Design of the Reverse Auction in the FCC Incentive Auction

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Appendix: Detailed description of the simulation model and analysis

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System architecture

Performing large-scale simulations of the incentive auction and analyzing the results involves a wide range of software and hardware resources. In addition to running simulations on local machines, we harness the power of the cloud to achieve simulations at a much larger scale than would have been otherwise possible. Scaling up the simulations, however, also scales up the volume of data that needs to be analyzed. Our analysis made use of many powerful software tools to convert this large volume of data into meaningful results. We give a brief description of the key hardware and platform resources our simulations required.

Hardware

Our simulations make use of both local and cloud computing resources. While most development and analysis work is completed locally, cloud resources are critical to simulating a wide variety of alternatives and scenarios in a timely manner.

Local resources

Our local computing resources consisted of two Dell PowerEdge T620 servers, each with dual 6-core/12-thread 3.50GHz Intel Xeon E5-2643 v2 processors, and 64GB of 1866MHz RAM.

Cloud resources

Our cloud computing resources are obtained from the Amazon Elastic Compute Cloud (EC2) service. We primarily run our simulations on their compute optimized C3 instances; these provide virtual machines running on servers equipped with 2.80GHz Intel Xeon E5-2643 v2 Intel Xeon E5-2680 v2 processors. Virtual machines are available with the ability to execute a range of numbers of execution threads, and have access to 3.75GB of RAM for every two execution threads.

Software

Our software resources fell into three broad categories: those required to implement our simulation; those required to scale our simulation up to the cloud; and those required to analyze and assess the large volume of data produced by our simulations.

Simulation platform

Our simulation software uses the PicoSAT solver (see Biere 2008) when determining feasibility of station repacking; we use Gurobi to solve optimization problems arising in the RZR process.

Cloud computing

When running simulations in the cloud, we use the StarCluster project from MIT for creating and launching of computing clusters on Amazon's EC2 service. StarCluster provides a distributed, Linux-based computing environment with management and balancing of computing jobs via the Open Grid Engine batch-queuing system.

Post-processing and analysis

The large volumes of data produced by our cloud simulations requires the use of sophisticated tools to produce meaningful results and derive insights. Initial processing of the data is handled

via Stata and the Pandas library for Python. Visualization and analysis is largely performed in Tableau Desktop.

Auction simulator

Simulations are critical to our evaluation of scoring rules under a wide variety of conditions. We outline here the key details of our simulator. We focus on details or assumptions that went into our particular simulator, and refer the reader to the Comment PN for the full details of the proposed reverse auction design. Many of the key technical details of our auction simulator were involved in our implementation of RZR and of feasibility checking, which we discuss in detail later.

The main logic of the auction simulator is simple. Initially, the RZR process is run on all auction participants, determining the initial set of repacked stations, the set of stations receiving RZR prices, and the set of stations that will remain active in the bidding. Once the RZR process has completed, the main auction simulation begins with the initial repacking and set of active bidders that result from RZR. The auction process itself involves repeated rounds of bidding, where each round involves a decrement of the clock, and any station exits or freezes that result from lowering prices. The process of a single round can be seen in Figure A1. This repeated lowering of the auction clock continues until all stations have either exited or been frozen, at which point the simulation concludes.

In order to focus our results on the aspects of the auction we felt most critical to our analysis, we made several key assumptions. First, all of our simulations focused on UHF stations; the interactions and complexity of bids to move to VHF complicate the auction process without much return. Similarly, we assume straightforward bidding on the part of participants: all prices are evaluated simply by comparison to reservation values, and accepted or rejected accordingly if they are above or below those values. Finally, we note that all feasibility checks performed over the course of the auction and RZR simulations are performed using our feasibility checker (described in detail later).

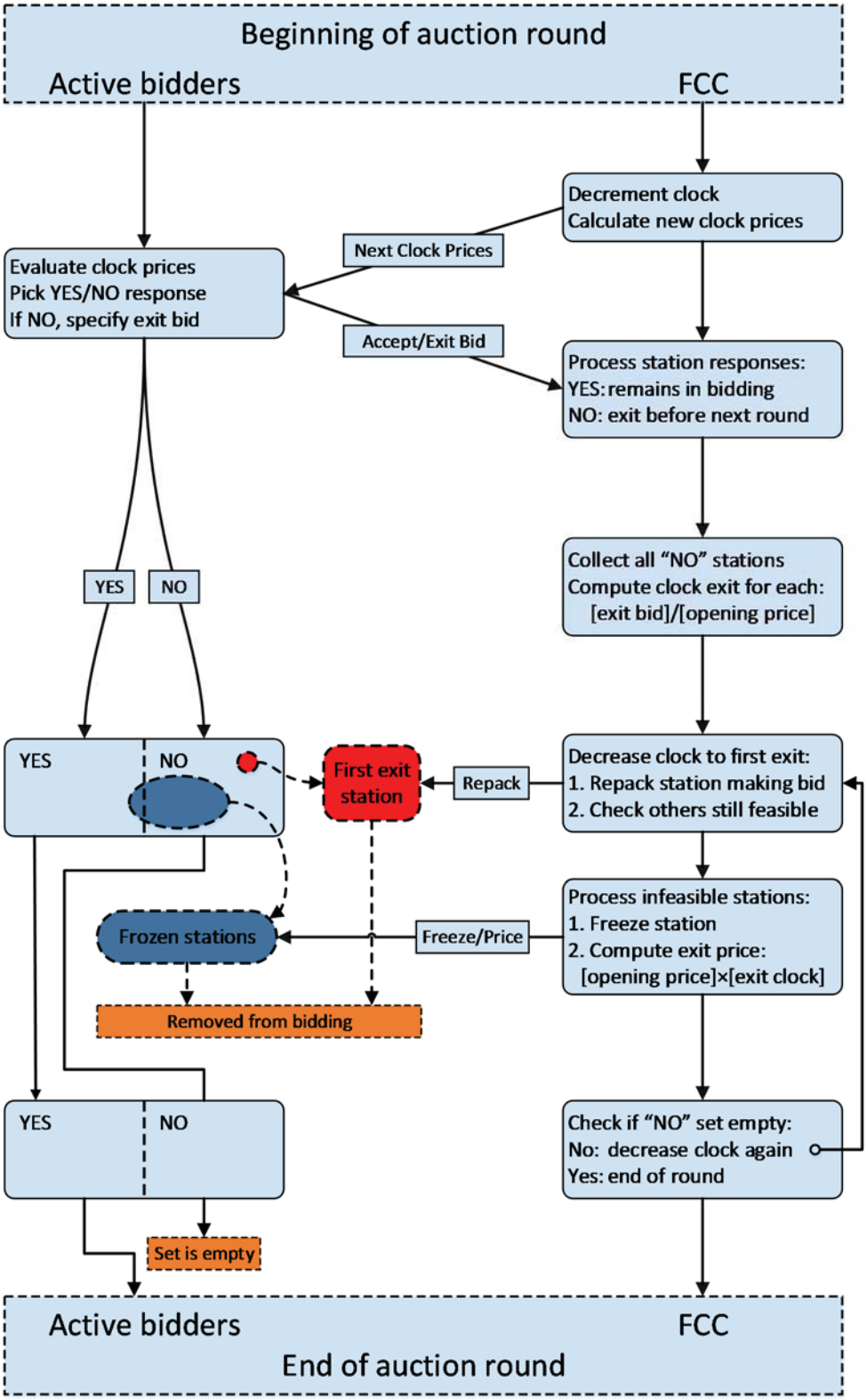


Figure A1: Flow chart of single round of reverse auction simulation

Feasibility checker

Determining the feasibility of repacking stations is a key technical challenge that a successful incentive auction must overcome. This difficulty arises from fundamental computational issues: the repacking problem is readily shown to belong to a class of known hard problems in computer science, which have no known generally efficient solutions. While theoretically intractable, the outlook is fortunately much less bleak in practice. As noted, for example, by Leyton-Brown (2014), the repacking problem lends itself to being expressed as a classical hard problem, namely the Boolean satisfiability problem (SAT). Many heuristics have been developed for the SAT problem that provide efficient solutions in practical applications much of the time. Additionally, solution methods can be tuned to the particular problem instances arising during the incentive auction. For example, both the structure of the interference constraints limiting how stations may be repacked, and the fact that most of the problems we solve are incremental—whether a single station can be added to a set we already know can be repacked—provide natural avenues for improvement.

Improvements such as mentioned above can boost performance in practice, but they cannot completely avoid the fundamental difficulty of the underlying problem. Despite reducing the complexity of the SAT instances that must be solved, and even completely avoiding the need to solve a SAT problem in some cases, it is inevitable that not all instances will admit efficient solutions. In the context of the auction, choosing to repack a station is a commitment to assign it a channel once the auction completes, and so whenever we cannot efficiently determine the feasibility of a station we must assume the worst and consider it infeasible to repack. As such, the goal becomes to develop a feasibility checker that runs efficiently while solving as many of the problems presented to it as possible.

Just as a successful incentive auction must overcome the challenges of the repacking problem, so must any successful simulation of the auction. In the rest of this section, we outline the technical details of how we handle feasibility checking. Our overall approach combines an established SAT solver with several pre-solving routines designed to minimize the complexity and number of problems the SAT solver must handle. We begin by briefly describing the interpretation of the repacking problem as an instance of the SAT problem, and how we solve these instances, and then go on to describe our pre-solving routines. As our pre-solving routines are based on the interpretation of the interference constraints in the repacking problem as a mathematical graph, we discuss this structure before describing the details of the pre-solving routines themselves.

Satisfiability

We will briefly sketch the interpretation of the repacking problem as an instance of SAT. A more complete description of this interpretation can be found in materials from the FCC LEARN Workshops (see, for example, Leyton-Brown 2014). Instances of the SAT problem ask whether a logical statement can be satisfied. In particular, SAT focuses on statements that can be viewed as

asking whether a sequence of claims can all be true at the same time, where each claim is that at least one of a specific set of conditions is true. This naturally captures the repacking problem, since a successful repack must:

1. Choose, for every station, a channel consistent with its domain and the clearing target.
2. Choose, for any pair of interfering channel assignments that could be made for two different stations, at least one of these channels to leave open.

Although we do not provide the details of the reduction, the above does lead to a natural formal expression of checking the feasibility of repacking a set of stations as an instance of SAT.

Our feasibility checker utilizes this reduction to express repacking problems as SAT instances. We then use the freely-available solver PicoSAT to determine the feasibility of each problem encountered. As previously mentioned, it is an impossible goal to efficiently solve every instance of SAT, and one must balance resources expended trying to solve instances against quality of solution. Our chosen solver, PicoSAT, provides various parameters for adjusting how much time is spent attempting to determine feasibility before declaring failure. The parameter we use to cut off execution in our simulations is the propagation limit. Although the details of how this parameter affects solver behavior depends on internal aspects of the PicoSAT solver's implementation that are outside the scope of this paper, we use this parameter as it gives each problem instance the most consistent amount of time. The solver performance is shown in Table A1. We can see that on average, both infeasible and feasible solutions are found significantly before the average SAT solver cut-off occurs.

Table A1: Feasibility check performance

	Feasible	**Frozen**	
	Feasible	Infeasible	Unknown
Average number of solutions	557,821.48	168.79	321.09
Average solution time per SAT instance (seconds)	0.01	0.84	7.53
Max solution time (seconds)	13.86	26.45	28.73

It is not possible to exactly solve every feasibility check. Rather, it is necessary to "give up" after some threshold is reached. In the PicoSAT solver the propagation limit is a suitable parameter to quite the check and conservatively classify an "unknown" instance as "infeasible".

We experimented with various propagation limits and found that a limit of 10 million worked well for our purposes. This conclusion is supported by Figure A2, which shows the cumulative number of feasibility checks by solution type, excluding feasible solutions. In this figure, we can

see that the solution rate has largely plateaued by the time our execution cut-off begins to have an effect. This motivates our choice of cut-off, as we must consider marginal benefit from a higher cut-off. In particular, since every simulation involves about ½ million feasibility checks, even small increases have a large cost in computation time. This cost must be weighed against the amount of extra information obtained. Figure A2 suggests that the additional information of running the feasibility checker for long periods of time results in little additional information. Especially for our purpose of evaluating alternative scoring rules, setting a propagation limit of 10 million appears appropriate. This results in spending about 7 seconds on unknown instances before giving up.

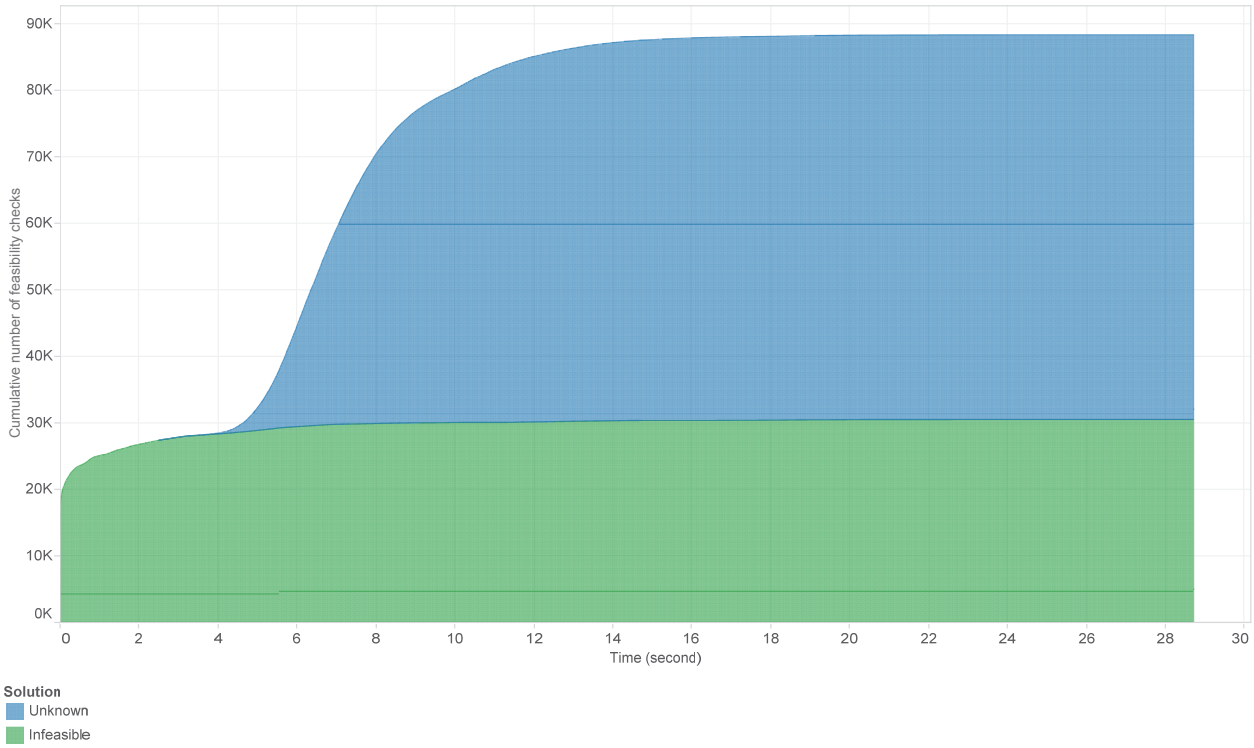


Figure A2: Cumulative number of feasibility checks

Structure of interference constraints

In developing the pre-solving routines in our feasibility checker, we leverage special structure present in the interference constraints between stations. In particular, one can think of interference between two stations as linking them together. Understanding the links of interference between various groups of stations yields critical insights into the structure of the repacking problem. As seen in previous work, this structure can be leveraged to significantly improve feasibility checking routines. Formally, the links that interference creates between stations induces a graph structure on the set of stations, and several key insights into the interactions between stations during the repacking process can be obtained from graph theory and other areas of mathematics.

Locality-based pre-solving

One class of pre-solving routines our feasibility checker implements is based on determining how closely interference links pairs of stations. As suggested by Leyton-Brown (2014), a key observation is that, when packing a set of stations, these interference links identify which pairs of stations directly affect each other when choosing a channel assignment. Furthermore, although stations not directly linked can impact each other's channel assignment, for this to happen the two stations must be linked by a daisy-chain of intermediate stations, each one directly linked to the last. As noted in previous work, the longer the daisy-chain needed to link two stations by interference, the less likely it is that their channel assignments have any effect on each other. Intuitively, the more links separate two stations, the larger the set of stations that must all be involved if the two are to have an effect on each other. For example, in Figure A3, for station A to affect station E, station C must be involved, whereas for station A to affect station G, at least two additional stations must be involved. When combined with the fact that most of the problems we encounter in the context of the incentive auction ask whether a single station can be added to a repacking, this observation leads to several natural pre-solving approaches that can greatly reduce the size and complexity of the problems that we must submit to the SAT solver.

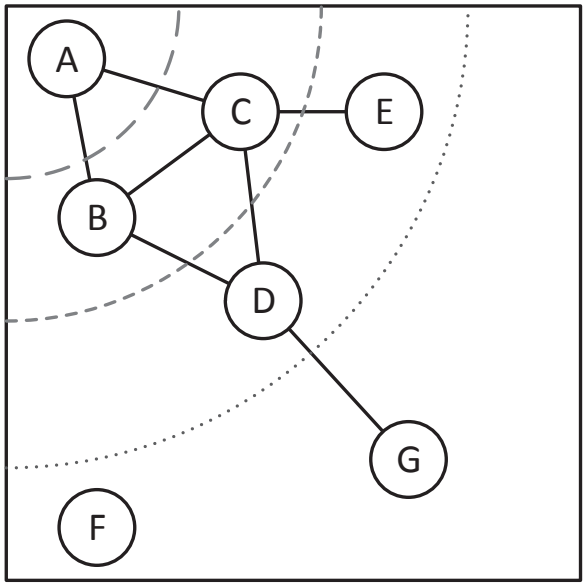


Figure A3: Locality of interference constraints

One immediate improvement that can be made comes from the fact that if two stations are not connected—either directly or by some daisy-chain—through the currently repacked set, the feasibility of repacking one of these stations is completely independent of the other. See Figure A3 for an example of this. When determining whether a station can be repacked, we limit our problem to just the stations that are linked by interference to the station in question, either directly or through a daisy-chain. This can lead to significant improvements; for example, the East and West coasts are completely disconnected in terms of interference, and so any repacking of a station on one coast can completely ignore stations on the other.

