use of allocated costs to proxy marginal costs can be problematic. Accounting procedures and economic causality do not always mesh. In addition, incremental cost may not be constant over the range of output considered. But the approximation has several advantages. The major advantage is that, despite the limitations of the cost allocation exercise, unit allocated costs can be expected to satisfy much more closely the proportionality requirement than prices, given the very large cross-subsidization from toll to local services, which is at the centre

9 All references to data submitted in evidence will use the notation adopted by the CRTC to identify documents. The basic Bell Canada productivity data can be found in Bell(Unitel)28Dec90-253. This document is Item No. 253 of the responses to interrogatories posed by Unitel to Bell on 28 December 1990. The basic B.C. Tel data can be found in B.C. Tel(CRTC)28Dec90-2214, Attachment 1.

10 A detailed categorization of outputs and inputs into the aggregates presented in the productivity data can be found in Bell(CRTC)15Feb91–3201, Supplemental. The categorization of outputs presented in the text is the current categorization. There have been some changes in terminology over the years, which are detailed in the cited Bell response to a CRTC interrogatory.

11 Further information on the methodology used to construct the B.C. Tel productivity data can be found in Olley and Le (1984).

12 These studies were called 'Five-Way Split' studies prior to 1986 and since 1986 have been called 'Phase III' studies.

13 The cost allocation data have also been used by Curien (1991) to study the pattern of cross-subsidies in the Canadian telecommunications industry. His paper contains an example of the kind of information that is available from typical cost-allocation studies of Bell Canada and B.C. Tel (see table 1 of his paper).

14 The two papers of which I am aware that estimate cost functions using Canadian data from the 1980s (Gentzoglanis and Cairns (1989), Ngo (1990)) are plagued with lack of regularity and/or cost elasticity estimates that are negative. Highly trended output data and inadequate technical change indicators appear to be particularly problematic with respect to the 1980s data. For discussions of difficulties with the U.S. data see Waverman (1989), Roller (1990) and Diewert and Wales (1991).

15 As discussed below, I only use three cost elasticities in this study due to limitations in the productivity data which are currently available.

of Canadian public policy towards telecommunications.[16] Hence TFP growth rates constructed from allocated cost weights will provide a more accurate picture of efficiency changes that TFP growth rates constructed from revenue weights.[17] As noted earlier, under the Phase III system currently in use by the CRTC, cost allocation can provide weights for as many as seven outputs, in contrast to the two or three potentially available from econometric cost functions.[18]

TFP estimates are often used by regulators and by the management of telecommunications carriers as one of the indicators of firm performance. (For a discussion of the use of TFP by management for planning purposes see Denny, de Fontenay, and Werner 1983b.) In this context, estimation of cost weights from cost allocation data as opposed to cost function estimation has a number of practical advantages. Like prices, unit allocated costs are auditable as part of the ongoing process of regulation. Allocated costs are available to the firm on a timely basis (unlike econometric estimates), so TFP measures constructed from cost weights can be used to track current improvements in a firm's efficiency. Finally, the regulatory authorities do not have to be involved in the difficult problems associated with the evaluation of econometric cost functions.

The cost elasticity shares M_{it} can be expressed in terms of original costs as

$$M_{it} = (Q_{it} \cdot \mathrm{MC}_{it})/(\Sigma Q_{it} \cdot \mathrm{MC}_{it}), \tag{6}$$

where MC_{it} is the marginal cost of the ith output at time t. Replacement of MC_{it} in (6) with a constant of proportionality times the average allocated cost (service category i) yields the alternative expression for M_{it},

16 I have compared, for Bell Canada, price/marginal cost ratios calculated for 1978 by Fuss and Waverman (1981b) with unit allocated cost/marginal cost ratios for 1979 calculated from Fuss and Waverman (1981b) and the cost allocation studies. The results were as follows:

	Local	*Toll*
Price / marginal cost	0.73	3.17
Unit allocated cost / marginal cost	1.04	1.33

17 One of the referees makes the valid point that the regulatory process may bias the cost weights based on cost-allocation studies away from the true cost weights in such a way as to lead to an overstatement of the bias associated with using revenue weights. Since rates for telecommunications services are based on allocated costs, the desire of the regulated firm (and perhaps the regulator) in recent years to rate rebalance (increase local rates and decrease toll rates) provides an incentive for the process to allocate excessive costs to local services. An example consistent with this incentive is the change in the procedure to allocate the gross receipts tax that occurred after 1981. This change in procedure resulted in a transfer to local services of costs that had previously been allocated to message toll services. When I made an approximate reallocation of this tax to try to maintain consistency over the whole 1979–89 period, however, the result was only a small change in the cost-weighted TFP growth rate. This reverse bias possibility, while certainly real, is likely to cause only a minor adjustment to the large biases that I calculate.

