Slide 1



# E911 PHASE 2 ACCURACY DEFINITION MODIFICATION PROPOSAL

## MEAN RADIAL ERROR (MRE)

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Slide 2



## Accuracy Proposals

- RMS
  - Root Mean Square of all location attempt errors
  - Current FCC sanctioned accuracy definition
  - Assessment:
    - Gives undue weight to outliers
    - Includes all location attempts in accuracy calculation
- 67%
  - Abscissa value at which the location error CDF crosses the 67% level
  - Assessment:
    - Very insensitive to outliers
    - Magnitude of largest 33% errors not counted in accuracy calculation
- 90% RMS
  - Root Mean Square of location attempt errors with largest 10% excluded
  - Assessment:
    - Insensitive to outliers
    - Magnitude of largest 10% errors not counted in accuracy calculation
- All definitions *except RMS* exclude some fraction of the errors from the accuracy calculation

Slide 3



# Mean Radial Error (MRE) Proposal

- **Proposal development criteria**
  - all location attempts should be counted as part of the accuracy definition
  - a small fraction of location failures or "outliers" should not unduly skew the results of an otherwise excellent ALI technology
  - the original accuracy goals of the Commission should not be relaxed
- **MRE proposal**
  - Calculate accuracy as the Mean of the Radial Error
  - Comparison: RMS and MRE
    - RMS error equation (assumes actual location at origin)

$$A_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i^2 + y_i^2)}$$

*Errors accumulated as magnitude squared*

    - MRE error equation (same assumption)

$$A_{MRE} = \frac{1}{N}\sum_{i=1}^{N}\sqrt{x_i^2 + y_i^2}$$

*Errors accumulated as magnitude*

    - Errors accumulated by magnitude for MRE and magnitude squared for RMS
    - MRE includes all errors, but does not unduly magnify "outliers"
- **MRE provides for more equitable error weighting**

Slide 4



## ACCURACY EQUIVALENCE

- **Probability theory**
  - **Gaussian probability density function (PDF)**

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \longrightarrow A_{RMS} = \sqrt{2\sigma^2}$$

  - **Transform this density by the function:** $r = \sqrt{x^2 + y^2}$
    - **Resulting random variable has a Rayleigh PDF**

$$f_r(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} U(r)$$

    - **The mean of this transformed Gaussian process is well known to be:**

$$A_{MRE} = \sigma\sqrt{\pi/2}$$

- **Accuracy equivalence**
  - **$A_{RMS}$ = 125 m for $\sigma$ = 88.39 m**
  - **For $\sigma$ = 88.39 m, $A_{MRE}$ = 110 m**
  - **Thus, setting a 110 meter MRE accuracy goal is equivalent to an RMS goal of 125 meters**
- **MRE accuracy goal of 110 meters is equivalent to 125 meter RMS goal**

Slide 5

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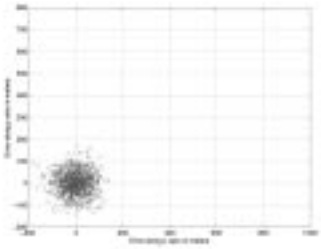
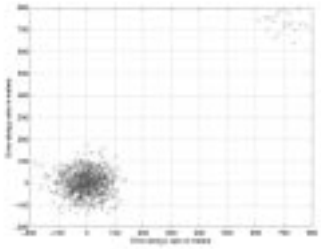
## ACCURACY DEFINITION CASE STUDIES



- Case 1
  - 1000 2-D Gaussian draws
  - $\sigma$ = 50 m (both dimensions)
  - Radial error $\sigma$ = 71 m

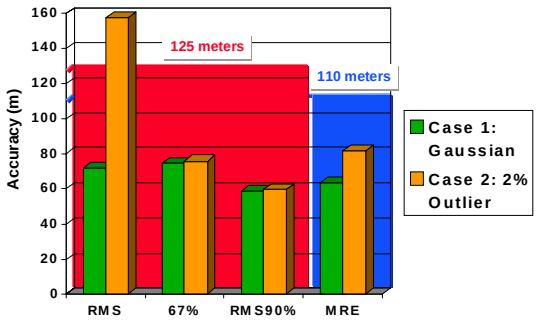


- Case 2
  - 980 points identical to Case 1
  - 20 "outliers" created
    - offset points by 1000 m
    - 2% outlier probability

Slide 6



## ACCURACY RESULTS




- Accuracy impact of 2% outlier as compared to Gaussian case
  - RMS accuracy degraded by 120%, non-compliant
  - RME accuracy degraded by 30%, compliant
  - 67%, CEP & RMS90% accuracy's degraded by less than 2%, compliant

Slide 7



## CONCLUSIONS

- The MRE definition:
  - Assures that all errors are counted in the accuracy calculation
  - Reduces sensitivity to outliers
  - Assures equivalent accuracy to the 125m/RMS goal (Gaussian assumption)
    - By setting MRE accuracy goal to 110 meters
- Win/Win for the interested parties
  - Public safety assured that all location errors counted in accuracy calculation
  - Carriers and vendors assured relief from excessive outlier impact on accuracy calculation
- We now have at least four accuracy definition proposals
  - RMS
  - 67%
  - 90% RMS
  - MRE
- Which one represents the best compromise between public safety requirements and wireless industry compliance difficulty?
- Let's work together to expeditiously decide