Another improvement we can make is based on the observation that, the longer the daisy-chain it takes to link two stations together, the less likely it is that the presence of one will impact the feasibility of repacking the other. Thus, before asking whether it is feasible to pack a station, we ask simpler questions about whether it can be repacked when only considering the stations closest to it. When determining whether a station can be feasibly repacked, we first consider whether we can repack the station itself; the station and those directly linked to it; and the station and those at most two links from it, in isolation from all other stations. For example, in Figure A3, we can see that station A is one link away from each of stations B and C, two links away from each of stations D and E, three links away from station G, and not connected at all to station F.

One subtle issue about the above approach is what these local problems tell us about the overall problem. In particular, we must be careful how we formulate these local repacking problems, and what we infer about the overall feasibility of repacking a station from the smaller problems we formulate. The key question is how we handle stations that are outside the local set of our focus.

One approach is to simply ignore them, leaving them out of the repacking problem entirely. Notice, however, that if we determine it is feasible to repack the local set of stations, there is no guarantee that our local channel assignment is compatible with the channel assignment we currently have for other stations. We can, however, be sure that if we cannot repack this small set of stations while ignoring the larger set, there is no hope to find a repacking that works for both the local set and the larger set at the same time. Thus, we can extend an infeasibility result from the local set to the full one, but not a feasibility result.

Another approach is to include the larger set of stations, but keep the problem local by forcing those stations to remain on their currently assigned repack channels. This has the opposite problem: any repacking of the local set that we find is compatible with our existing repack by construction. But if we cannot repack the local set there is no guarantee that we could not have done so if we had the flexibility to adjust the channel assignments made for the larger set of stations.

Due to the above considerations, our pre-solving routines try both approaches for each of the local sets we consider. As soon as we see a result that allows us to infer an answer for the full set, we stop our search and report the answer; if none of the local problems yield a general answer—either because they return no answer, or one without implications for the larger set—we then expand our efforts to ask the question for the full set of stations linked by interference to the station we wish to repack.

Clique-based pre-solving

Another class of pre-solving routines that our feasibility checker implements is based on identifying groups of stations that are all strongly linked to each other by interference constraints. Specifically, we are interested in sets of stations where every pair of stations within

the set interfere with each other; in the language of graph theory, such a set of stations forms a *clique* with respect to the interference links between stations. Critically, if we have such a set of stations, then each must be assigned a channel different from all other stations in the set, and so we cannot feasibly repack the entire set if there are fewer channels available than there are stations in the set. As Kearns and Dworkin (2014) observed, many infeasible repacking instances can be tied to just such a set of stations blocking the repacking.

Our pre-solving routines leverage this notion of blocking cliques to identify stations that become infeasible to repack due to such a configuration. In particular, we use the fact that as soon as the number of stations we have repacked in a clique reaches the total number of channels available at our current clearing target, we can immediately declare all remaining stations in this clique as being infeasible to repack. By tracking when such cliques reach their capacity, and identifying the stations that become infeasible when this occurs, we are able to avoid the need to perform any further computation—notably, any need to solve SAT instances—to determine that these stations must be frozen.

One challenge in implementing the pre-solving technique described above is that identifying all of the cliques in a graph is a computationally hard problem. In fact, even the simpler task of determining just the size of the largest clique belongs to the same class of problems as SAT. One approach to surmounting this challenge seen in prior work is to perform random sampling to get a large subset of cliques. We take a different approach, and implement the Bron-Kerbosch algorithm for finding all maximal cliques, and in particular the variant proposed by Eppstein et al. (2010). Although in theory the runtime of this algorithm can be high, we found that in practice it computes all maximal cliques in the interference graph in a matter of hours. Furthermore, most of this time is spent on identifying small cliques. Since only cliques with more stations than there are channels available under a clearing target are relevant to our pre-solving approach, we prune the search space of the algorithm to remove such small cliques. We found that doing so cut the computation time down to be on the order of minutes.

One subtlety in implementing this pre-solving routine is how the links between stations should be defined, especially since two stations that interfere with each other on one channel may not interfere on a different channel. To avoid the chance of misidentifying stations that are feasible to repack as being blocked by a clique, we use the following criteria: we consider two stations to be linked if, given a particular clearing target, there is no channel available on which both stations can be simultaneously placed. We include both cases caused by co-channel interference, and cases where one (or both) stations are prevented by domain restrictions from being placed on a channel.

The above definition guarantees the correctness of all produced results; however, the strength of the definition brings with it some shortcomings. Most significantly, it ignores the fact that in addition to co-channel interference, it is also possible for stations to experience adjacent channel interference. Such interference can, in some cases, result in the need to space stations in a clique more widely among available channels. In such cases, we will never reach the theoretical

maximum number of stations the clique can tolerate, and so our pre-solving routine will fail to identify the fact that the remaining stations are impossible to repack. Another shortcoming of the approach is that since our definition takes both interference and station domains into account, links between stations are dependent on the clearing target. Thus, we need to compute our lists of maximal cliques once for every possible clearing target.

In implementing this pre-solving routine, we first precompute the full list of cliques induced by interference between stations, for each possible clearing target. At the start of a simulation run, the feasibility checker initializes a list of cliques from the appropriate computed list. Over the course of the simulation, the feasibility checker tracks the number of repacked stations in every clique. Each time a station exits and is repacked, every clique it belongs to has its count updated, and for any clique that reaches capacity, any participating stations not already repacked are frozen.

RZR simulator

Our simulations included a full implementation of our proposed RZR procedure. We refer the reader to Figure A4 for an overview of the RZR procedure itself. Here we describe our implementation of RZR for the purposes of simulation. Our focus is on the details of the optimization processes used both to select clearing targets and to repack stations that reject opening or RZR prices. Discussing these optimizations requires us to first describe how we chose to model impairment in the context of our simulations. To put these issues in context, we briefly outline the steps in the RZR process below.

1. We identify all stations rejecting opening prices, and place them in the initial repack set.
2. We use the initial repack set to determine a clearing target for the auction.
3. We repeatedly apply the RZR procedure, until either no new stations freeze in round zero, or all such station accept the offered RZR price.
4. We determine the final pre-auction state of all participants.

Since our simulations assume straightforward bidding by participants, step 1 is simply a matter of comparing reservation values against opening prices; similarly, step 4 simply requires checking whether each station accepting a RZR prices is still frozen by the final repack, and either making a RZR payment to them or allowing them to continue to the auction proper as appropriate. As steps 2 and 3 involve the most complexity, we focus on those steps—as well as our model of impairment—in the following sections.

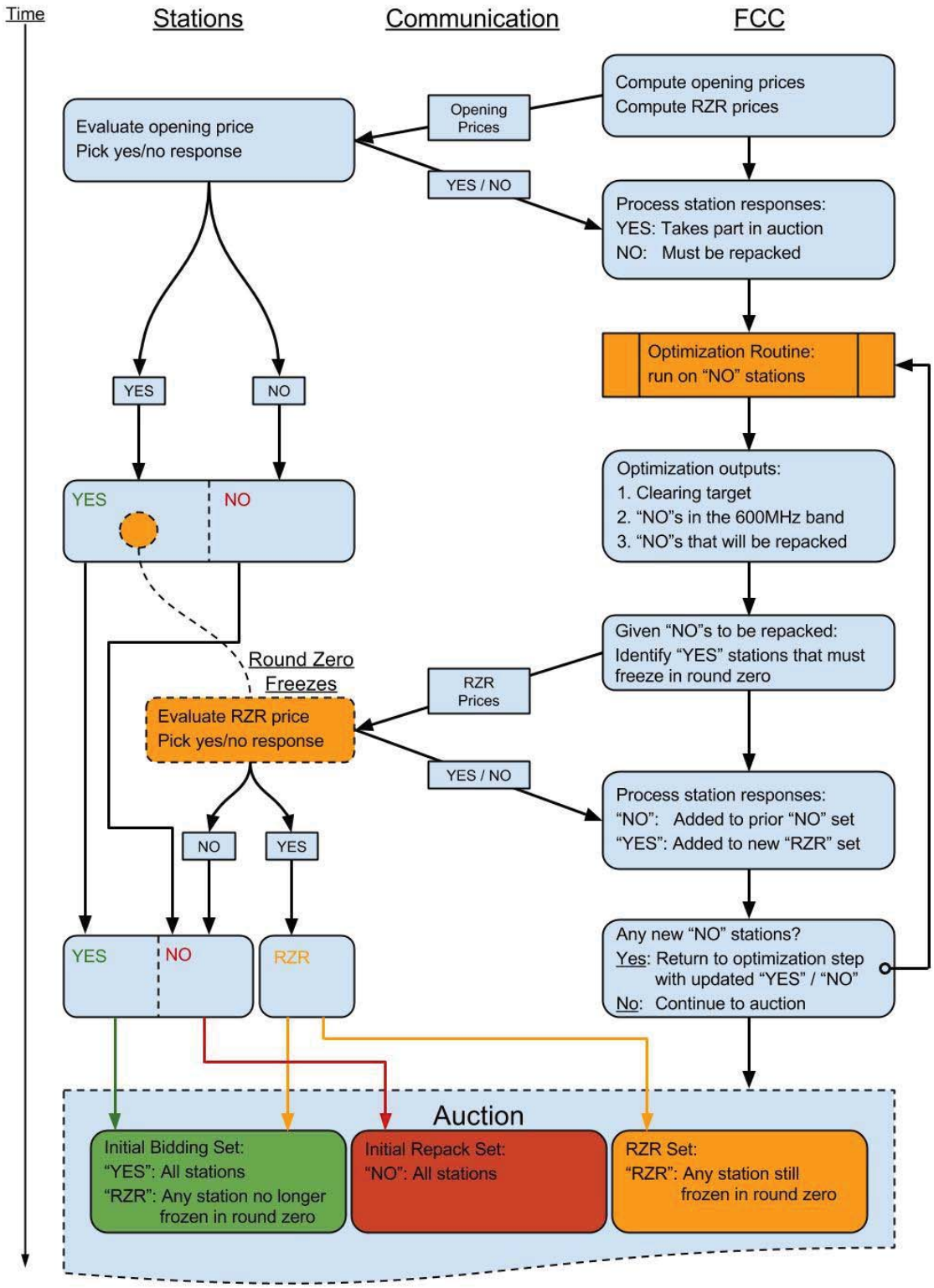


Figure A4: Overview of RZR process

Impairment model

We adopt a simplified model of impairment when implementing RZR and evaluating the quality of the solutions it produces. We model impairment in a straightforward manner, and say that assigning a station to a particular channel will impair a particular wireless license whenever both of the following hold:

1. The PEA served by the license contains one or more points predicted by TVStudy to receive service from the television station in question; and
2. The license block occupies spectrum that overlaps with, or is separated by 3 or fewer MHz from, the spectrum occupied by the assignment channel.

We place weights on impaired licenses in a manner consistent with that described in the Comment PN: we assign each PEA a weight equal to the population times a PEA-specific index value (the third and fourth columns in the table presented in Appendix F of the Comment PN). Further, we give an impairment of just the uplink portion of a license one-half weight.

Optimizing the clearing target

Our procedure for optimizing the clearing target is extremely simple. We identify the largest target that is achieved with zero impairment in each of the New York and Los Angeles PEAs, and then set the larger of these as our clearing target. The main detail that must be specified is exactly how we implement finding the maximum possible clearing target in each of these two critical PEAs. The approach we use follows directly from our impairment model.

First, we identify all stations which, if repacked into the 600MHz band, would result in impairment of licenses in the New York PEA or in the Los Angeles PEA. We then form a list of all PEAs that contain at least one such station, and consider them as possible sources of impairment to New York and Los Angeles. Under our impairment model, we find that stations in the New York, Philadelphia, Boston, Springfield, Harrisburg, Binghamton, and Albany PEAs; and the Los Angeles, San Diego, Las Vegas, San Francisco, Fresno, and Yuma PEAs are potential sources of impairment for the New York and Los Angeles PEAs, respectively.

To determine the largest target we can achieve with zero impairment in the New York PEA, we identify all non-participants from any of the PEAs we identify as a potential sources of impairment for New York. We then submit this set of stations to our feasibility checker with progressively higher targets, in order to determine the largest one we can achieve without the need to impair any licenses. We use the same process to determine the largest target we can achieve with zero impairment in the Los Angeles PEA, by considering non-participating stations in those PEAs we identified as potential sources of impairment for Los Angeles. Finally, we take the larger of the two targets from New York and LA, and adopt it as our clearing target for the auction itself.

Applying RZR to participating stations

Each round of RZR has three main steps: first, all stations that have rejected either opening prices or RZR prices are repacked, potentially on channels in the 600MHz band if necessary; second, all

participating stations that have yet to receive a RZR offer are checked to see if this new repack causes them to freeze in round zero; and finally, all stations newly frozen in round zero are offered their RZR price. The above process continues until we have a round where either no new stations are frozen in round zero, or all stations that are newly frozen accept their offered RZR price.

The main technical challenge in implementing the RZR process comes from finding a repack of the stations which have rejected either opening or RZR prices. Our implementation seeks a repack of the stations that achieves the minimum possible impairment. It does so by formulating the interference constraints, impairment model, and impairment weights as an Integer Program, which is then submitted to the Gurobi Optimizer. We give the formal definition of the optimization problem we solve in Figure A5.

The objectives and constraints in this optimization program follow naturally from our objective of finding an impairment-minimizing repacking. The objective function is precisely to minimize impairment, adjusted by weighted population; the factor of two is to account for the fact that an uplink-only impairment is considered to only impair one-half of the license. Constraints (1), (2), and (3) capture that we find a valid repacking; that is, one that respects co-channel interference, adjacent-channel interference, and domain constraints, respectively. Constraint (4) ensures that we do not allow stations in our repack set (those that have rejected prices) to be assigned to go off-air. Constraints (5) and (6) require that the downlink and uplink portions of a license be impaired if a conflicting channel assignment is made, while Constraint (7) requires that the entire license be impaired if the downlink portion is.

$$\min_{\boldsymbol{x},\boldsymbol{y}} \sum_{\ell \in L} \frac{w_\ell}{2}\left(y_\ell^D + y_\ell^U\right) \text{ subject to:}$$

$$x_{s,c} + x_{s,c'} \leq 1 \qquad \forall\{(s,c),(s,c')\} \in \text{CoPairs} \qquad (1)$$

$$x_{s,c} + x_{s',c'} \leq 1 \qquad \forall\{(s,c),(s',c')\} \in \text{AdjPairs} \qquad (2)$$

$$\sum_{c \in C_s} x_{s,c} = 1 \qquad \forall s \in S \qquad (3)$$

$$x_{s,0} = 0 \qquad \forall s \in R \qquad (4)$$

$$x_{s,c} \leq y_\ell^D \qquad \forall (s,c) \in \text{SC}_\ell^D \qquad (5)$$

$$x_{s,c} \leq y_\ell^U \qquad \forall (s,c) \in \text{SC}_\ell^U \qquad (6)$$

$$y_\ell^D \leq y_\ell^U \qquad \forall \ell \in L \qquad (7)$$

$$x_{s,c} \in \{0,1\} \quad \forall s \in S, c \in C_s \qquad (8)$$

$$y_\ell^D, y_\ell^U \in \{0,1\} \quad \forall \ell \in L \qquad (9)$$

where:

- $x_{s,c}$ is a binary decision variable indicating whether station s has been assigned to channel c, where $c = 0$ corresponds to going off-air;
- y_ℓ^D and y_ℓ^U are binary decision variables indicating whether the downlink or uplink portion of license ℓ has been impaired;
- w_ℓ is the weighted population of the PEA served by license $\ell \in L$;
- S is the set of all UHF stations;
- R is the set of all UHF stations that have rejected either RZR or opening prices;
- C_s is the set of all channels c that station s may be assigned to, plus 0;
- L is the set of all licenses available under the current clearing target, where each license $\ell \in L$ specifies a spectrum block and a PEA;
- CoPairs and AdjPairs contain all pairs of assignment pairs that conflict due to co-channel and adjacent-channel interference; and
- SC_ℓ^D and SC_ℓ^U are the sets of all channel assignments (s,c) which cause impairment to the downlink and uplink portions of license $\ell \in L$.

Figure A5: Optimization program used in RZR to minimize impairment

Once we have repacked the stations that have rejected prices so far, we proceed to identify any newly frozen stations, and simulate the process of making them a RZR offer. We handle the process of identifying newly frozen stations via our feasibility checker: we initialize it with the channel assignment found by the Gurobi Optimizer, and then query it to see whether each of the remaining stations can be feasibly added to this repack. If not, we must offer the station their RZR price. As our implementation assumes straightforward bidding by participants, evaluating a

bidder's response to such an offer is a simple matter of comparing the offered price to the station's reservation value.

Scoring rules

The scoring rule determines the opening prices in the reverse auction. These are critical to motivate participation of broadcasters, as the opening price is the maximum price that a station can receive, and a commitment to participate in the auction is a commitment to accept the opening price.

The scoring rule consists of two components, the base clock price and volume, in particular:

$$\text{Score} = (\text{base clock price}) \times (\text{volume})$$

For the base clock price, we considered two alternatives in addition to the FCC price of $900: $1250 and $1500. These two base clock prices increase the FCC base clock price to encourage participation and thereby make the auction more robust to high broadcaster reservation values.

For volume, we focus on these measures in our analysis:

$$\text{FCC volume} = (\text{Broadcast population})^{1/2} \times (\text{Interference count})^{1/2}$$

$$\text{Reweighted volume} = (\text{Broadcast population})^{1/4} \times (\text{Interference count})^{1/2}$$

$$\text{FCC–value volume} = \max(\text{FCC volume, Value-based})$$

$$\text{Reweighted-value volume} = \max(\text{Reweighted volume, FCC volume, Value-based})$$

$$\text{Freeze-value volume} = \max(\text{Freeze volume, FCC volume, Value-based})$$

where

Broadcast population = a station's interference-free broadcast population (IF). This is the FCC's population measure defined in ¶96 of the Comment PN. We use "broadcast population" rather than "interference-free population" to highlight that this population is referring to broadcast coverage.

Interference count = a count of the station's pairwise interference constraints (IC). This is the FCC's interference measure also defined in ¶96 of the Comment PN.

Value-based = a measure of the spectrum value generated by the exit of a station. It is calculated as (Precluded population) × (AWS-3 price).

Precluded population = the population that cannot be served by any other station if the specified station is repacked (PC). This is a new measure of population that better reflects the population that the station interferes with if repacked.