18 In practice, it appears that when well-behaved econometric cost functions can be obtained, the cost allocation and econometric cost function procedures result in similar adjustments to the revenue-weighted TFP numbers. In the empirical results presented below, I compare Bell Canada cost elasticity shares for 1976 derived from an econometric cost function, where marginal cost elasticities are estimated directly, with 1979 shares derived from cost allocation. The results are quite close.

$$M_{it} = \frac{\text{costs allocated to service category } i}{\text{total costs allocated (excluding Common)}} \tag{7}$$

$$= \text{allocated cost share (service category } i\text{)}. \tag{8}$$

Cost elasticity shares for the broad service categories toll, local (including access) and miscellaneous (other) were calculated, using equation (7), from Bell Canada's cost allocation studies for the years 1979–84 (Five Way Split) and 1986–9 (Phase III) and from B.C. Tel's cost allocation studies for the years 1980 and 1983 (Five Way Split) and 1986–9 (Phase III). The missing data in the 1979–89 period were obtained by interpolation. A more disaggregated breakdown of cost elasticity estimates corresponding to a finer division of outputs was not calculated because the telephone companies' public productivity accounts do not provide the necessary output data.[19]

I now turn to some of the details involved in the calculation of cost elasticity weights for Bell Canada. Bell Canada's cost allocation data are constructed so that total cost is equal to total revenue. This fact implies that capital expenditures are valued at the residual rate of return.[20] In order to construct cost elasticity shares appropriate for the long-run model, where capital should be priced at the user costs, capital expenditures were multiplied by the ratio of the user cost of capital to the per unit (of capital) residual return. The construction of the cost data appropriate for the short-run model was more problematic. Berndt and Fuss (1986) demonstrate that the expected shadow value is the correct price of capital to be used. In the case of competitive behaviour and constant returns to scale, Berndt and Fuss (1986) and Hulten (1986) show that the per unit residual return is a reasonable approximation to the expected shadow price. For telecommunications, constant returns to scale and competitive behaviour are not reasonable assumptions. Nevertheless an argument can be made that use of the residual rate of return will place a bound on the difference between the TFP growth rates calculated using the long-run and short-run models. This result is due to the nature of telecommunications regulation in Canada, in which the regulatory commission seeks to guarantee both the regulated firm and its customers that the firm will, over time, earn a rate of return close to its cost of capital. If the firm earns too high a return relative to its cost of capital, it can expect steps to be taken by the regulators to reduce that return (such as the rebates to customers required of Bell Canada in the late 1980s). If it earns too low a return, the firm will expect relief to be granted in the form of higher prices

19 The most important disaggregation for correcting the revenue-weighted TFP estimates is the toll/local split. The potentially next most important disaggregation (not available in the public data) is the split of local service into the categories of monopoly local, competitive terminal, and access. Monopoly local is probably priced above marginal cost and access is almost surely priced below marginal cost. Competitive terminal may be priced close to marginal cost.

20 This procedure is in contrast to Bell's productivity data, where the user cost of capital is the price of capital services and total revenue does not necessarily equal total cost.

for its services. As a consequence, the expected shadow value will be somewhere between the realized ex post return and the user cost. For Bell Canada, as I show below, the results are insensitive to whether we use the user cost of capital or the residual return to value capital. Hence the TFP growth rate and bias calculations over the 1980–9 period are robust to the model specification (short run versus long run).

For the more distant historical period, cost allocation data are not available. I have utilized cost elasticity share estimates for Bell Canada for the period 1952–76 from Denny, Fuss, and Waverman (1981), who used the econometric cost function methodology to estimate the elasticities. Since their model is a long-run model, TFP growth rate results are provided prior to 1980 only for this case.

Tables 1 and 2 contain the revenue shares and estimated cost elasticity shares for Bell Canada for the long-run and short-run models, respectively. In table 1 there are only two outputs for the period 1952–78, since Denny, Fuss and Waverman (1981) aggregated local and miscellaneous services into a single output category and I have had to do the same pre-1979 in order to use their estimates. For the period 1979–89, it is clear from a comparison of tables 1 and 2 that the cost elasticity shares are quite insensitive to whether the user cost or the residual return is used to value capital.

Of important for the TFP growth rate results to follow is the fact that the cost elasticity shares for local output exceed the revenue shares, and vice-versa for toll, throughout the period 1952–89. This fact, along with the additional fact that toll output grew at a faster rate than local output (see table 5) throughout the period, is the source of the upward bias in the conventional measure of TFP. Use of revenue shares puts too high a weight on the faster-growing output and leads to an overestimate of the rate of growth of aggregate output and hence TFP.

Notice from the cost elasticity shares in table 1 the close correspondence between the results for 1976 from Denny, Fuss, and Waverman (1981), who obtain as estimates of cost elasticity shares: toll = 0.302, local + miscellaneous = 0.698; and my results for 1979 using the cost allocation procedure (toll = 0.285, local + miscellaneous = 0.715). This close correspondence lends support to the basic assumption of the cost allocation methodology; that shares based on average allocated costs are more reasonable approximations to the theoretically correct cost elasticity shares based on marginal costs than are shares based on prices.

Table 3 contains additional information which can be used to evaluate the reasonableness of the allocation estimates of cost elasticity shares. This table contains all of the econometric estimates of the toll cost elasticity share for Bell Canada that I could find in the literature. (The local cost elasticity share is simply one minus the toll elasticity share.)

Two things stand out from this table. First, the Denny, Fuss, and Waverman (1981) estimate is quite close to the corresponding estimate of Kiss et al. (1981, 1983) for the one year (1967) when both estimates are available. Second, both the Denny, Fuss and Waverman and Kiss et al. estimates of the toll cost elasticity share are the largest estimates in this table. The correction to the revenue share-weighted

TABLE 1
Bell Canada output shares – cost weights based on long-run model

	Local		Toll		Misc.	
Year	Revenue[a]	Cost[b]	Revenue[a]	Cost[b]	Revenue[a]	Cost[b]
1952	0.688	0.907	0.312	0.093		
1953	0.690	0.904	0.310	0.097		
1954	0.690	0.903	0.310	0.098		
1955	0.677	0.893	0.323	0.107		
1956	0.670	0.886	0.330	0.114		
1957	0.673	0.884	0.327	0.116		
1958	0.679	0.880	0.322	0.119		
1959	0.680	0.876	0.320	0.123		
1960	0.680	0.870	0.320	0.129		
1961	0.682	0.866	0.318	0.135		
1962	0.674	0.857	0.325	0.143		
1963	0.673	0.850	0.327	0.150		
1964	0.654	0.832	0.346	0.168		
1965	0.644	0.825	0.356	0.175		
1966	0.640	0.818	0.360	0.182		
1967	0.633	0.812	0.367	0.188		
1968	0.626	0.800	0.374	0.200		
1969	0.611	0.787	0.389	0.213		
1970	0.594	0.780	0.405	0.220		
1971	0.602	0.778	0.397	0.222		
1972	0.589	0.759	0.412	0.241		
1973	0.568	0.740	0.431	0.260		
1974	0.559	0.727	0.441	0.273		
1975	0.550	0.710	0.450	0.290		
1976	0.547	0.698	0.454	0.302		
1977	0.547	0.704	0.453	0.296		
1978	0.540	0.709	0.459	0.291		
1979	0.492	0.703	0.469	0.285	0.039	0.012
1980	0.485	0.686	0.474	0.302	0.041	0.011
1981	0.476	0.770	0.481	0.217	0.043	0.012
1982	0.466	0.763	0.491	0.224	0.043	0.013
1983	0.450	0.774	0.497	0.212	0.053	0.014
1984	0.426	0.784	0.502	0.203	0.072	0.013
1985	0.403	0.763	0.516	0.213	0.081	0.023
1986	0.380	0.738	0.532	0.226	0.088	0.036
1987[c]	0.446	0.765	0.516	0.226	0.038	0.010
1988	0.445	0.752	0.515	0.239	0.040	0.009
1989	0.446	0.744	0.512	0.245	0.041	0.012