AWS-3 Price = each station's weighted average Price/MHzPop as observed in the AWS-3 auction (AP). Each station's weights are the fraction of its precluded population in each partial economic area (PEA).

Freeze volume = the expected population precluded as a result of a station's broadcast, calculated as (Precluded population) × (Freeze probability), where

Freeze probability = the long-run frequency with which the station freezes in simulations with a random order of station exits (FR). This is a measure of repacking difficulty that follows directly from the interference constraint and domain files. It was initially proposed in Kearns and Dworkin (2014).

Following the FCC's approach, we scale all volumes to have a maximum value of one million. This uniform scaling method provides solid ground for comparisons in auction results with alternative volume metrics.

Volume comparisons

In this section we compare the five different volumes under analysis, all scaled to have a maximum value of one million.

Figure A6 shows the value of each volume metric. Stations are in a decreasing order based on their FCC volume. Both the FCC-value and Freeze-value volumes are attempts to combine the virtues of the FCC metric with a value-based metric and a freeze-based metric. Both combined metrics (FCC-value and Freeze-value) follow the FCC curve very closely, with the exception of a handful of stations in New York, Los Angeles and San Diego.

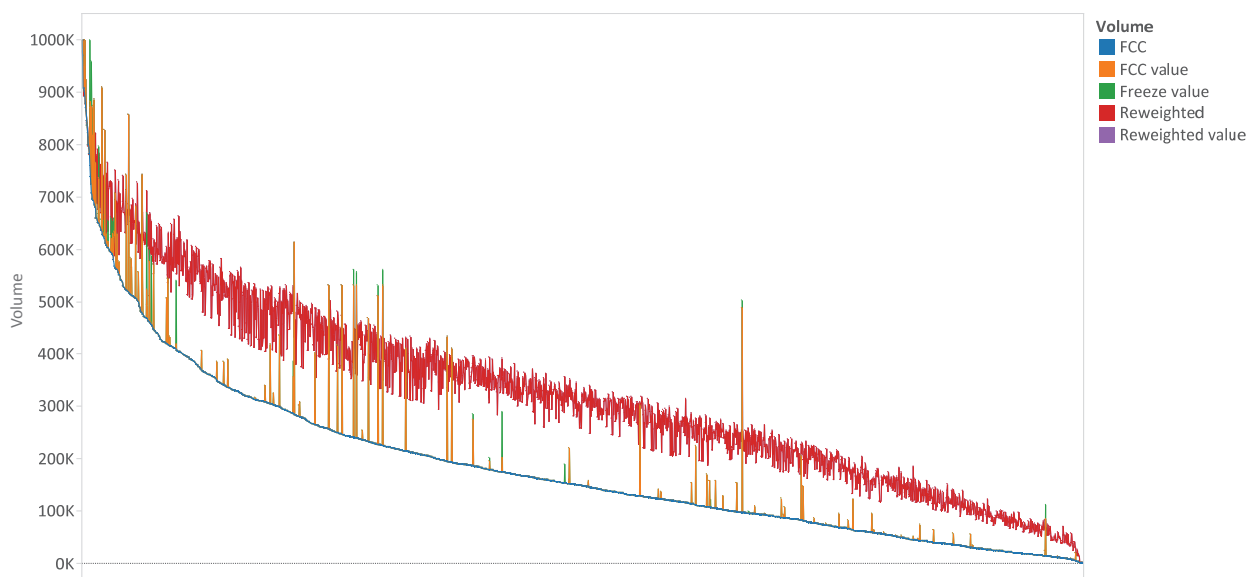


Figure A6: Volume curves

The Reweighted and the Reweighted-value volumes are almost identical with differences for only about 30 stations spread across the nation. The most important difference among the various

volumes is the steepness of the volume curve. Reweighted and Reweighted-value are much flatter than the other volumes. This characteristic cannot be stressed enough. A flatter volume curve means that high-value stations are more apt to be resolved first. Resolution from big stations to small stations tends to promote efficiency, maximizing the value of the repack, as small stations are less apt to get in the way of more valuable larger stations. This intuition is demonstrated in our simulation results. Auctions using Reweighted volume tend to perform better than the alternatives.

Figure A7 provides the cumulative frequency for each volume. The flatter curve with Reweighted volume (red) again means that relative to FCC volume smaller stations have a higher opening price, motivating participation and competition in the auction.

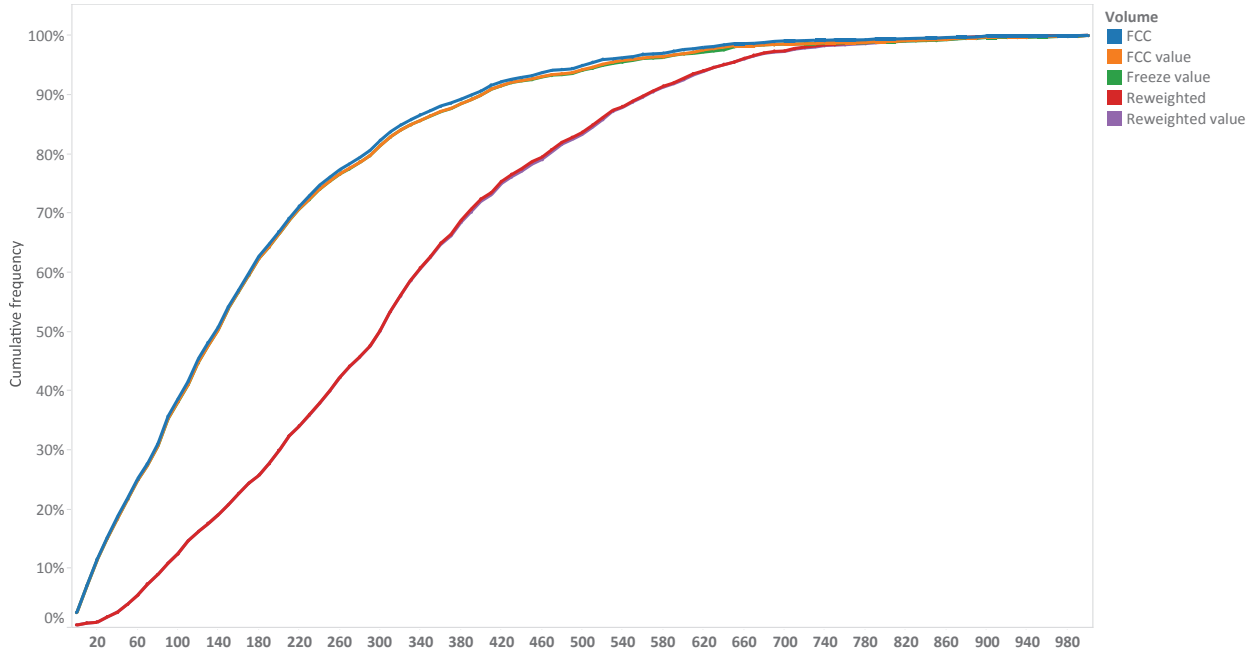


Figure A7: Distribution of volumes

Figures A8-12 show each volume metric on a map. The most important difference is seen in the first two figures which show FCC and Reweighted volume. The Reweighted volumes have a more gradual transition from high value markets such as New York to mid-value markets in the center of the country.

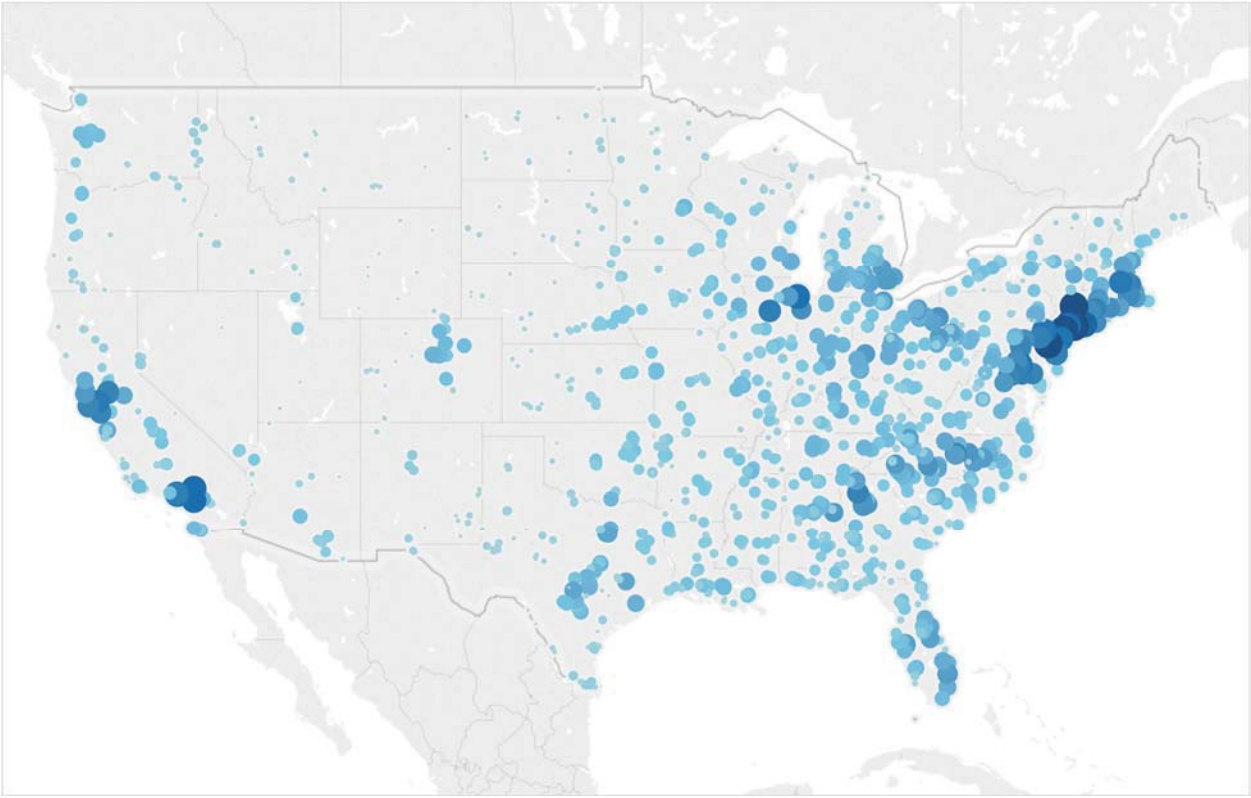


Figure A8: FCC volume

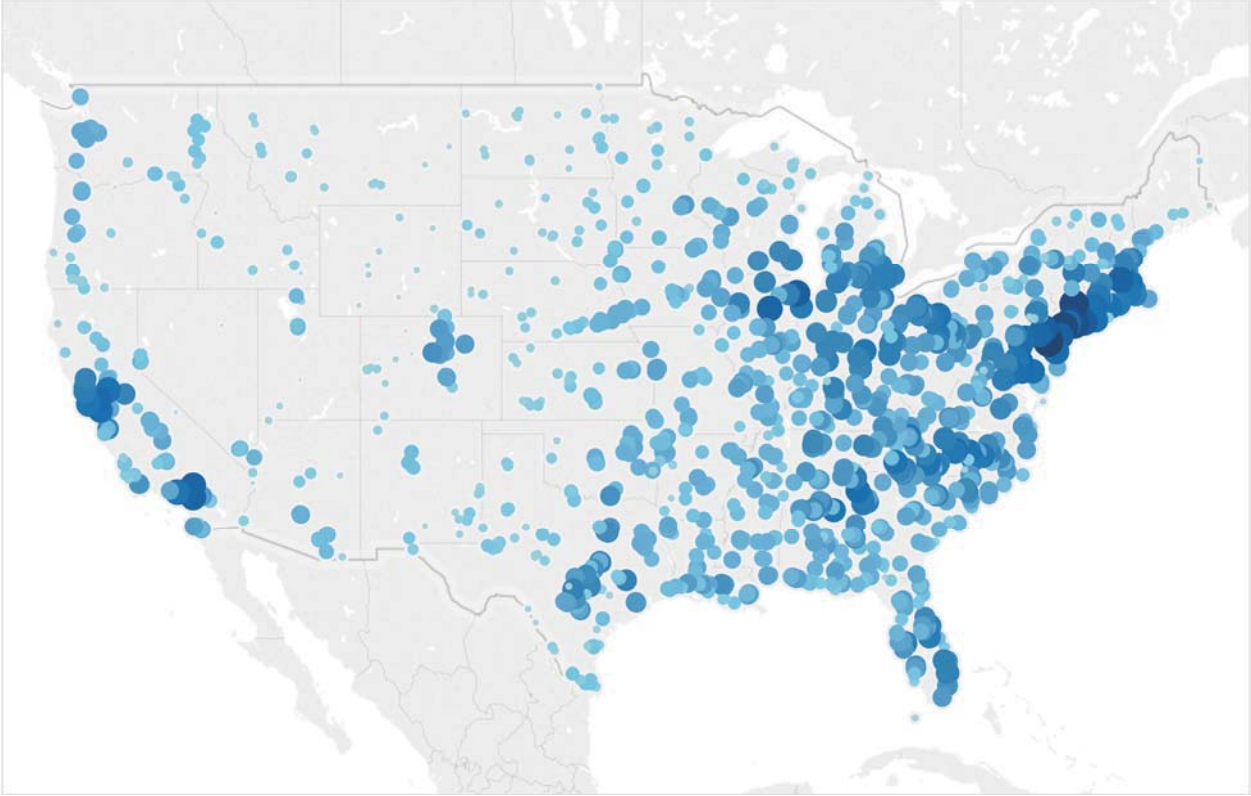


Figure A9: Reweighted volume

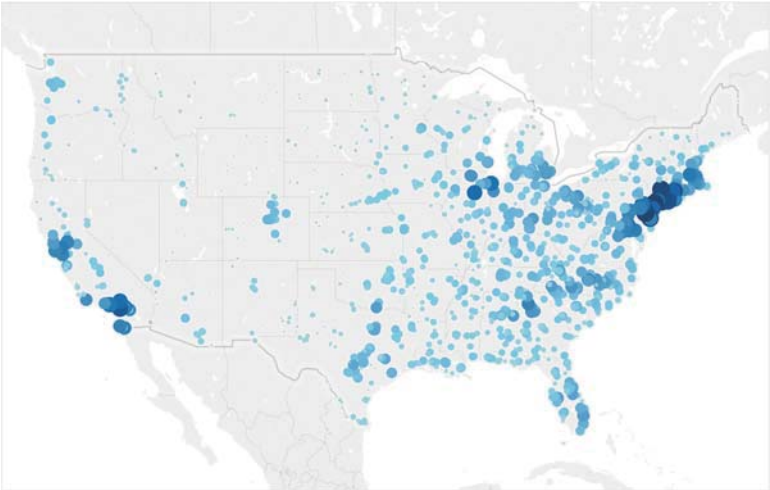


Figure A10: FCC-value volume

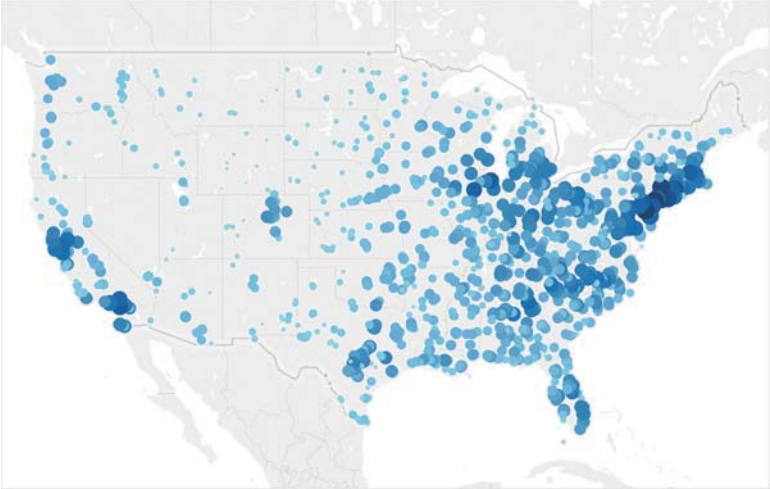


Figure A11: Reweighted-value volume

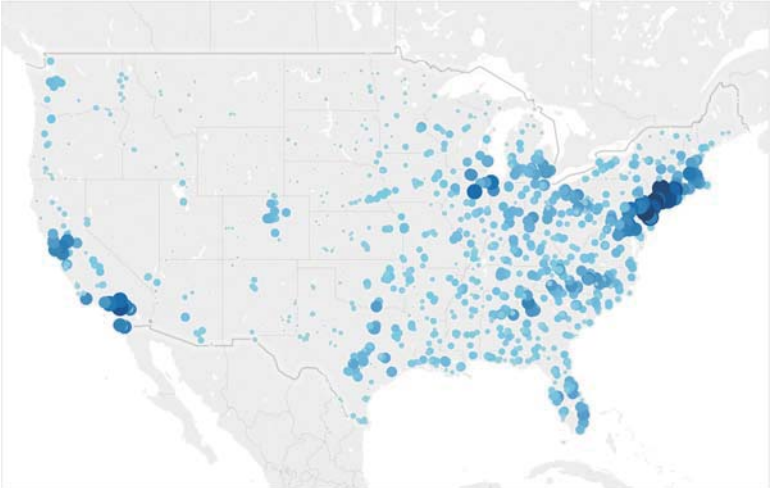


Figure A12: Freeze-value volume

Table A2 shows the average opening price for each volume with a $900 base clock price. In top markets the difference between FCC opening prices and Reweighted is relatively small. For example, the differences are 8.13% for New York and 9.86% for Los Angeles. In smaller markets the difference increases, but our simulations show that this does not imply a higher clearing cost since competition drives opening prices down to competitive levels.