a SOURCES: Bell(Unitel)28Dec.90-253 and Bell(Unitel)28Dec.90-255, Supplemental; Exhibit Bell Canada 182

b SOURCES: 1952–76 – Denny, Fuss, and Waverman (1981, 204)
1977–8 – interpolated
1979–84 – Five-Way Split Study (annual)
1985 – interpolated
1986–9 – Phase III Study (annual)

c The revenue shares listed are to be compared with 1988+ data. For comparison with 1952–86 data, the share of local = 0.384, the share of toll = 0.519, and the share of misc. = 0.097.

TABLE 2
Bell Canada output shares – cost weights based on short-run model

Year	Local		Toll		Misc.	
	Revenue[a]	Cost[b]	Revenue[a]	Cost[b]	Revenue[a]	Cost[b]
1979	0.492	0.703	0.469	0.286	0.039	0.012
1980	0.485	0.692	0.474	0.297	0.041	0.012
1981	0.476	0.774	0.481	0.214	0.043	0.012
1982	0.466	0.769	0.491	0.218	0.043	0.013
1983	0.450	0.772	0.497	0.214	0.053	0.014
1984	0.426	0.781	0.502	0.206	0.072	0.013
1985	0.403	0.758	0.516	0.218	0.081	0.024
1986	0.380	0.739	0.532	0.229	0.088	0.033
1987[c]	0.446	0.763	0.516	0.227	0.038	0.010
1988	0.445	0.751	0.515	0.241	0.040	0.008
1989	0.446	0.741	0.512	0.248	0.041	0.011

a SOURCES: Bell(Unitel)28Dec.90-253 and Bell(Unitel)28Dec.90-255, Supplemental; Exhibit Bell Canada 182
b SOURCES: 1979–84 – Five-Way Split Study (annual)
1985 – interpolated
1986–9 – Phase III Study (annual)
c The revenue shares listed are to be compared with 1988+ data. For comparison with 1952–86 data, the share of local = 0.384, the share of toll = 0.519, and the share of misc. = 0.097.

TFP measure implied by the use of allocated costs appears to be a conservative one. Utilization of most of the econometric studies contained in table 2 would result in greater adjustments to the TFP growth rate than those contained in this paper. In summary, the evidence in table 3 suggests that the use of allocated costs has not resulted in an underestimation of TFP growth.

I now turn to a consideration of the B.C. Tel data. Table 4 contains revenue and cost elasticity share data for B.C. Tel for a period 1980–89. As with Bell Canada, B.C. Tel's cost allocation data contain residual rate of return capital valuation. However, in contrast to the Bell Canada data, the B.C. Tel productivity data base also contains a price of capital services based on the residual rate of return. Hence only TFP growth rates corresponding to the short-run model could be calculated for B.C. Tel. Local and miscellaneous outputs were aggregated (using revenue shares), since I did not feel that I could accomplish a reliable division of the allocated costs between the non-toll outputs in the period before 1986.[21]

The relationship between cost elasticity shares and revenue shares for B.C. Tel is very similar to that found for Bell Canada. Cost shares exceed revenue shares for local and the reverse is true for toll. Since toll output grows faster than local output over the periods portrayed in table 4 (see table 5), we can once again expect that revenue-weighted TFP indices will overestimate the actual rate of TFP growth.

21 It should be noted that for B.C. Tel no econometric cost function estimates exist that could provide cost elasticity weights.