Table A2: Average Score for top-20 DMAs in million $

DMA name	FCC	Reweighted	FCC value	Reweighted value	Freeze value
New York, NY	640	691	684	703	685
Los Angeles, CA	508	549	547	564	558
Baltimore, MD	469	601	469	601	469
Philadelphia, PA	468	599	516	612	531
Chicago, IL	464	578	478	578	478
Boston, MA	372	514	372	514	372
San Francisco-Oakland-San Jose, CA	368	495	369	495	369
Atlanta, GA	350	490	350	490	350
Greensboro-High Point-Winston Salem, NC	350	544	350	544	350
Providence, RI-New Bedford, MA	348	498	348	498	348
Washington, DC	342	482	350	482	352
Dayton, OH	334	517	334	517	334
Youngstown, OH	333	532	333	532	333
Harrisburg-Lancaster-Lebanon-York, PA	327	508	336	508	370
Detroit, MI	325	464	325	464	325
Hartford-New Haven, CT	323	493	358	505	358
Sacramento-Stockton-Modesto, CA	307	447	307	447	307
Tampa-St Petersburg-Sarasota, FL	306	453	306	453	306
Lansing, MI	275	464	275	464	275
Rockford, IL	214	409	376	413	376

Precluded population (PC)

The definition of precluded population is the *population that cannot be served by any other station if a certain station is repacked*. It is a quantity that can be derived from the pairwise interference file, together with the associated output from TVStudy. It has many attractive properties, as discussed in the main text. For example,

- For a station that causes no interference, precluded population is its interference-free broadcast population.
- Blocked population is only counted once. Unlike some metrics which grow to large numbers with no intuitive meaning, precluded population produces numbers that still represent real population counts. They are higher than the broadcast population counts because they include blocked populations that are outside a station's service contour or on adjacent channels. So for the KAMU-TV example shown in Table A3, the broadcast population is only 330,386, but the precluded population is 8.5 million. The interpretation of these numbers is simply that if KAMU-TV is assigned to channel 25, it will make it impossible for any other station to provide service on channel 25 to 8.5 million people, including 330,386 inside KAMU's contour, and 8.2 million people outside of KAMU's contour. KAMU should be priced equivalently in the auction to other stations in the same area that block service to 8.5 million people when repacked.
- Simulations show that the sum of the precluded populations of all stations that can be packed onto a single channel across the country averages about 300 million—close to the national population. Intuitively this is right because in a tight repack almost the entire national population should be precluded, otherwise there would be open spaces available for repacking more stations.
- The sum of precluded populations of repacked stations is much less variable in our simulations than the sum of broadcast populations, suggesting it is a better indicator of volume—when optimally packing a trunk with suitcases the sum of the volumes of the packed suitcases is roughly a constant equal to the volume of the trunk.

Precluded population is easily calculated using the following method:

- The FCC paired interference file lists all the stations with which a given station is mutually exclusive ("blocked stations").
- Any point that can receive service from a blocked station, but cannot receive service from any unblocked station is "precluded" from service if the given station is repacked.
- With the detailed cell-level output files from TVStudy, each precluded point can be identified, and the population associated with those points can be added up to determine the total precluded population.
- The same method is repeated for a given channel and each of the adjacent channels.
- We weight adjacent channel preclusion at 50%, because our analysis indicates adjacent channel interference had approximately 1/2 the significance of co-channel interference.

We did this calculation using a proxy channel (25), but this could easily be done on every channel and averaged, or on some other basis to reflect varying preclusion across a channel range.

To get some intuition for the calculation, it is easiest to see an example. The full detail is available in the code and the resulting measure for each station. We focus on KAMU. Table A3 shows the interference free population and the precluded population for KAMU in each PEA using a proxy channel 25. The population measures are then found by summing over all PEAs. KAMU has 330,386 interference free pops and 8.551 million precluded pops.

Table A3: Precluded population by PEA for KAMU-TV (using proxy channel 25)

facid	PEA	IntFreePop	Precluded Pop CO	Precluded Pop Adj+	Precluded Pop Adj-	Total Precluded Pop Count
65301	Houston, TX	12,662	5,821,376	-	-	5,834,038
65301	Austin, TX	11,787	1,085,088	-	-	1,096,875
65301	Waco, TX	-	621,992	-	-	621,992
65301	Nacogdoches, TX	30,794	333,720	44,144	36,914	405,043
65301	Beaumont, TX	-	4,320	-	-	4,320
65301	Victoria, TX	52,077	105,250	189	24	157,434
65301	Eagle Pass, TX	-	225	-	-	225
65301	Bryan, TX	220,054	1,022	-	-	221,076
65301	Brownwood, TX	-	32,309	3,990	1,729	35,169
65301	Corsicana, TX	-	22,209	-	-	22,209
65301	Lockhart, TX	3,012	57,693	-	-	60,705
65301	Jacksonville, TX	-	152	-	-	152
65301	Natchitoches, LA	-	17	-	-	17
65301	Mineral Wells, TX	-	699	-	-	699
65301	Gonzales, TX	-	88	-	-	88
65301	Marble Falls, TX	-	62,051	-	-	62,051
65301	Del Rio, TX	-	14	-	-	14
65301	Lampasas, TX	-	25,078	-	-	25,078
65301	Brady, TX	-	3,897	-	-	3,897
65301	(blank)	-	-	-	-	-
Total		330,386	8,177,200	48,323	38,667	8,551,081

Here are the steps of the calculation. The calculations are done for every 2 km × 2 km cell using detailed coverage data produced by TVStudy. However, the basic logic is easiest to understand graphically.

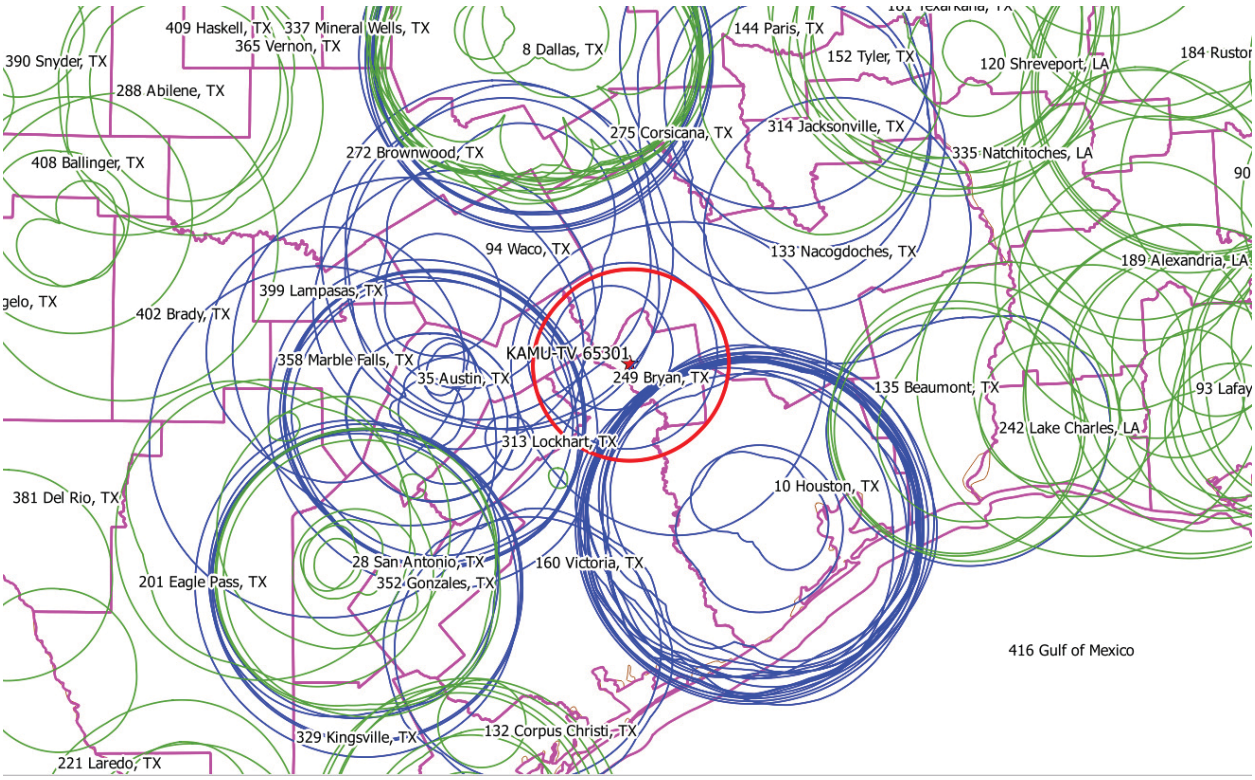
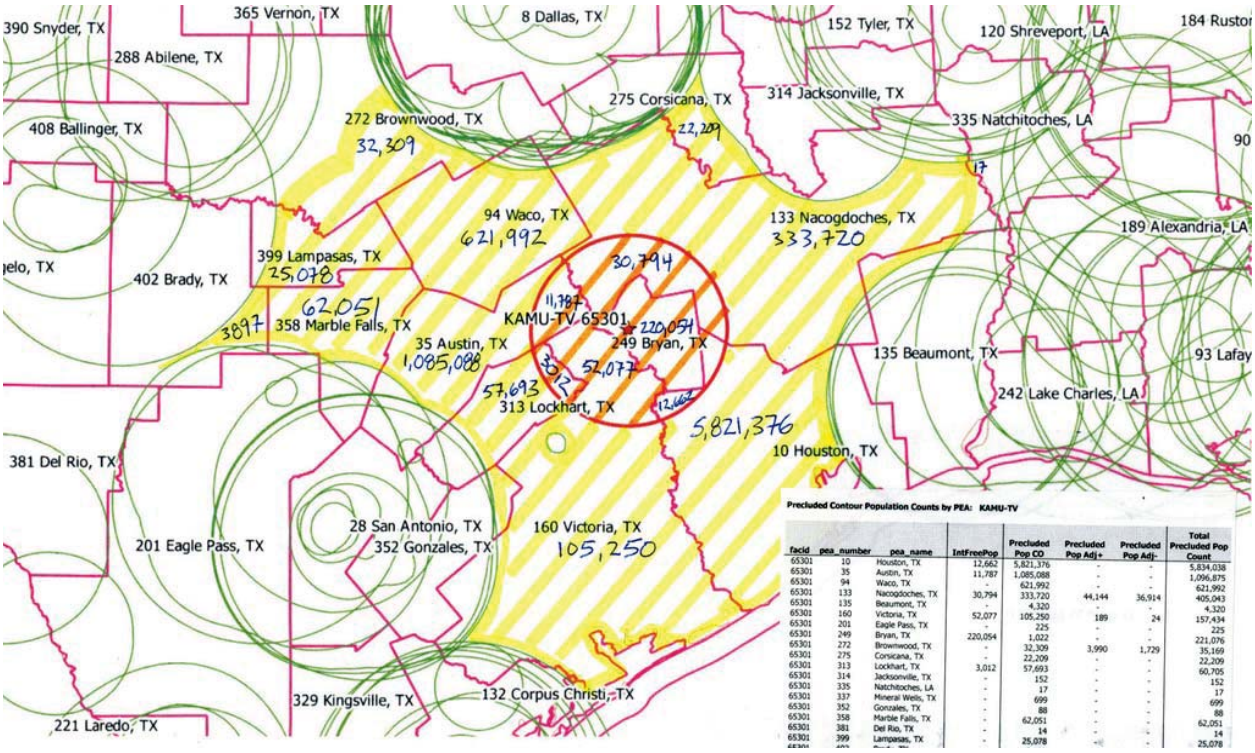


Figure A13: Contour of KAMU (red), contours it interferes with (blue) and does not (green)

Figure A13 shows the first step. For a given station (shown in red), we find all the contours it interferes with (in blue) and all the contours it does not interfere with (in green).



Precluded Contour Population Counts by PEA: KAMU-TV

facid	pea_number	pea_name	IntFreePop	Precluded Pop CO	Precluded Pop Adj+	Precluded Pop Adj-	Total Precluded Pop Count
65301	10	Houston, TX	12,662	5,821,376	-	-	5,834,038
65301	35	Austin, TX	11,787	1,085,088	-	-	1,096,875
65301	94	Waco, TX	-	621,992	-	-	621,992
65301	133	Nacogdoches, TX	30,794	333,720	44,144	36,914	405,043
65301	135	Beaumont, TX	-	4,320	-	-	4,320
65301	160	Victoria, TX	52,077	105,250	189	24	157,434
65301	201	Eagle Pass, TX	-	225	-	-	225
65301	249	Bryan, TX	220,054	1,022	-	-	221,076
65301	272	Brownwood, TX	-	32,309	3,990	1,729	35,169
65301	275	Corsicana, TX	-	22,209	-	-	22,209
65301	313	Lockhart, TX	3,012	57,693	-	-	60,705
65301	314	Jacksonville, TX	-	152	-	-	152
65301	335	Natchitoches, LA	-	17	-	-	17
65301	337	Mineral Wells, TX	-	699	-	-	699
65301	352	Gonzales, TX	-	88	-	-	88
65301	358	Marble Falls, TX	-	62,051	-	-	62,051
65301	381	Del Rio, TX	-	14	-	-	14
65301	399	Lampasas, TX	-	25,078	-	-	25,078

Figure A14: Precluded population (co-channel)

Figure A14 shows the second step. We remove all the blocked blue contours, leaving only the green contours that can potentially be repacked co-channel with the subject channel (KAMU). Any population that is inside a blue contour but is not inside a green contour is precluded from service if the station is repacked. The yellow and orange areas are precluded. The orange area is the station's own service area. If the station did not interfere with any other stations, then only the orange area would be precluded. The results can be aggregated in a variety of ways. Below the results are totaled by PEA as in Table A3.

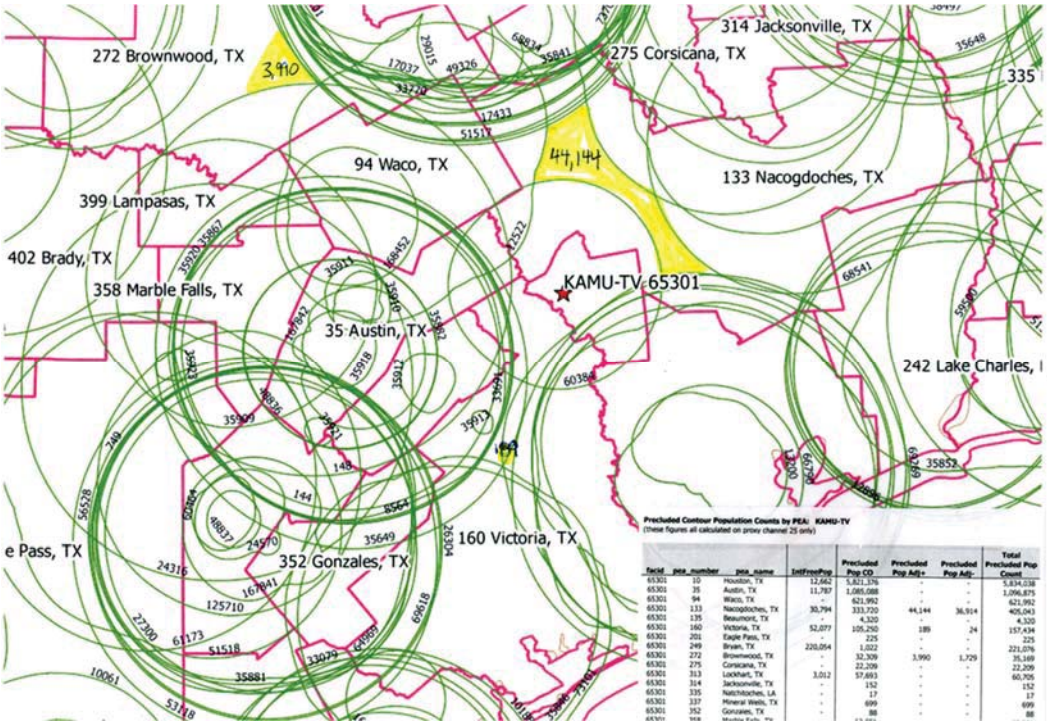


Figure A15: Precluded population (adjacent channel above)

Figure A15 shows the third and final step. The same method is applied to the adjacent channel above and the adjacent channel below (Figure A15 only shows the adjacent channel above calculation). KAMU has little adjacent channel blocking effect. The precluded population calculation includes only ½ of the adjacent channel blocking, because our analysis indicates adjacent channel interference had approximately ½ the significance of co-channel interference.

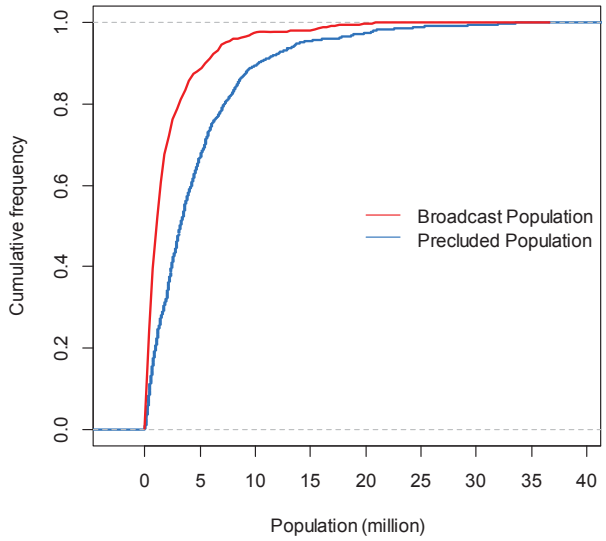


Figure A16: Cumulative distribution of precluded population and broadcast population

Figure A16 shows the cumulative frequency of precluded population and broadcast population. All stations have a higher precluded population than broadcast population, but the difference, as a proportion of the broadcast population, gets smaller as stations with higher populations are considered.