TABLE 3
Econometric estimates of toll service cost elasticity shares

Year	Study	Toll cost elasticity share
1963	Denny, Fuss, and Waverman (DFW)(1981)	0.15
	Fuss and Wavermann (1981b)	0.11
1967	DFW (1981)	0.19
	Kiss et al. (1981, 1983) (as reported in Kiss and Lefebvre (1987))	0.18
	Ngo (1990)	
	Model 1	0.12
	Model 2	0.06
	Model 3	0.03
1970	DFW (1981)	0.22
	Bernstein (1988)	0.13
1972	DFW (1981)	0.24
	Bernstein (1988)	0.17
1976	DFW (1981)	0.30
1978	Bernstein (1989)	0.12

TABLE 4
B.C. Tel output shares – cost weights based on short-run model

Year	Local		Toll	
	Revenue[a]	Cost[b]	Revenue[a]	Cost[b]
1980	0.417	0.772	0.583	0.228
1981[c]	0.434	0.762	0.566	0.239
1982	0.468	0.762	0.532	0.239
1983	0.472	0.758	0.528	0.242
1984	0.469	0.762	0.531	0.239
1985	0.452	0.762	0.548	0.239
1986	0.442	0.778	0.558	0.222
1987	0.437	0.748	0.563	0.252
1988	0.449	0.760	0.551	0.240
1989	0.480	0.753	0.520	0.247

a SOURCES: 1980–1 – B.C.Tel(CRTC)28Dec.90-2214, Attachment 2
1981–9 – B.C.Tel(CRTC)28Dec.90-2214, Attachment 1
b SOURCES: 1981–2; 1984–5 – interpolated
1980, 1983 – Five-Way Split Study (annual)
1986–9 – Phase III Study (annual)
c 1981 revenue shares listed are from B.C.Tel(CRTC)28Dec.90-2214, Attachment 1. Revenue shares for 1981 from Attachment 2 of 0.423 for local and misc. and 0.577 for toll were used in calculating TFP growth over the 1980–1 period.

TABLE 5
Average annual rates of growth of outputs (per cent)[a]

Time period	Local	Toll	Misc.
Bell Canada			
1953–9	8.7	9.9	8.6
1960–9	7.0	10.7	4.7
1970–9	6.3	10.1	10.8
1980–9	2.8	9.2	8.8
1985–9	3.7	11.4	12.2
B.C. Tel			
1980–9	4.0	10.1	11.4
1985–9	3.6	11.5	5.1

a Computed as average of logarithmic differences.

V. COST-WEIGHTED TOTAL FACTOR PRODUCTIVITY GROWTH RATES FOR BELL CANADA AND B.C. TEL

In this section I estimate TFP growth rates for Bell Canada and B.C. Tel using the cost-weighted formula (4), and compare these estimates with revenue-weighted estimates. The two sets of estimates are presented in tables 6 and 7 (long-run and short-run models respectively) for selected historical periods.

The bias discussed previously is readily apparent from tables 6 and 7. The cost-weighted TFP growth rates are all less than the conventional revenue-weighted measures. The bias appears to be particularly important in the 1980s. That result is not surprising, since the 1980s was the period of most rapid growth in toll output relative to local output. While the revenue-weighted index apparently indicates that the period 1985–9 was a period of especially rapid TFP growth, the conceptually more correct cost-weighted index indicates this was not the case. For Bell Canada, a TFP growth rate of 3.3–3.5 per cent per annum over the 1985–9 period, while greater than the rate during the rest of the 1980s, is close to the average growth rate over the 1960s and 1970s.

In the case of Bell Canada, the results presented in tables 6 and 7 imply that the upward bias in the conventional TFP growth measure is 74–77 per cent over the period 1980–9 and 77–82 per cent over the period 1985–9. By way of contrast, the bias is considerably less in the earlier periods of less rapid relative growth of toll output. It can also be seen from tables 6 and 7 the Bell Canada's cost-weighted TFP growth rates and the bias associated with revenue weighting are quite robust to the choice of model (long run versus short run).

Table 7 also presents TFP growth rates for B.C. Tel. over the period 1980–9. As was the case with Bell Canada, use of the conventional revenue-weighted TFP index for B.C. Tel imparts an upward bias to the estimates of efficiency gains. The effect is particularly striking for the 1985–9 period. The revenue-weighted estimate of TFP growth of 7.1 per cent per annum is reduced to 4.8 per cent per annum when

TABLE 6
Average annual total factor productivity growth rates (per cent)[a] – based on long-run model

Time period	Revenue weights[b]	Cost weights[c]	Upward bias of revenue-weighted index (per cent)
Bell Canada			
1953–9	2.2	2.0	10
1960–9	4.1	3.4	21
1970–9	3.9	3.3	18
1980–9	4.6	2.6	77
1985–9	6.0	3.3	82

a Computed as average of logarithmic differences.
b Calculated from Bell(Unitel)28Dec.90-253 and B.C.Tel(CRTC)28 Dec.90-2214.
c Calculated from Bell(Unitel)28Dec.90-253 and using the cost weights from table 1.