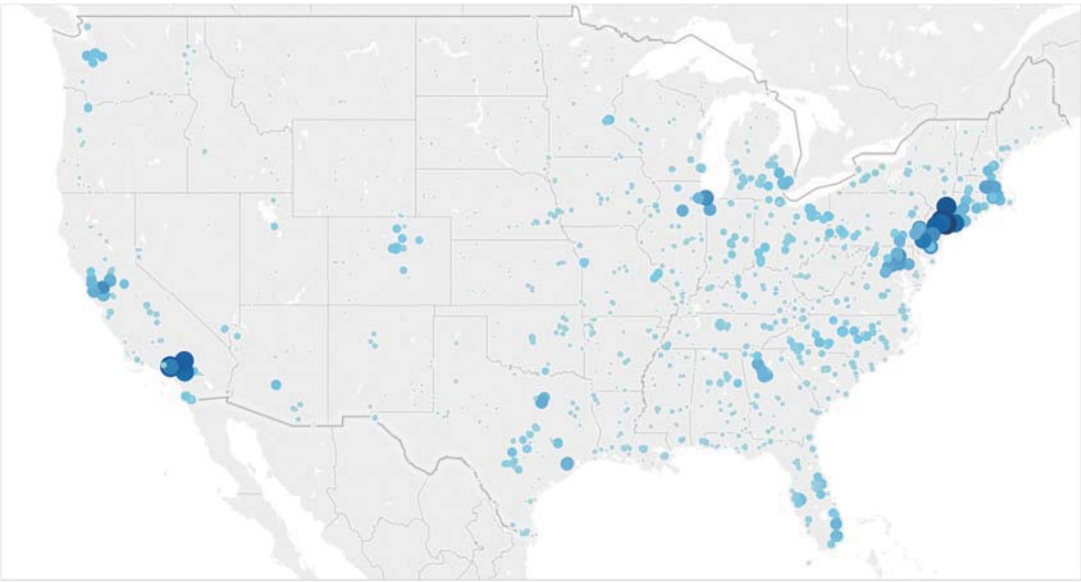


Figure A17: Map of broadcast population

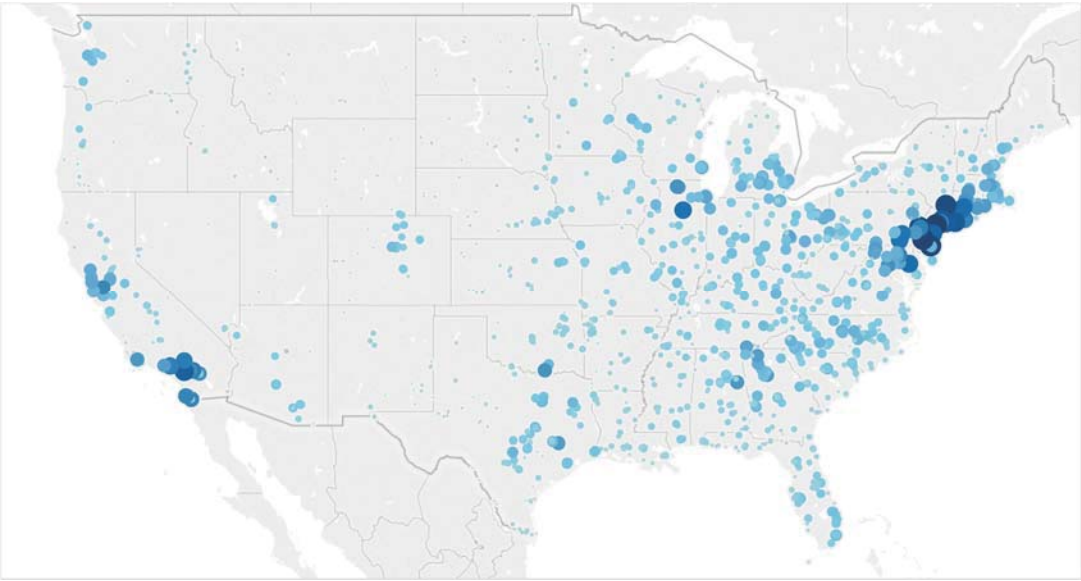


Figure A18: Map of precluded population

Figures A17 and A18 show maps of broadcast population and precluded population. The color and size represent the value for each measure; the larger and darker every mark is, the higher the associated population. Precluded population is more evenly distributed in all markets. This is especially important in New York and Los Angeles since relatively small stations, using the

broadcast population metric, are more apt to be repacked, preventing larger stations from serving nearby population.

Freeze probability (FR)

Repacking constraints interact in complex, hard-to-predict ways. Thus, we propose to study the difficulty of repacking stations via an outcome-based, rather than input-based, analysis of the repacking process. We propose the freeze probability (FR), the long-run frequency that a station freezes given a random exit of stations, as a measure of a station's importance in the repacking process. The freeze probability is readily calculated by simulating thousands of auctions with random station exits.

Our calculations (a) assume full participation by all eligible stations; and (b) use completely random exit sequences for the set of stations. This combination of assumptions ensures both that we include the effects of all interference and domain constraints and that our results are free of any bias from a particular valuation model.

We compute the freeze probability as follows. Each simulation follows the same basic steps:

1. Choose a uniformly random order over all UHF stations
2. Begin with an empty repack set
3. For each station in turn check whether it can be feasibly added to the repack set:
    a. If yes, repack the station (add it to the repack set)
    b. If no, freeze the station and leave the repack set unchanged
4. At the end of the above process, every station is either repacked or frozen.

We run the above simulation process for a predetermined number of repetitions. At the end, we compute the fraction of the total number of runs in which the station is frozen; this is the station's freeze probability.

One caveat is that the freeze probability analysis is sensitive to station domains; because of this, the measure varies as a function of the clearing target, and is strongly affected by border constraints. One approach we have tried is considering a domain-free variant of freeze probability, but this faces the challenge that it becomes difficult to capture natural market-level variance in number of available channels—for example, that due to land mobile constraints.

Another caveat is that the freeze probability is more difficult to define over the VHF bands than it is for the UHF bands. Repacking issues can only arise in the VHF band due to relocation of UHF stations, and so it is more difficult to find a simulation-based approach that estimates VHF freeze probabilities while remaining mechanism-free.

We can compute the freeze probabilities of all UHF stations with good accuracy by performing a sufficiently large number of simulations. We bound the maximum error seen across all stations as follows. For each station, freezing is a binary process—it either does or does not happen. Thus, for each station, our set of simulation runs yields a set of identical and independent Bernoulli

trails. To bound the error seen for a given station, we can use classical concentration bounds such as Chernoff. One complication is that we want to bound the *maximum* error across all stations; since whether two stations freeze can be highly correlated, we use a union bound to go from the error of individual stations to this overall maximum error.

We compute freeze probabilities using 11,500 trials. This number produces a maximum error of 2.36% with a confidence of 99%. Figure A19 shows the behavior of the maximum error and its probability for our selected number of trials. The upper bounds shown are all derived using the combination of Chernoff and union bounds discussed above. The (approximate) lower bound is derived by calculating the exact distribution of the maximum error under the assumption that freeze probabilities are independent. While freeze probabilities are not independent, this assumption allows us to get an estimate of where the true maximum error is likely to lie. Figure A20 shows the number of trials required for different error levels at a confidence of 99%.

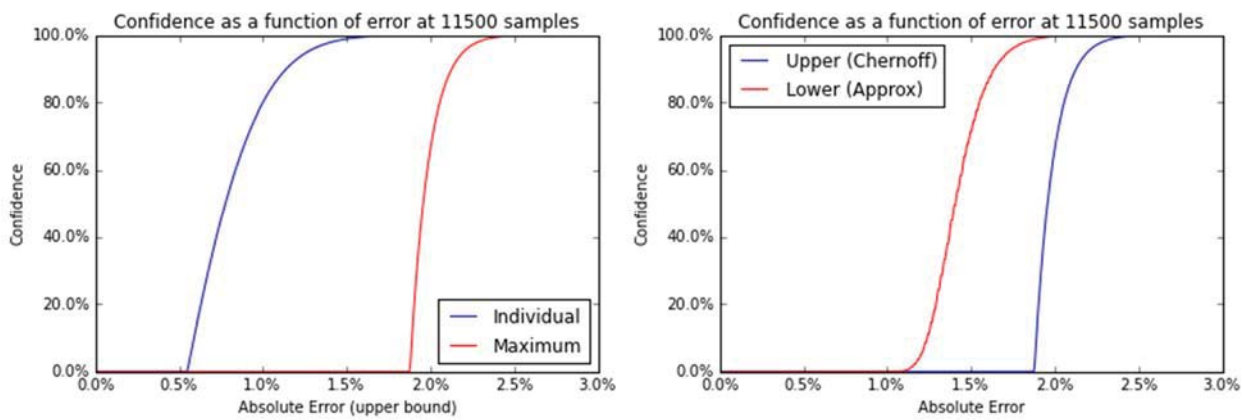


Figure A19: Confidence of FR estimation

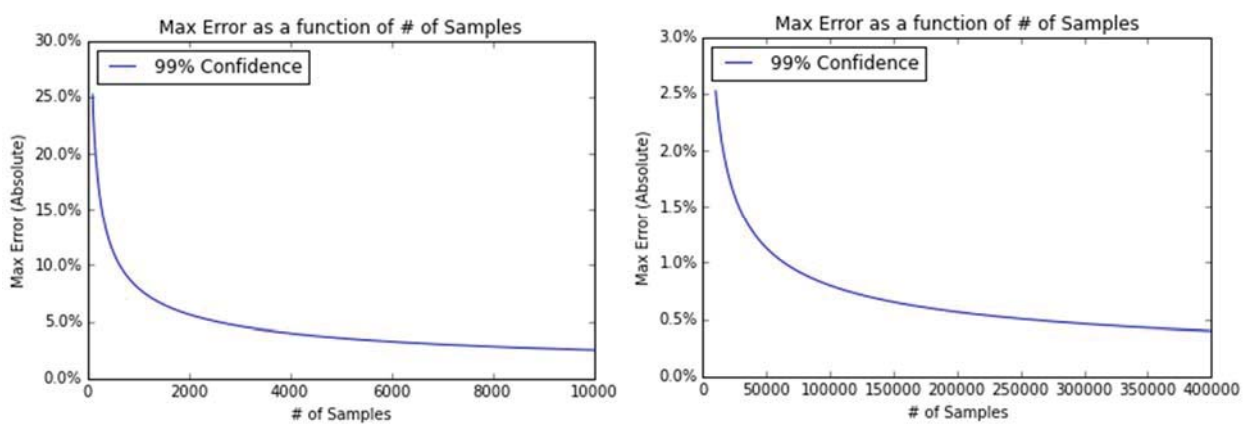


Figure A20: Trials and error of FR estimation

Figures A21 and A22 show maps of freeze probability and interference count. A key difference is that freeze probability correctly identifies the challenges in border markets; whereas, interference count does not. For example, in the Harlingen-Weslaco-Brownsville-McAllen DMA,

the interference count is relatively low, despite having a freezing probability of one due to domain constraints.

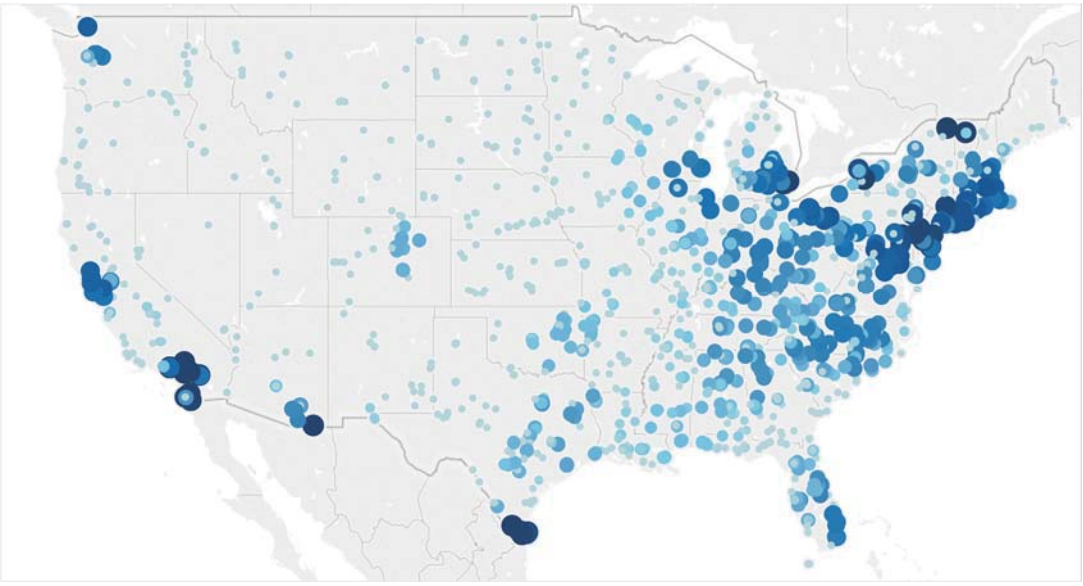


Figure A21: Map of Freeze probability

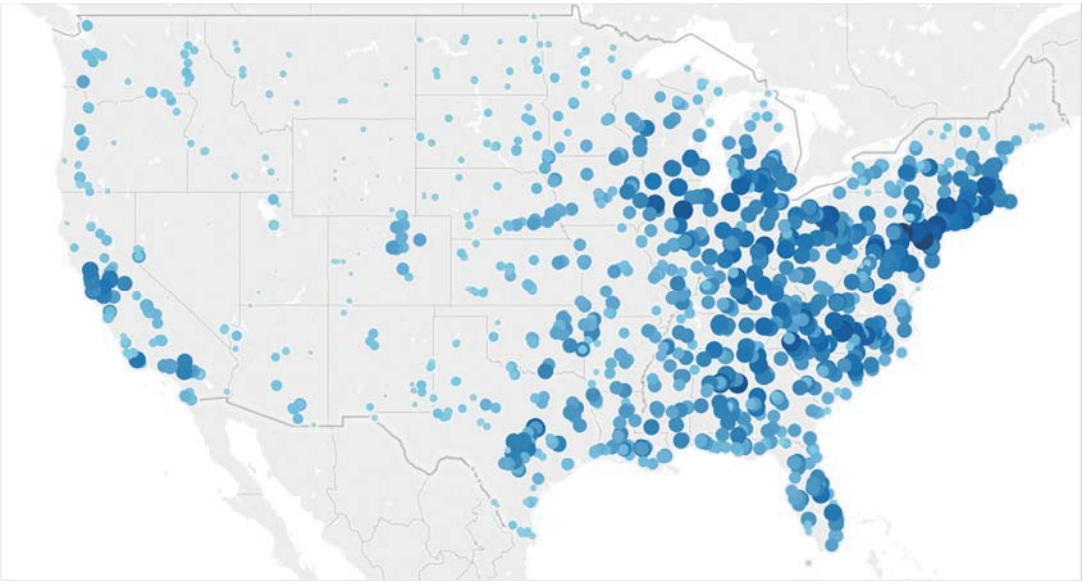


Figure A22: Map of Interference constraint count

When freeze probabilities are used to compute volumes they are bounded below by 0.1 and above by 0.8. The lower bound is introduced to obtain positive volumes for all stations, while the upper bound limits the volumes for stations which freeze in most cases. Figure A23 shows the distribution of freeze probability before and after bounds have been applied. The upper bound applies to a small number of stations.

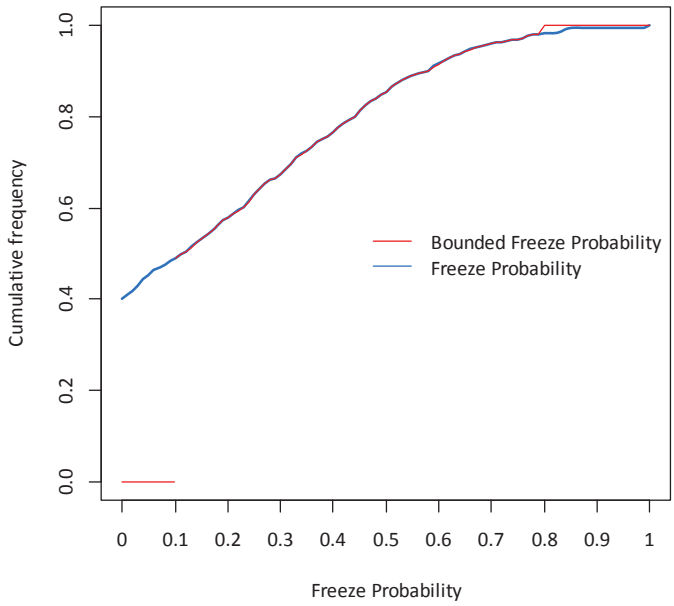


Figure A23: Distribution of Freeze probability with and without bounds

AWS-3 price (AP)

Forecasting demand in a spectrum auction typically is quite challenging. However, in this case, the challenge is greatly reduced as a result of the AWS-3 auction completed on 29 January 2015. The AWS-3 was a competitive auction for mid-band paired spectrum. Carriers will use this paired spectrum for mobile broadband using the same LTE technology as the 600 MHz spectrum. The key difference is that the mid-band AWS-3 spectrum has inferior propagation characteristics, especially for providing coverage in buildings, in difficult terrain, and in less densely populated areas. Nonetheless, the AWS-3 outcome does represent an excellent point of comparison with which to assess demand in the forward auction.

For each station we calculated the weighted average price/MHzPop as observed in the AWS-3 auction. Each station's weights are the fraction of its precluded population in each partial economic area (PEA). Finally, we use the maximum of the previously described calculation and the national average price in the AWS-3 auction of $2.71/MHzPop.

Table A4 details the calculation of the AWS-3 price for Komi-CD, located in Altus, OK. Komi-CD interferes with seven stations in seven PEAs. The precluded population for each of these interferences, as a fraction of the total precluded population of Komi-CD, is multiplied by its PEA price and then summed.

Table A4: AWS-3 price calculation for KOMI-CD

Facility Id	PEA	IF	PC-Co	PC-ADJp	PC-ADJm	PC	PEA Price	AP
87	Tulsa, OK	-	205	-	-	205	2.5465	0.00
87	Salina, KS	-	3,918	-	-	3,918	0.6884	0.01
87	Garden City, KS	-	1,679	-	-	1,679	0.678	0.01
87	Stillwater, OK	-	1,036	-	-	1,036	2.1031	0.01
87	Enid, OK	436	8,353	315	315	9,104	3.4955	0.17
87	Altus, OK	28,413	67,943	54,091	54,091	150,447	0.3751	0.29
87	Pampa, TX	-	13,108	13,108	13,108	26,216	0.4575	0.06
Total		28,849	96,242	67,514	67,514	192,605		**0.554**

For each PEA, the PEA price = 1/5 × (CMA average price) + 4/5 × (EA average price). Table A5 shows this calculation for Tulsa, OK.

Table A5: PEA Price for Tulsa, OK

PEA	CMA average price	EA average price	PEA Price
Tulsa, OK	2.6729	2.5149	2.5465

The EA average price is calculated taking the average prices over blocks H, I, and J for each Economic Area, weighted by MHz. Tulsa, OK belongs to Economic Area 124.

Table A6: EA average price for Tulsa, OK.

Economic Area	Market	Block	MHz	Population	Winning bid	Price/MHzPop
Tulsa OK-KS	BEA124	H	10	1,478,165	36,914,000	2.50
Tulsa OK-KS	BEA124	I	10	1,478,165	31,971,000	2.16
Tulsa OK-KS	BEA124	J	20	1,478,165	79,815,000	2.70
						2.5149

CMA average prices are calculated by adding the contribution each CMA has on each PEA. In Table A7, CMA population is the population that belongs to both the CMA and the PEA.