TABLE 7
Average annual total factor productivity growth rates (per cent)[a] – based on short-run model

Time period	Revenue weights[b]	Cost weights[c]	Upward bias of revenue-weighted index (per cent)
Bell Canada			
1980–9	4.7	2.7	74
1985–9	6.2	3.5	77
B.C. Tel			
1980–9	5.6	4.1	37
1985–9	7.1	4.8	48

a Computed as average of logarithmic differences.
b Calculated from Bell(Unitel)28Dec.90-253 and B.C.Tel(CRTC)28 Dec.90-2214.
c Calculated from Bell(Unitel)28Dec.90-253, B.C.Tel(CRTC)28Dec. 90-2214 and using the cost weights from tables 3 and 4.

the conceptually correct cost-weighted index is used. Once again the reason for the large gap between the estimates is the fact that the relative growth rate of toll versus local service is greatest in this period (see table 5).

VI. CONCLUSIONS

One of the enduring facts in Canadian Telecommunications regulation is the CRTC's social policy to use surplus revenues from toll services to subsidize local service prices. Under these conditions, the use of prices to approximate marginal costs results in misleading estimates of TFP growth when the growth rates of the two

outputs differ substantially. This situation occurred in the 1980s and resulted in Bell Canada and B.C. Tel's overestimating efficiency-related TFP growth by 74–82 per cent and 37–48 per cent, respectively, in their submissions to the Interconnect Hearings.

The debate over TFP measurement procedures played an important role in the Interconnect Hearings. The two companies' forecasts of TFP growth in the 1990s under a monopoly industry structure appeared to be a continuation of growth in the 1985–9 period when calculated using revenue weights, but they represented a radical acceleration of growth when cost-weighted TFP rates were compared. The CRTC concluded in its June 1992 decision that the two companies had overestimated future production efficiency growth and hence the ability to deliver toll price reductions under a monopoly structure. This conclusion appeared to be one of the elements that led the commission to the view that facilities-based competitors should be allowed to enter toll markets in Bell Canada and B.C. Tel's territories.

APPENDIX: DERIVATION OF THE CONCEPTUALLY CORRECT FORM OF THE TFP INDEX AND DEMONSTRATION THAT THE REVENUE SHARE-WEIGHTED INDEX IS INAPPROPRIATE

The first part of this appendix draws heavily on Denny, Fuss, and Waverman (1981), where a more extensive discussion is available.

Suppose we represent the underlying production process of a cost-minimizing telecommunications firm by the cost function

$$C = C(Q, w, t), \tag{A1}$$

where Q is a vector of outputs, w is a vector of input prices, and t (time) indexes the state of technology. Totally differentiating the cost function (A1) and applying Shephard's lemma we obtain the rate of change of cost equation,

$$\text{dlog}\, C/dt = \sum(ECQ_i \cdot \text{dlog}\, Q_i/dt) + \sum(S_i \cdot \text{dlog}\, w_i/dt) + \partial \log C/\partial t, \tag{A2}$$

where $ECQ_i = \partial \log C/\partial \log Q_i$ is the cost-output elasticity for the ith output, and $\partial \log C/\partial t$ is the rate of cost reduction due to technical change.

Denny, Fuss, and Waverman (1981, 210) demonstrate that the rate of change of total factor productivity for a cost-minimizing firm that incorporates *only* technical efficiency effects (hereafter denoted TFP^c) can be written as

$$\text{dlog}\, TFP^c/dt = (1 - ECQ) \cdot \text{dlog}\, Q^c/dt - \partial \log C/\partial t, \tag{A3}$$

where

$$ECQ = \sum ECQ_i \tag{A4}$$

is the aggregate cost elasticity (equal to the reciprocal of the aggregate scale elasticity), and

$$\text{dlog}\, Q^c/dt = [1/ECQ] \cdot \left(\sum ECQ_i \cdot \text{dlog}\, Q_i/dt\right). \tag{A5}$$

Equation (A5) can be rewritten as

$$\text{dlog}\, Q^c/dt = \sum M_i \cdot \text{dlog}\, Q_i/dt, \tag{A6}$$

where $M_i = ECQ_i/ECQ$ is the cost elasticity of the ith output relative to the sum of the cost elasticities; that is, it is the ith cost-elasticity share.

The most frequently used discrete approximation to (A6) is given by the 'Törnqvist' formula,

$$\Delta \log Q^c = \sum(1/2) \cdot (M_{it} + M_{i,t-1}) \cdot [\log Q_{it} - \log Q_{i,t-1}], \tag{A7}$$

where M_{it} is the cost elasticity share of the ith output at time t and Q_{it} is the amount of the ith output produced at time t. Comparing (A7) and equation (2) in the text, we see that the definitions of the rate of change of aggregate output differ according to whether revenue shares or cost elasticity shares are used in the aggregation procedure.

Following Denny, Fuss, and Waverman (1981), we can use equations (A2)–(A6) to obtain the conceptually correct expression for TFP growth for a cost-minimizing firm,

$$\text{dlog}\, TFP^c/dt = -\left\{\text{dlog}\, C/dt - \text{dlog}\, Q^c/dt - \sum S_i \cdot \text{dlog}\, w_i/dt\right\}. \tag{A8}$$

An alternative form of the conceptually correct expression for TFP growth is

$$\text{dlog}\, TFP^c/dt = \text{dlog}\, Q^c/dt - \text{dlog}\, X/dt. \tag{A9}$$

For the case of a single output, Fuss and Waverman (1990b, Technical Appendix) demonstrate that (A8) and (A9) are equivalent representations of TFP growth. The extension to the case of multiple outputs is straightforward.

A discrete approximation to (A9) is given by

$$\Delta \log TFP^c = \Delta \log Q^c - \Delta \log X, \tag{A10}$$

which is equation (4) in the text.

I have now demonstrated that equation (A10) is the conceptually correct form of the Törnqvist family of TFP growth rate indices. Under what conditions is the conventional measure (equation(1) in the text) equivalent to (A10) (equation (4)

in the text)? This equivalence will occur *only if* output prices in a given year are proportional to marginal costs of production. To see this, write

$$p_{it} = \theta_{it} \cdot (\partial C_t / \partial Q_{it}), \tag{A11}$$

where p_{it} is the output price for the ith output at time t. Now suppose $\theta_{it} = \theta_t$ for all i outputs (i.e., proportionality). Then

$$\begin{aligned} M_{it} &= ECQ_{it}/ECQ \\ &= (Q_{it} \cdot \partial C_t/\partial Q_{it}) / \left(\sum Q_{it} \cdot \partial C_t / \partial Q_{it}\right) \\ &= (Q_{it} \cdot p_{it}) / \left(\sum Q_{it} \cdot p_{it}\right) \\ &= R_{it} \end{aligned} \tag{A12}$$

However, the conditions underlying the calculations in (A12) are *not* valid in the case of Bell Canada and B.C. Tel, since $\theta_{it} > 1$ for toll services (and probably miscellaneous services), whereas $\theta_{it} < 1$ for local services. Hence θ_{it} cannot be the same for each output in a particular year and the revenue share – weighted Törnqvist TFP index is not valid conceptually. How important this conceptual error is remains an empirical question. As we can see from the empirical results in this paper, the inappropriate index biases the TFP estimates for both Bell Canada and B.C. Tel upward to a considerable degree in the 1980s.

The above development has been based on the continuous Divisia index and the Törnqvist discrete approximation to that index. Diewert (1991) has criticized this procedure for its reliance on the continuous form of the index. An alternative development can be obtained using the theory of discrete exact index numbers (see Diewert 1976, 1991, for descriptions of this theory). It turns out that the weights differ slightly from those obtained above, owing to the different discrete approximations used, but the differences are inconsequential for the times series data used in this paper