Table A7: CMA average price for Tulsa, OK.

PEA	PEA Population	CMA	CMA population	Price / MHzPop	CMA average price
Tulsa, OK	969,078	Tulsa, OK	922,091	2.79	2.66
Tulsa, OK	969,078	Oklahoma 4 – Nowata	46,987	0.30	0.01
					2.6729

Reweighted volume

The FCC has established in the Public Notice that, to fulfill its mandate, opening prices in the reverse auction will be calculated using a station-specific "volume" factor and an underlying base clock price for a UHF station going off air.

The FCC has proposed to calculate a station's volume using the formula

$$\text{FCC volume} = (\text{Broadcast population})^{1/2} \times (\text{Interference count})^{1/2}$$

Furthermore, the FCC has established that the interference component should measure a station's potential impact on repacking. The FCC proposed the Interference Count (IC) as a measure of interference and Interference-free broadcast population (IF) as a measure of population.

As explained above, the empirical freezing probability (FR) of a station directly measures its impact on repacking. Thus, a volume based on FR would better address the FCC's stated objectives.

Considering that a volume based on FR could be perceived as more complicated than the originally proposed volume, several other alternatives were studied. In particular, we studied variations on the exponents proposed by the FCC.

Improving the FCC volume formula by changing only its exponents have two major advantages:

i) Uses the same inputs (IC and IF), hence minimizing the cost and time of analyzing the change for interested parties, and
ii) Can approximate the effect a station has on repacking.

In order to select which coefficients better reflect the impact each station has on repacking we did a regression analysis on the coefficients of the FCC formula. Specifically we analyzed regressions of the form:

$$\ln(FR_i) = c + a * \ln(IF_i) + b \ln(IC_i) + e_i$$

where FR_i is the empirical freezing probability of station i, IC_i is the Interference count for station i and IF_i is the interference-free broadcast population of station i and e_i is an error. The following table contains basic descriptive measures of each variable. IF and IC measures statistics are reported for the complete set of stations considered by the FCC. FR is reported only for UHF stations.

Table A8: Summary statistics of IF, IC and FR

	Min.	1st Qu.	Median	Mean	3rd Qu	Max
IF	0	379,000	1,039,000	2,088,000	2,428,000	21,190,000
IC	0	41	71	74.49	106	220
FR	0	0.02855	0.24460	0.27180	0.45100	1.0000

The set of station on which the regression is run is of major importance. On the one hand, stations with freezing probabilities close to 0% or close to 100% do not affect the auction outcome. On the other hand, stations with freezing probabilities close to 50% can have significant impacts on auction outcomes.

Specifically, stations with FR=0% can always be repacked, hence the results of the auction do not change if these stations do not participate; if they participate these stations will remain active until they exit the auction. Stations with FR=100% can never be repacked. If one of these stations participates in the auction, it will be frozen for sure and cannot take the place of any station in the repacked set.

Stations with FR away from the extremes of 0 and 1 determine the outcome of the auction as variations in initial score among these stations determine the order of exit and hence change the set of frozen and repacked stations. Thus, there is a trade-off between the quantity and the relevance of the data considered in each regression.

We considered a series of scenarios to determine the appropriate set of coefficients that would achieve a good overall fit. We only consider stations with positive IC and IF in the analysis. Table A9 shows the results of several regressions for different subsets of data. Each scenario only considers stations that satisfy $Min\ FR < FR < Max\ FR$. Regression results are in ascending order by the sum of coefficients, column "a+b".

Table A9: Selected Regression Scenarios

a	b	a+b	a%	b%	Max FR	Min FR
0.21	0.49	0.70	30%	70%	60%	10%
0.23	0.52	0.75	31%	69%	70%	10%
0.25	0.47	0.71	35%	65%	80%	10%
0.26	0.44	0.71	37%	63%	90%	10%
0.26	0.44	0.71	37%	63%	100%	10%
0.30	0.56	0.85	35%	65%	100%	6%
0.29	0.49	0.77	37%	63%	100%	7%
0.28	0.48	0.76	37%	63%	100%	8%
0.27	0.46	0.73	37%	63%	100%	9%

Based on our findings, we propose to use the following formula for volume:

$$\text{Reweighted volume} = (\text{Broadcast population})^{1/4} \times (\text{Interference count})^{1/2}$$

In order to select an appropriate trade-off between the quantity and relevance of stations to be included in the regression analysis we used the following measures for every subset considered:

$Fiterror = \frac{1}{n}\sum(\ln(FR_i) - \ln(PFR_i))^2$, where n is the number of stations, FR_i is the freezing probability of station i and PFR_i is the fitted value.

$Totalerror = \frac{1}{N}\sum(\ln(FR_i) - \ln(PFR_i))^2$, where N is the total number of stations, FR_i is the freezing probability of station i and PFR_i is the fitted value. Stations with $FR = 0$ were assigned a value of $FR = 0.0001$

Figure A24 shows the trade-off between normalized versions of these two measures for all subsets of data considered. Fit error was normalized by dividing Fit error in each scenario by the Maximum Fit error; that is, Fit error of scenario Max FR =100% and Min FR=0%. Total error was normalized by dividing Total error in each scenario by the Maximum of Total error; that is, Total error of scenario Max FR =60% and Min FR=20%.

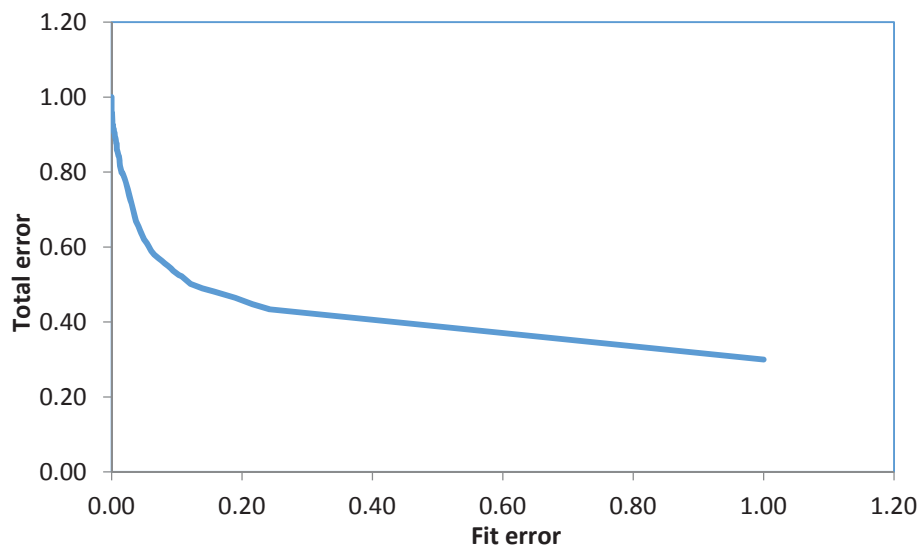


Figure A24: Tradeoff between quantity and relevance

In order to select a best scenario, a point along the curve displayed in Figure A24, we assigned a "price" of one to each normalized error measure; that is, we give them the same weight. We selected scenarios with the lowest "expenditure"—those scenarios that are "tangent" to the line in Figure A24 when equal weight is put on quantity and relevance.

Table A10: Regression Scenarios

a	b	Fit Error	Total Error	nFit Error (1)	nTotal Error (2)	(1)+(2)	Max FR	Min FR
0.295	**0.558**	**0.186**	**9.652**	**0.12**	**0.81**	**0.93**	**100%**	**6%**
0.232	**0.520**	**0.127**	**10.238**	**0.08**	**0.86**	**0.94**	**70%**	**10%**
0.286	**0.487**	**0.165**	**10.039**	**0.11**	**0.84**	**0.95**	**100%**	**7%**
0.282	**0.480**	**0.158**	**10.111**	**0.10**	**0.84**	**0.95**	**100%**	**8%**
0.206	**0.490**	**0.126**	**10.399**	**0.08**	**0.87**	**0.95**	**60%**	**10%**
0.269	**0.461**	**0.146**	**10.307**	**0.10**	**0.86**	**0.96**	**100%**	**9%**
0.247	**0.466**	**0.136**	**10.414**	**0.09**	**0.87**	**0.96**	**80%**	**10%**
0.263	**0.444**	**0.140**	**10.444**	**0.09**	**0.87**	**0.96**	**100%**	**10%**
0.263	**0.444**	**0.140**	**10.444**	**0.09**	**0.87**	**0.96**	**90%**	**10%**
0.256	0.431	0.133	10.571	0.09	0.88	0.97	100%	11%
0.220	0.408	0.088	10.952	0.06	0.91	0.97	70%	15%
0.255	0.410	0.125	10.698	0.08	0.89	0.98	100%	12%
0.252	0.388	0.118	10.835	0.08	0.91	0.98	100%	13%
0.192	0.377	0.085	11.142	0.06	0.93	0.99	60%	15%
0.250	0.368	0.110	10.973	0.07	0.92	0.99	100%	14%
0.235	0.360	0.096	11.119	0.06	0.93	0.99	80%	15%
0.251	0.338	0.100	11.148	0.07	0.93	1.00	100%	15%
0.251	0.338	0.100	11.148	0.07	0.93	1.00	90%	15%
0.242	0.320	0.094	11.330	0.06	0.95	1.01	100%	16%
0.235	0.310	0.091	11.450	0.06	0.96	1.02	100%	17%
0.187	0.331	0.063	11.708	0.04	0.98	1.02	70%	20%
0.228	0.301	0.088	11.571	0.06	0.97	1.02	100%	18%
0.221	0.286	0.083	11.736	0.05	0.98	1.03	100%	19%
0.155	0.293	0.058	11.970	0.04	1.00	1.04	60%	20%
0.204	0.279	0.072	11.897	0.05	0.99	1.04	80%	20%
0.221	0.251	0.076	11.941	0.05	1.00	1.05	100%	20%
0.221	0.251	0.076	11.941	0.05	1.00	1.05	90%	20%

Note: Selected scenarios in bold.

RZR prices

RZR prices are fundamental in determining the quantity and location of impaired licenses to be offered in the forward auction. Stations that are deemed essential to meet a clearing target before the auction begins determine impairment by accepting or rejecting the RZR price offered in the reverse auction. The basic trade-off is that high RZR prices will reduce impairment, but increase the clearing cost somewhat. Clearly RZR prices should depend on carriers' preferences for avoiding impairment.

To balance impairment and clearing costs of round-zero-frozen stations we have produced two different RZR prices based on two different measures of spectrum value: (1) Greenhill-based and (2) Value-based. These measures are used to determine the RZR prices to be offered to all stations in case they are frozen in round zero. As discussed in the main text, the RZR prices are scaled so that for the station with the highest score, the RZR price is equal to the opening price.

RZR prices are calculated as follows:

$$\text{Greenhill-based} = (\text{Adjusted coverage population}) \times (\text{Domain capacity adjustment factor})^{1/2}$$

$$\text{Value-based} = (\text{Precluded population}) \times (\text{AWS-3 price})$$

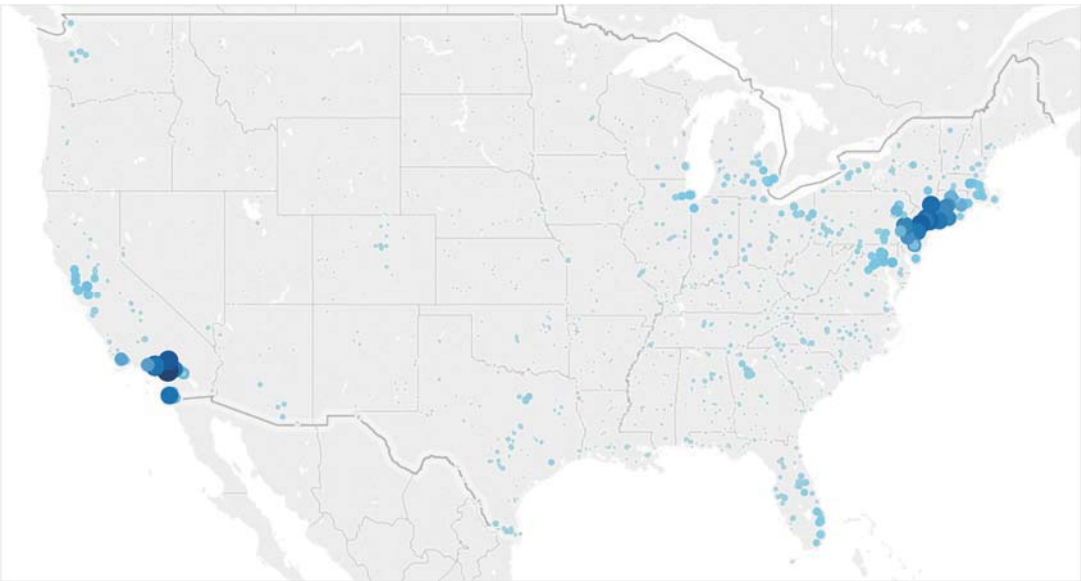


Figure A25: Greenhill RZR prices

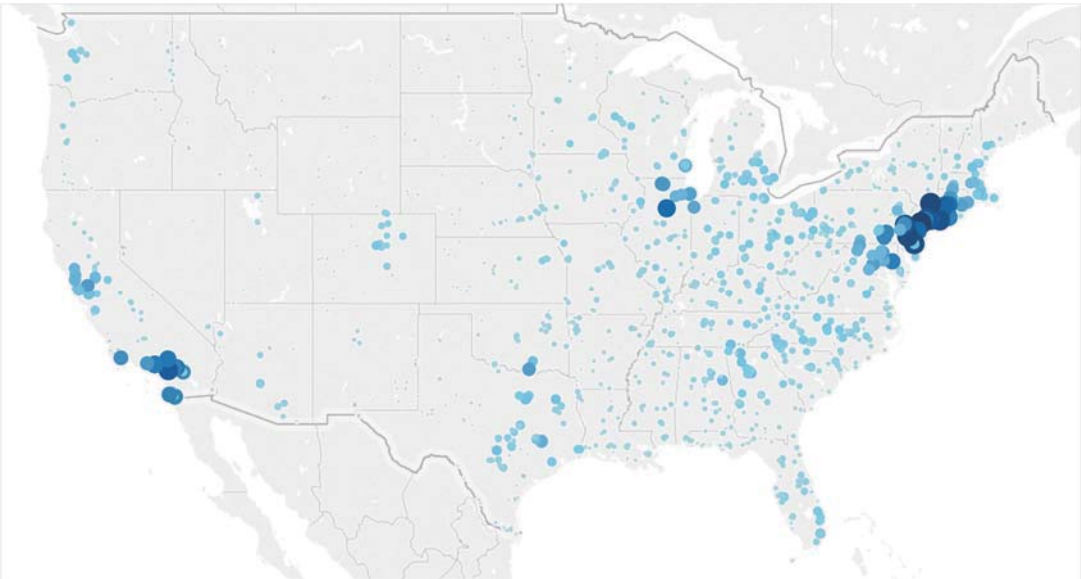


Figure A26: Value-based RZR prices

Figures A25 and A26 show maps of Greenhill-based and Value-based RZR prices. RZR prices are identical in the max-score station as a result of scaling to the highest opening price, but differ for other stations. However, in most markets value-based RZR prices tend to be higher. This flatter RZR price curve should tend to reduce impairments relative to the Greenhill-based RZR prices.

It is also important to compare RZR prices and opening prices. Figures A27 and A28 do this for the FCC and Reweighted volume, respectively, and a base price clock of $900.

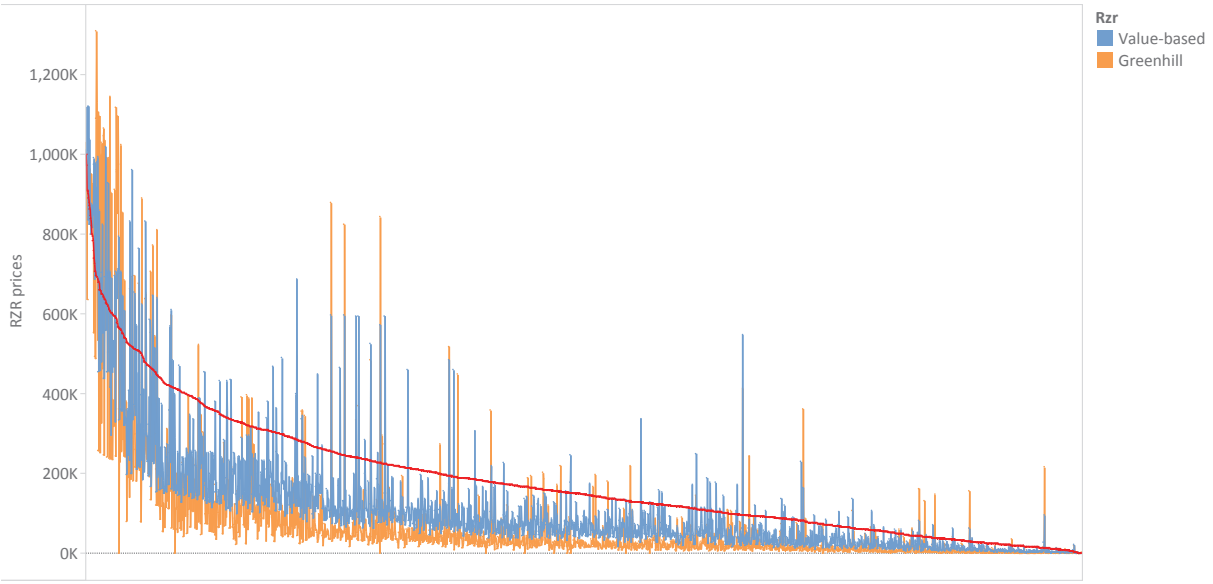


Figure A27: RZR prices for FCC $900

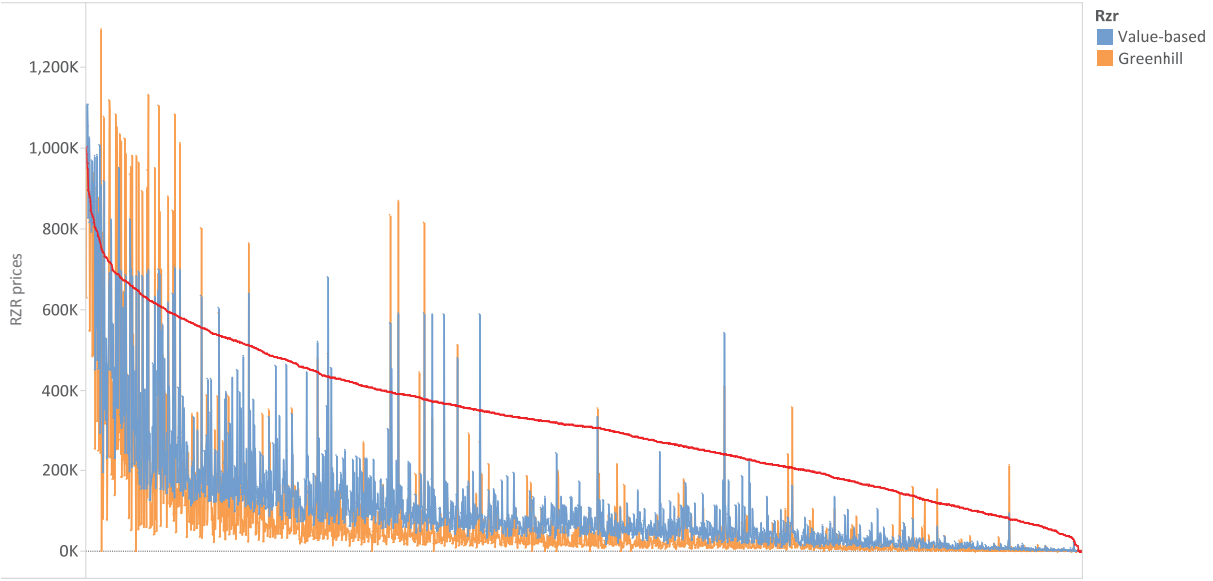


Figure A28: RZR prices for Reweighted $900

Table A11 shows RZR prices for both the FCC volume and Reweighted volume in the DMAs where stations are more likely to be offered a RZR price using a base clock price of $900. Both RZR prices

are identical for the max-score station, which happens to be in New York. However, Value-based RZR prices tend to be higher in most DMAs.

Table A11: RZR prices in DMAs with round zero freezes in million $

		All				RZR Stations			
		FCC		Reweighted		FCC		Reweighted	
DMA name	RZR Frequency	Greenhill	Value-based	Greenhill	Value-based	Greenhill	Value-based	Greenhill	Value-based
Harlingen-Weslaco-Brownsville-McAllen, TX	39.2%	62	23	61	22	69	23	68	23
San Diego, CA	34.7%	357	322	353	318	256	230	253	227
Laredo, TX	32.3%	13	6	13	6	9	6	9	6
Buffalo, NY	28.0%	70	59	69	59	97	74	96	73
Burlington, VT-Plattsburgh, NY	18.2%	36	46	35	46	29	20	29	20
Los Angeles, CA	17.3%	846	595	836	588	906	617	894	609
Detroit, MI	15.0%	164	133	162	132	193	156	191	154
Tucson, AZ	8.3%	44	80	43	79	15	3	15	3
Palm Springs, CA	8.1%	190	142	187	141	565	504	558	498
Philadelphia, PA	3.3%	390	478	385	472	407	433	402	428
Cleveland-Akron, OH	2.9%	108	112	107	110	153	104	151	102
New York, NY	2.0%	730	723	721	715	728	688	719	680
Wilkes Barre-Scranton, PA	1.8%	170	110	168	109	315	202	311	200
Harrisburg-Lancaster-Lebanon-York, PA	1.3%	121	317	119	313	151	396	149	392
Rochester, NY	0.3%	79	60	78	59	54	72	53	71
Seattle-Tacoma, WA	0.1%	81	118	80	116	88	102	87	101

Note: RZR frequency = likelihood of round zero freeze with value multipliers of 1, 1.5, and 2

Greenhill prices

Greenhill & Co., LLC ("Greenhill") prepared estimates of potential auction compensation that were published by the FCC in October 2014. However, the released information was aggregated by DMA maximums and medians, without disclosure of the underlying formula used to arrive at the values. In this section we present a formula which we believe is a reasonable approximation of the Greenhill method; it was developed after multiple conversations with FCC staff, who shared some basic information about the structure of the formula. We obtained an R^2 of 0.975 between this formula and a small set of station-specific Greenhill values that various broadcasters received directly from the FCC, suggesting that this formula is a good, but not perfect match for the Greenhill method.

Greenhill price = (Scale factor)×(Adjusted coverage pop)×(Domain capacity adjustment factor)$^{1/2}$

where

Adjusted coverage pop = ½ (Broadcast population) + ½ (Interfered population/62)

Interfered population = the sum of all population of all stations that a given station interferes with. Each interfered station's population is counted once for each type of interference that exists. A station's population can be counted up to three times if it has CO, ADJ+1 and ADJ-1 interference. This metric is divided by 62 in the formula; the FCC advised 62 represented an average number of interference constraints for a single station.

Domain capacity adjustment factor = a population-weighted average of each station's "PEA congestion factors," calculated across all PEAs that a station's signal serves.

PEA congestion factor = number of stations having a service area that reaches a PEA, divided by the number of channels available for repacking a particular station, according to its domain file. Calculations are restricted to a station's home band, so for UHF stations, this factor is calculated relative to channels 14 to 51. Low VHF stations are considered only in the channel range 2 to 6, while High VHF stations are considered only in the channel range 7 to 13.

Scale factor = a value chosen such that the sum of expected payments to broadcasters required to clear 126 MHz would total approximately $36 billion, reflecting total available forward auction revenue of $45 billion, less $7 billion to fund FirstNet, less $2 billion in auction costs (including $1.75 billion relocation costs.) This scale factor is irrelevant in our simulations, since RZR prices are scaled so that the RZR price is equal to the Opening price for the station with the maximum volume.

Simulation results

We have simulated 180 reverse auction scenarios. These include three variations of the base clock price ($900, $1,250 and $1,500) and two volume metrics (FCC and Reweighted). We also conducted several robustness checks, raising reservation values by a multiplicative factor, equal to 1, 1.5, 2, 2.5 or 3, and adding unbiased random error terms to reservation values, using six different random seeds. This section presents detailed simulation results.

Figures A30.1-3 show impaired PEA in each scenario. Only 19 PEAs that are impaired in at least one scenario are shown. Of these 19 PEAs, Rochester, Buffalo, Jamestown, NY and Erie, PA are most frequently impaired.

					PEA / PEA name																		
					1	2	6	12	14	18	44	47	54	61	66	81	113	123	147	188	221	324	329
Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	New York, NY	Los Angeles, CA	Philadelphia, PA	Detroit, MI	Cleveland, OH	San Diego, CA	Rochester, NY	Brownsville, TX	Buffalo, NY	Toledo, OH	Lansing, MI	Saginaw, MI	Erie, PA	Mansfield, OH	Salisbury, MD	Jamestown, NY	Laredo, TX	Honesdale, PA	Kingsville, TX
1.0x	$1,500	FCC	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz																			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
		Reweighted	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz																			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
	$1,250	FCC	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz																			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
		Reweighted	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz																			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
	$900	FCC	1	126 MHz																			
			2	126 MHz				●						●	●	●							
			3	126 MHz							●		●				●			●			
			4	126 MHz				●						●	●	●							
			5	126 MHz																			
			6	126 MHz							●		●				●			●			
		Reweighted	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz							●		●				●			●			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz							●		●				●			●			
1.5x	$1,500	FCC	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz							●		●				●			●			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
		Reweighted	1	126 MHz																			
			2	126 MHz																			
			3	126 MHz							●		●				●			●			
			4	126 MHz																			
			5	126 MHz																			
			6	126 MHz																			
	$1,250	FCC	1	126 MHz							●		●				●			●			
			2	126 MHz				●						●	●	●							
			3	126 MHz							●		●				●			●			
			4	126 MHz				●			●		●	●	●	●	●			●			
			5	126 MHz																			
			6	126 MHz							●		●				●			●			
		Reweighted	1	126 MHz							●		●				●			●			
			2	126 MHz																			
			3	126 MHz							●		●				●			●			
			4	126 MHz							●		●				●			●			
			5	126 MHz																			
			6	126 MHz							●		●				●			●			
	$900	FCC	1	126 MHz				●			●		●	●	●	●	●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●	●	●				●			●	●		●
			6	126 MHz							●	●	●				●			●	●		●
		Reweighted	1	126 MHz							●		●				●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●	●	●				●			●	●		●
			6	126 MHz							●	●	●				●			●	●		●

Figure A30.1: Impaired PEA

					PEA / PEA name																		
					1	2	6	12	14	18	44	47	54	61	66	81	113	123	147	188	221	324	329
Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	New York, NY	Los Angeles, CA	Philadelphia, PA	Detroit, MI	Cleveland, OH	San Diego, CA	Rochester, NY	Brownsville, TX	Buffalo, NY	Toledo, OH	Lansing, MI	Saginaw, MI	Erie, PA	Mansfield, OH	Salisbury, MD	Jamestown, NY	Laredo, TX	Honesdale, PA	Kingsville, TX
2.0x	$1,500	FCC	1	126 MHz							●		●				●			●			
			2	126 MHz				●						●	●	●							
			3	126 MHz							●		●				●			●			
			4	126 MHz				●			●		●	●	●	●	●			●			
			5	126 MHz							●		●				●			●			
			6	126 MHz							●		●				●			●			
		Reweighted	1	126 MHz							●		●				●			●			
			2	126 MHz				●						●	●	●							
			3	126 MHz							●		●				●			●			
			4	126 MHz							●		●				●			●			
			5	126 MHz							●		●				●			●			
			6	126 MHz							●		●				●			●			
	$1,250	FCC	1	126 MHz				●			●		●	●	●	●	●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●		●				●			●			
			6	126 MHz							●	●	●				●			●	●		●
		Reweighted	1	126 MHz							●		●				●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●		●				●			●			
			6	126 MHz							●	●	●				●			●	●		●
	$900	FCC	1	126 MHz	●	●	●	●		●	●	●	●	●	●	●	●		●	●	●	●	●
			2	126 MHz	●		●	●	●		●	●	●	●	●	●	●	●	●	●	●	●	●
			3	126 MHz	●		●	●			●	●	●	●	●	●	●			●	●	●	●
			4	126 MHz				●	●		●		●	●	●	●	●	●		●			
			5	126 MHz	●		●	●	●		●	●	●	●	●	●	●		●	●	●	●	●
			6	126 MHz	●		●	●			●	●	●	●	●	●	●		●	●	●	●	●
		Reweighted	1	126 MHz	●	●	●	●		●	●	●	●	●	●	●	●		●	●	●	●	●
			2	126 MHz	●		●	●	●		●	●	●	●	●	●	●	●	●	●	●	●	●
			3	126 MHz	●		●	●	●		●	●	●	●	●	●	●			●	●	●	●
			4	126 MHz				●	●		●		●	●	●	●	●	●		●			
			5	126 MHz				●	●		●	●	●	●	●	●	●			●			●
			6	126 MHz	●		●				●	●	●				●		●	●			●
2.5x	$1,500	FCC	1	126 MHz				●			●		●	●	●	●	●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●	●	●				●			●	●		●
			6	126 MHz							●	●	●				●			●	●		●
		Reweighted	1	126 MHz							●		●				●			●			
			2	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			3	126 MHz							●		●				●			●			
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz							●	●	●				●			●	●		●
			6	126 MHz							●	●	●				●			●	●		●
	$1,250	FCC	1	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			2	126 MHz				●	●		●	●	●	●	●	●	●			●	●		●
			3	126 MHz	●		●	●			●	●	●	●	●	●	●			●	●	●	●
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			6	126 MHz	●		●	●			●	●	●	●	●	●	●		●	●	●	●	●
		Reweighted	1	126 MHz							●	●	●				●			●	●		●
			2	126 MHz				●	●		●	●	●	●	●	●	●			●	●		●
			3	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			6	126 MHz	●		●				●	●	●				●		●	●	●		●
	$900	FCC	1	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			2	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			3	126 MHz	●		●	●	●		●	●	●	●	●	●	●			●	●	●	●
			4	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			5	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			6	114 MHz	●		●				●	●	●				●		●	●	●	●	●
		Reweighted	1	126 MHz	●	●	●	●	●	●	●	●	●	●	●	●	●		●	●	●	●	●
			2	114 MHz							●	●	●				●			●	●		●
			3	126 MHz	●		●	●	●		●	●	●	●	●	●	●		●	●	●	●	●
			4	114 MHz							●	●	●				●			●	●		●
			5	114 MHz							●	●	●				●			●	●		●
			6	114 MHz	●		●				●	●	●				●		●	●	●	●	●

Figure A30.2: Impaired PEA

					PEA / PEA name																		
					1	2	6	12	14	18	44	47	54	61	66	81	113	123	147	188	221	324	329
Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	New York, NY	Los Angeles, CA	Philadelphia, PA	Detroit, MI	Cleveland, OH	San Diego, CA	Rochester, NY	Brownsville, TX	Buffalo, NY	Toledo, OH	Lansing, MI	Saginaw, MI	Erie, PA	Mansfield, OH	Salisbury, MD	Jamestown, NY	Laredo, TX	Honesdale, PA	Kingsville, TX
3.0x	$1,500	FCC	1	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			2	126 MHz				●	●		●	●	●	●	●	●	●			●	●		●
			3	126 MHz	●		●	●			●	●	●	●	●	●	●			●	●	●	●
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			6	126 MHz	●		●	●			●	●	●	●	●	●	●		●	●	●	●	●
		Reweighted	1	126 MHz							●	●	●				●			●	●		●
			2	126 MHz				●	●		●	●	●	●	●	●	●			●	●		●
			3	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			4	126 MHz				●	●		●		●	●	●	●	●			●			
			5	126 MHz				●			●	●	●	●	●	●	●			●	●		●
			6	126 MHz	●		●				●	●	●				●		●	●	●		●
	$1,250	FCC	1	126 MHz	●	●	●	●		●	●	●	●	●	●	●	●		●	●	●	●	●
			2	126 MHz	●		●	●	●		●	●	●	●	●	●	●	●	●	●	●	●	●
			3	126 MHz	●		●	●	●		●	●	●	●	●	●	●			●	●	●	●
			4	114 MHz							●	●	●				●			●	●		●
			5	126 MHz	●		●	●	●		●	●	●	●	●	●	●		●	●	●	●	●
			6	126 MHz	●		●	●			●	●	●	●	●	●	●		●	●	●	●	●
		Reweighted	1	126 MHz	●	●	●	●		●	●	●	●	●	●	●	●		●	●	●	●	●
			2	126 MHz	●		●	●	●		●	●	●	●	●	●	●	●	●	●	●	●	●
			3	126 MHz	●		●	●	●		●	●	●	●	●	●	●			●	●	●	●
			4	126 MHz		●		●	●	●	●	●	●	●	●	●	●	●		●	●		●
			5	126 MHz	●	●	●	●	●	●	●	●	●	●	●	●	●		●	●	●	●	●
			6	126 MHz	●		●				●	●	●				●		●	●	●	●	●
	$900	FCC	1	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			2	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			3	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			4	114 MHz	●		●		●		●	●	●				●	●	●	●	●	●	●
			5	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			6	108 MHz		●				●	●	●	●				●			●	●		●
		Reweighted	1	114 MHz							●	●	●				●			●	●		●
			2	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			3	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			4	114 MHz	●		●				●	●	●				●		●	●	●	●	●
			5	114 MHz							●	●	●				●			●	●		●
			6	114 MHz	●	●	●			●	●	●	●				●		●	●	●	●	●

Figure A30.3: Impaired PEA

In the reverse auction, each station out of the 1,648 UHF stations in the contiguous United States may be either (1) a non-participant, (2) an impairing station, (3) frozen in round zero, (4) frozen during the auction, and (5) exited during the auction. A non-participant is a station that rejects its opening price or RZR price and it can be repacked. An impairing station is one that rejects its opening price or RZR price but it cannot be repacked. A station is frozen at round zero if it accepts its RZR price. During the auction, a station can either be frozen or exited.

Figures A31.1-3 shows the number of stations for each of the five statuses. As the value multiplier rises, the number of non-participants increases and there are more impairing stations or lower optimized clearing target. The higher base clock price can help avoid costly impairments. Even at value multiplier of 3, both FCC and Reweighted volumes with $1,500 base clock price can achieve 126 MHz clearing target with reasonable amount of impairments. Moreover, the Reweighted volume encourages more participations and is thereby less prone to impairments.

Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	Non-participant	Impairing	Round-0 frozen	Frozen	Exited
1.0x	$1,500	FCC	1	126 MHz	35		13	492	1,108
			2	126 MHz	34		13	490	1,111
			3	126 MHz	28		13	498	1,109
			4	126 MHz	32		13	486	1,117
			5	126 MHz	29		13	491	1,115
			6	126 MHz	35		13	483	1,117
		Reweighted	1	126 MHz	10		13	497	1,128
			2	126 MHz	10		13	487	1,138
			3	126 MHz	9		13	491	1,135
			4	126 MHz	8		13	487	1,140
			5	126 MHz	9		13	500	1,126
			6	126 MHz	9		13	494	1,132
	$1,250	FCC	1	126 MHz	54		13	488	1,093
			2	126 MHz	46		13	498	1,091
			3	126 MHz	44		13	481	1,110
			4	126 MHz	44		13	482	1,109
			5	126 MHz	44		13	501	1,090
			6	126 MHz	47		13	488	1,100
		Reweighted	1	126 MHz	13		13	498	1,124
			2	126 MHz	10	1	13	488	1,136
			3	126 MHz	11		13	492	1,132
			4	126 MHz	14		13	492	1,129
			5	126 MHz	11		13	500	1,124
			6	126 MHz	15		13	487	1,133
	$900	FCC	1	126 MHz	86		13	480	1,069
			2	126 MHz	87	1	19	486	1,055
			3	126 MHz	81	1	16	487	1,063
			4	126 MHz	84	1	19	486	1,058
			5	126 MHz	80		17	487	1,064
			6	126 MHz	79	1	15	480	1,073
		Reweighted	1	126 MHz	25		13	491	1,119
			2	126 MHz	26		13	487	1,122
			3	126 MHz	22	1	12	495	1,118
			4	126 MHz	29		13	495	1,111
			5	126 MHz	25		13	488	1,122
			6	126 MHz	24	1	12	494	1,117
1.5x	$1,500	FCC	1	126 MHz	73		13	487	1,075
			2	126 MHz	80		20	492	1,056
			3	126 MHz	71	1	12	488	1,076
			4	126 MHz	72		13	490	1,073
			5	126 MHz	69		17	481	1,081
			6	126 MHz	66		16	486	1,080
		Reweighted	1	126 MHz	21		13	496	1,118
			2	126 MHz	20		13	491	1,124
			3	126 MHz	17	1	12	495	1,123
			4	126 MHz	22		13	491	1,122
			5	126 MHz	18		13	482	1,135
			6	126 MHz	21		13	504	1,110
	$1,250	FCC	1	126 MHz	98	1	12	495	1,042
			2	126 MHz	96	1	19	489	1,043
			3	126 MHz	88	1	16	482	1,061
			4	126 MHz	91	2	18	479	1,058
			5	126 MHz	95		17	488	1,048
			6	126 MHz	94	1	15	493	1,045
		Reweighted	1	126 MHz	29	1	12	488	1,118
			2	126 MHz	31		13	487	1,117
			3	126 MHz	26	1	12	487	1,122
			4	126 MHz	32	1	12	488	1,115
			5	126 MHz	30		13	488	1,117
			6	126 MHz	30	1	12	504	1,101
	$900	FCC	1	126 MHz	156	3	16	487	986
			2	126 MHz	154	4	19	485	986
			3	126 MHz	158	2	23	475	990
			4	126 MHz	161	4	20	479	984
			5	126 MHz	160	2	22	487	977
			6	126 MHz	157	3	14	488	986
		Reweighted	1	126 MHz	57	2	11	489	1,089
			2	126 MHz	56	4	17	487	1,084
			3	126 MHz	48	2	15	489	1,094
			4	126 MHz	54	4	17	484	1,089
			5	126 MHz	55	2	19	484	1,088
			6	126 MHz	58	3	14	500	1,073
					0 50 100 150	0 2 4	10 15 20	480 490 500	1,000 1,050 1,100
					No of stations	No of stations	No of stations	No of stations	No of stations

Figure A31.1: Station status

Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	Non-participant	Impairing	Round-0 frozen	Frozen	Exited
2.0x	$1,500	FCC	1	126 MHz	113	2	11	483	1,039
			2	126 MHz	113	1	22	483	1,029
			3	126 MHz	102	1	21	475	1,049
			4	126 MHz	107	2	22	464	1,053
			5	126 MHz	115	1	20	479	1,033
			6	126 MHz	113	1	15	486	1,033
		Reweighted	1	126 MHz	39	2	11	487	1,109
			2	126 MHz	39	1	20	481	1,107
			3	126 MHz	33	1	12	493	1,109
			4	126 MHz	40	1	12	485	1,110
			5	126 MHz	39	1	16	486	1,106
			6	126 MHz	36	1	15	500	1,096
	$1,250	FCC	1	126 MHz	146	3	16	493	990
			2	126 MHz	146	4	19	484	995
			3	126 MHz	143	2	20	481	1,002
			4	126 MHz	153	3	21	477	994
			5	126 MHz	148	1	22	476	1,001
			6	126 MHz	150	3	14	485	996
		Reweighted	1	126 MHz	54	2	11	492	1,089
			2	126 MHz	54	4	17	479	1,094
			3	126 MHz	44	2	15	483	1,104
			4	126 MHz	52	3	18	488	1,087
			5	126 MHz	52	1	16	499	1,080
			6	126 MHz	53	3	14	488	1,090
	$900	FCC	1	126 MHz	235	9	63	439	902
			2	126 MHz	231	10	60	430	917
			3	126 MHz	237	6	72	426	907
			4	126 MHz	232	6	40	458	912
			5	126 MHz	231	9	63	440	905
			6	126 MHz	247	7	52	440	902
		Reweighted	1	126 MHz	97	9	59	437	1,046
			2	126 MHz	94	11	58	440	1,045
			3	126 MHz	88	7	64	436	1,053
			4	126 MHz	96	6	39	464	1,043
			5	126 MHz	90	5	40	457	1,056
			6	126 MHz	92	6	39	467	1,044
2.5x	$1,500	FCC	1	126 MHz	156	3	16	480	993
			2	126 MHz	153	4	19	493	979
			3	126 MHz	158	2	23	485	980
			4	126 MHz	161	4	20	481	982
			5	126 MHz	160	2	22	485	979
			6	126 MHz	158	3	14	484	989
		Reweighted	1	126 MHz	57	2	11	500	1,078
			2	126 MHz	56	4	17	493	1,078
			3	126 MHz	48	2	15	493	1,090
			4	126 MHz	54	4	17	490	1,083
			5	126 MHz	54	2	19	483	1,090
			6	126 MHz	58	3	14	496	1,077
	$1,250	FCC	1	126 MHz	202	4	20	477	945
			2	126 MHz	198	5	39	457	949
			3	126 MHz	204	6	69	429	940
			4	126 MHz	208	4	41	453	942
			5	126 MHz	202	4	44	460	938
			6	126 MHz	208	6	48	455	931
		Reweighted	1	126 MHz	79	3	10	489	1,067
			2	126 MHz	77	5	39	467	1,060
			3	126 MHz	74	4	43	457	1,070
			4	126 MHz	81	4	40	463	1,060
			5	126 MHz	80	4	40	465	1,059
			6	126 MHz	76	4	32	476	1,060
	$900	FCC	1	114 MHz	301	6	55	347	939
			2	114 MHz	311	6	35	378	918
			3	126 MHz	308	8	70	437	825
			4	114 MHz	301	6	34	366	941
			5	114 MHz	306	6	34	373	929
			6	114 MHz	307	5	38	364	934
		Reweighted	1	126 MHz	133	12	57	437	1,009
			2	114 MHz	123	3	13	403	1,106
			3	126 MHz	122	9	63	438	1,016
			4	114 MHz	137	3	16	391	1,101
			5	114 MHz	125	3	13	407	1,100
			6	114 MHz	132	5	35	368	1,108
					100 200 300	0 5 10	20 40 60	350 400 450 500	900 1,000 1,100
					No of stations	No of stations	No of stations	No of stations	No of stations

Figure A31.2: Station status

Value Multiplier	Base clock price	Volume	Value Seed	Optimized Clearing Target	Non-participant	Impairing	Round-0 frozen	Frozen	Exited
3.0x	$1,500	FCC	1	126 MHz	202	4	20	476	946
			2	126 MHz	199	5	39	458	947
			3	126 MHz	204	6	69	432	937
			4	126 MHz	208	4	41	455	940
			5	126 MHz	202	4	44	456	942
			6	126 MHz	207	6	48	447	940
		Reweighted	1	126 MHz	80	3	10	494	1,061
			2	126 MHz	77	5	39	468	1,059
			3	126 MHz	74	4	43	455	1,072
			4	126 MHz	81	4	40	466	1,057
			5	126 MHz	79	4	40	464	1,061
			6	126 MHz	76	4	32	465	1,071
	$1,250	FCC	1	126 MHz	255	9	66	425	893
			2	126 MHz	258	11	65	420	894
			3	126 MHz	258	8	71	423	888
			4	114 MHz	261	3	16	386	982
			5	126 MHz	256	9	71	433	879
			6	126 MHz	259	7	53	444	885
		Reweighted	1	126 MHz	108	9	63	438	1,030
			2	126 MHz	102	11	58	441	1,036
			3	126 MHz	99	8	64	431	1,046
			4	126 MHz	105	8	40	458	1,037
			5	126 MHz	100	10	62	440	1,036
			6	126 MHz	98	6	39	465	1,040
	$900	FCC	1	114 MHz	370	9	55	341	873
			2	114 MHz	380	9	37	362	860
			3	114 MHz	367	7	53	349	872
			4	114 MHz	368	8	56	346	870
			5	114 MHz	367	6	35	371	869
			6	108 MHz	311	60	30	304	943
		Reweighted	1	114 MHz	169	3	33	371	1,072
			2	114 MHz	172	6	33	376	1,061
			3	114 MHz	175	6	51	356	1,060
			4	114 MHz	175	6	54	348	1,065
			5	114 MHz	165	3	14	397	1,069
			6	114 MHz	176	7	50	354	1,061

Figure A31.3: Station status

Figures A32.1-3 show population coverage loss, clearing cost, block loss and impairment in each scenario. To make the comparisons among alternatives more meaningful, population loss and clearing cost is relative to the average of the FCC $900 instances for cases where there is no change in the clearing target and holding the value-multiplier fixed. On average, the Reweighted volume reduces viewer loss and creates less impairments than the FCC $900 scoring rule. These significant improvements from Reweighted volume and a higher base clock price increase clearing cost only slightly in percentage terms holding the clearing target fixed.

Value Multiplier	Base clock price	Volume	Value Seed	Population loss relative to FCC $900 (-50.0M 0.0M 50.0M)	Percentage change in clearing cost relative to FCC $900 (-10% 0% 10% 20%)	Blocks cleared (0)	Impairment (0% 2% 4% 6%)
1.0x	$1,500	FCC	1	-9.5M	-1.0%	10	0.0%
			2	-5.5M	6.6%	10	0.0%
			3	2.5M	12.3%	10	0.0%
			4	12.5M	3.4%	10	0.0%
			5	3.5M	5.9%	10	0.0%
			6	35.5M	-1.5%	10	0.0%
		Reweighted	1	-22.5M	2.7%	10	0.0%
			2	-45.5M	11.6%	10	0.0%
			3	-34.5M	8.9%	10	0.0%
			4	-42.5M	3.7%	10	0.0%
			5	-37.5M	3.9%	10	0.0%
			6	-33.5M	-0.5%	10	0.0%
	$1,250	FCC	1	-18.5M	-2.4%	10	0.0%
			2	-0.5M	6.2%	10	0.0%
			3	-16.5M	11.0%	10	0.0%
			4	12.5M	2.5%	10	0.0%
			5	8.5M	5.4%	10	0.0%
			6	29.5M	-2.3%	10	0.0%
		Reweighted	1	-23.5M	1.7%	10	0.0%
			2	-40.5M	10.6%	10	0.0%
			3	-36.5M	8.3%	10	0.0%
			4	-41.5M	2.8%	10	0.0%
			5	-37.5M	2.9%	10	0.0%
			6	-36.5M	-1.8%	10	0.0%
	$900	FCC	1	-30.5M	-4.1%	10	0.0%
			2	-14.5M	1.4%	10	2.6%
			3	-2.5M	7.4%	10	0.7%
			4	17.5M	-1.4%	10	2.6%
			5	1.5M	1.2%	10	0.0%
			6	28.5M	-4.5%	10	0.7%
		Reweighted	1	-44.5M	0.3%	10	0.0%
			2	-44.5M	9.3%	10	0.0%
			3	-30.5M	6.5%	10	0.7%
			4	-44.5M	1.4%	10	0.0%
			5	-32.5M	1.6%	10	0.0%
			6	-35.5M	-3.0%	10	0.7%
1.5x	$1,500	FCC	1	-22.4M	0.8%	10	0.0%
			2	-2.4M	6.9%	10	0.0%
			3	-10.4M	14.0%	10	0.7%
			4	5.6M	5.9%	10	0.0%
			5	-17.4M	4.8%	10	0.0%
			6	27.6M	0.4%	10	0.0%
		Reweighted	1	-28.4M	4.1%	10	0.0%
			2	-31.4M	14.9%	10	0.0%
			3	-40.4M	11.5%	10	0.7%
			4	-47.4M	5.7%	10	0.0%
			5	-42.4M	5.7%	10	0.0%
			6	-26.4M	2.4%	10	0.0%
	$1,250	FCC	1	-17.4M	-0.2%	10	0.7%
			2	-1.4M	4.9%	10	2.6%
			3	-5.4M	10.5%	10	0.7%
			4	-7.4M	1.3%	10	3.3%
			5	12.6M	4.8%	10	0.0%
			6	40.6M	-0.7%	10	0.7%
		Reweighted	1	-39.4M	3.5%	10	0.7%
			2	-32.4M	14.1%	10	0.0%
			3	-47.4M	10.8%	10	0.7%
			4	-55.4M	4.4%	10	0.7%
			5	-36.4M	5.4%	10	0.0%
			6	-28.4M	1.5%	10	0.7%
	$900	FCC	1	-33.4M	-3.6%	10	3.3%
			2	-6.4M	0.3%	10	3.6%
			3	-16.4M	7.8%	10	0.7%
			4	0.6M	-2.6%	10	5.1%
			5	23.6M	1.1%	10	1.0%
			6	31.6M	-2.9%	10	1.0%
		Reweighted	1	-63.4M	2.0%	10	0.7%
			2	-55.4M	6.5%	10	3.6%
			3	-38.4M	6.4%	10	0.7%
			4	-61.4M	-1.3%	10	5.1%
			5	-37.4M	0.1%	10	1.0%
			6	-38.4M	-0.9%	10	1.0%

Figure A32.1: Population loss, clearing cost, blocks cleared, and impairment by scoring rule

Value Multiplier	Base clock price	Volume	Value Seed	Population loss relative to FCC $900	Percentage change in clearing cost relative to FCC $900	Blocks cleared	Impairment
2.0x	$1,500	FCC	1	17.9M	15.3%	10	0.7%
			2	35.9M	20.0%	10	2.6%
			3	38.9M	26.9%	10	0.7%
			4	35.9M	14.9%	10	3.3%
			5	47.9M	20.1%	10	0.7%
			6	70.9M	14.4%	10	0.7%
		Reweighted	1	4.9M	19.3%	10	0.7%
			2	-5.1M	26.3%	10	2.6%
			3	14.9M	28.3%	10	0.7%
			4	-12.1M	20.9%	10	0.7%
			5	7.9M	17.6%	10	0.7%
			6	8.9M	16.7%	10	0.7%
	$1,250	FCC	1	25.9M	12.1%	10	3.3%
			2	25.9M	16.2%	10	3.6%
			3	38.9M	25.5%	10	0.7%
			4	37.9M	13.2%	10	5.1%
			5	21.9M	15.9%	10	0.7%
			6	80.9M	13.4%	10	1.0%
		Reweighted	1	-8.1M	18.6%	10	0.7%
			2	-13.1M	23.7%	10	3.6%
			3	-0.1M	23.9%	10	0.7%
			4	-8.1M	15.0%	10	5.1%
			5	16.9M	17.5%	10	0.7%
			6	2.9M	14.4%	10	1.0%
	$900	FCC	1	-34.1M	-0.9%	10	64.2%
			2	-30.1M	-1.6%	10	55.5%
			3	12.9M	2.8%	10	33.5%
			4	44.9M	5.5%	10	5.4%
			5	-1.1M	-1.6%	10	55.2%
			6	7.9M	-4.3%	10	43.4%
		Reweighted	1	-54.1M	-4.1%	10	64.2%
			2	-55.1M	-5.8%	10	65.5%
			3	-47.1M	-0.1%	10	35.3%
			4	-17.1M	5.3%	10	5.4%
			5	-15.1M	7.3%	10	5.3%
			6	-28.1M	-2.0%	10	40.9%
2.5x	$1,500	FCC	1	N/A	N/A	10	3.3%
			2	N/A	N/A	10	3.6%
			3	N/A	N/A	10	0.7%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	1.0%
			6	N/A	N/A	10	1.0%
		Reweighted	1	N/A	N/A	10	0.7%
			2	N/A	N/A	10	3.6%
			3	N/A	N/A	10	0.7%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	1.0%
			6	N/A	N/A	10	1.0%
	$1,250	FCC	1	N/A	N/A	10	3.6%
			2	N/A	N/A	10	5.3%
			3	N/A	N/A	10	33.5%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	3.6%
			6	N/A	N/A	10	33.5%
		Reweighted	1	N/A	N/A	10	1.0%
			2	N/A	N/A	10	5.3%
			3	N/A	N/A	10	3.6%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	3.6%
			6	N/A	N/A	10	20.9%
	$900	FCC	1	N/A	N/A	9	40.9%
			2	N/A	N/A	9	40.9%
			3	N/A	N/A	10	45.2%
			4	N/A	N/A	9	40.9%
			5	N/A	N/A	9	40.9%
			6	N/A	N/A	9	30.9%
		Reweighted	1	N/A	N/A	10	81.4%
			2	N/A	N/A	9	1.0%
			3	N/A	N/A	10	55.2%
			4	N/A	N/A	9	1.0%
			5	N/A	N/A	9	1.0%
			6	N/A	N/A	9	30.9%

Figure A32.2: Population loss, clearing cost, blocks cleared, and impairment by scoring rule

Value Multiplier	Base clock price	Volume	Value Seed	Population loss relative to FCC $900	Percentage change in clearing cost relative to FCC $900	Blocks cleared	Impairment
3.0x	$1,500	FCC	1	N/A	N/A	10	3.6%
			2	N/A	N/A	10	5.3%
			3	N/A	N/A	10	33.5%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	3.6%
			6	N/A	N/A	10	33.5%
		Reweighted	1	N/A	N/A	10	1.0%
			2	N/A	N/A	10	5.3%
			3	N/A	N/A	10	3.6%
			4	N/A	N/A	10	5.1%
			5	N/A	N/A	10	3.6%
			6	N/A	N/A	10	20.9%
	$1,250	FCC	1	N/A	N/A	10	64.2%
			2	N/A	N/A	10	65.5%
			3	N/A	N/A	10	45.2%
			4	N/A	N/A	9	1.0%
			5	N/A	N/A	10	55.2%
			6	N/A	N/A	10	43.4%
		Reweighted	1	N/A	N/A	10	64.2%
			2	N/A	N/A	10	65.5%
			3	N/A	N/A	10	45.2%
			4	N/A	N/A	10	16.4%
			5	N/A	N/A	10	66.0%
			6	N/A	N/A	10	40.9%
	$900	FCC	1	N/A	N/A	9	41.0%
			2	N/A	N/A	9	41.0%
			3	N/A	N/A	9	41.0%
			4	N/A	N/A	9	41.5%
			5	N/A	N/A	9	40.9%
			6	N/A	N/A	8	11.5%
		Reweighted	1	N/A	N/A	9	1.0%
			2	N/A	N/A	9	40.9%
			3	N/A	N/A	9	40.9%
			4	N/A	N/A	9	40.9%
			5	N/A	N/A	9	1.0%
			6	N/A	N/A	9	51.7%

Figure A32.3: Population loss, clearing cost, blocks cleared, and impairment by scoring rule