Suppose the cost function (A1) is approximated to the second degree by a function that is quadratic in logarithms. Then a quadratic lemma (Diewart 1976) can be used to obtain the following expression for the discrete change in the logarithm of cost, $\Delta \log C = \log C_t - \log C_{t-1}$:

$$\begin{aligned} \Delta \log C = {} & 1/2 \sum_i \left(\frac{\partial \log C_t}{\partial \log Q_{it}} + \frac{\partial \log C_{t-1}}{\partial \log Q_{i,t-1}} \right) \cdot \Delta \log Q_i \\ & + 1/2 \sum_i \left(\frac{\partial \log C_t}{\partial \log w_{it}} + \frac{\partial \log C_{t-1}}{\partial \log w_{i,t-1}} \right) \cdot \Delta \log w_i \\ & + 1/2 \left(\frac{\partial \log C_t}{\partial t} + \frac{\partial \log C_{t-1}}{\partial (t-1)} \right) \end{aligned} \tag{A13}$$

$$\begin{aligned} \Delta \log C = {} & \left[\sum_i 1/2(ECQ_{it} + ECQ_{i,t-1}) \right] \\ & \times \left[\sum_i \left\{ \frac{ECQ_{it} + ECQ_{i,t-1}}{\sum_i (ECQ_{it} + ECQ_{i,t-1}} \right\} \cdot \Delta \log Q_i \right] \\ & + 1/2 \sum_i (S_{it} + S_{i,t-1}) \cdot \Delta \log w_i + 1/2 \left(\frac{\partial \log C_t}{\partial t} + \frac{\partial \log C_{t-1}}{\partial (t-1)} \right). \end{aligned} \tag{A14}$$

Define

$$\overline{ECQ_i} = \frac{ECQ_{it} + ECQ_{i,t-1}}{\sum_i (ECQ_{it} + ECQ_{i,t-1})} = \text{average discrete elasticity shares}$$

$$\bar{S}_i = 1/2(S_{it} + S_{i,t-1}) \tag{A15}$$

$$\Delta \log B = -1/2 \left(\frac{\partial \log C_t}{\partial t} + \frac{\partial \log C_{t-1}}{\partial (t-1)} \right).$$

Substituting (A15) into (A14) yields

$$\begin{aligned} \Delta \log C = {} & \left[\sum_i 1/2(ECQ_{it} + ECQ_{i,t-1}) \right] \cdot \left[\sum_i \overline{ECQ_i} \cdot \Delta \log Q_i \right] \\ & + \sum_i \bar{S}_i \Delta \log w_i - \Delta \log B. \end{aligned} \tag{A16}$$

Define

$$\Delta \log Q^c = \sum_i \overline{ECQ_i} \cdot \Delta \log Q_i \tag{A17}$$

$$\Delta \log \text{TFP}^c = - \left\{ \Delta \log C - \Delta \log Q^c - \sum_i \bar{S}_i \Delta \log w_i \right\}. \tag{A18}$$

Combining (A16)–(A18) yields

$$\Delta \log \text{TFP}^c = [1 - \overline{ECQ}] \cdot \Delta \log Q^c + \Delta \log B, \tag{A19}$$

where

$$\overline{ECQ} = \sum_i 1/2(ECQ_{it} + ECQ_{i,t-1}). \tag{A20}$$

Note that $\Delta \log Q^c$ in (A17) differs from $\Delta \log Q^c$ in (A7) by the way in which the average cost elasticities are calculated. In practice, both averaging methods will give very similar estimates. There is a second difference in the two approximating procedures. In the case of the exact index number procedure, there is no counterpart to the equality between (A8) and (A9). The TFP formula (A10), where definition (A17) is used for $\Delta \log Q^c$, differs from (A19) by terms that are of second-order differential smallness (see Denny and Fuss 1980 for a demonstration of this fact in the case of revenue-weighted TFP). Once again, in the current case the differences will be inconsequential.

Finally, the analysis in this appendix is for the case of the long-run model. It is not difficult to carry out the same analysis for the short-run case, replacing the long-run cost function (A1) with a variable (restricted) cost function. This procedure is done in Berndt and Fuss (1989), who show that equation (A9) remains the correct output aggregation equation as long as the shadow values of the quasi-fixed inputs replace the user costs in the calculation of the output cost elasticities. Similarly, they demonstrate that equation (A9) remains the correct TFP growth rate equation as long as the shadow values of the quasi-fixed inputs also replace the user costs in the calculation of the growth rate of the aggregate input $d\log X/dt$.

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