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July 24, 2001

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FEDERAL COMMUNICATIONS COMMISSION
OFFICE OF THE SECRETARY

VIA HAND DELIVERY

Ms. Magalie Roman Salas
Secretary
Federal Communications Commission
445 12th Street, S.W.
Room TW-B204
Washington, D.C. 20554

Re: **EX PARTE**
FCC Docket No. 94-102 – In the Matter of the Commission's Rules to Ensure Compatibility with Enhanced 911 Emergency Calling Systems,

Dear Ms. Salas:

Attached hereto is SigmaOne Communications Corporation's response to AT&T Wireless' July 13, 2001 *ex parte presentation* and U.S. Wireless' *ex parte* comments of July 11, 2001.

In accordance with Section 1.1206 of the Commission's rules, an original and one copy of this letter and the associated materials are being submitted. Please contact the undersigned if there are any questions in connection with this matter.

Sincerely yours,

Henry M. Rivera
Counsel to SigmaOne Communications Corporation

cc: Thomas Sugrue, Chief
Wireless Telecommunications Bureau
Blase Sento, Esquire
Wireless Telecommunications Bureau

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FEDERAL COMMUNICATIONS COMMISSION
OFFICE OF THE SECRETARY

Ms. Magalie Roman Salas
Secretary
Federal Communications Commission
445 12th Street, NW
Washington, DC 20554

RE: *Ex Parte* Communication in CC Docket 94-102

Dear Ms. Salas:

SigmaOne Communications Corporation (hereinafter "SigmaOne") hereby submits the following *ex parte* comments in response to AT&T Wireless' (hereinafter "AT&T") July 13th *ex parte* presentation and submits exhibit B to this *ex parte* in response to U.S. Wireless *ex parte* comments from July 11th.

Network Based Location Solutions Exist Today That Meet Or Very Closely Meet The Commission's Rules On Accuracy

AT&T's now bases its waiver request on the assertion that there are no viable alternatives to MNLS, and that "MNLS has an accuracy range comparable to that of the other network based solutions ..."[1] SigmaOne emphatically rejects both claims. In fact, there are several viable and far more accurate alternatives to "expected" performance of MNLS. Results from tests recently conducted by NENA[2] and SigmaOne's carrier audited AMPS/TDMA field trials (carrier identity withheld due to NDA) demonstrate that the SigmaOne location system, even in its beta stage, clearly met the 67% FCC requirement in suburban environments, fully met the 95% FCC requirement across all environments and achieved 110 meters– 67% accuracy when test results were compiled across all environments (suburban, urban and rural). These results represent an accuracy that is more than two and a half times better than those "expected" by AT&T and will be eight times better than the theoretical lower bound accuracy of MNLS.

[1] AT&T Wireless Services, Ex Parte Presentation, Revision of the Commission's Rules to Ensure Compatibility with Enhanced 911 Emergency Calling Systems, July 13, 2001, Page 8 of 13.
[2] Comments of APCO and NENA as Public Safety Organizations, in response to request for waiver of AT&T Wireless Services Inc, May 7th, 2001.

AT&T cannot dismiss SigmaOne's test results by simply trying to paint all technology providers with the same broad brush. Suggesting that all AOA-TDOA technology solutions must by definition deliver the same level of performance is clearly unfounded. We do not fault AT&T for not testing SigmaOne. But we do object to AT&T's unsubstantiated conclusion about the accuracy of SigmaOne's TDOA-AOA location system.

AT&T Has Yet To Provide A Theoretical Basis For MNLS Accuracy Performance Claims

In all the filings submitted to the commission, neither AT&T nor its infrastructure vendors have provided a theoretical foundation for the accuracy claims of MNLS. SigmaOne has performed and submitted for public review such an analysis for both technologies (RSSI triangulation and "Contour Matching") identified by AT&T as being used in MNLS. The optimal performance achievable by any location system will fundamentally be limited by a lower bound. The existence of such a bound, called the Cramer-Rao (CR) lower bound, implies that *irrespective of the method* used to quantify the parameters from the data (pre/post processing), there is a lower bound on the accuracy performance that *can not be superseded.* The CR lower bound has been the key analytical tool used by all location technology vendors to evaluate the performance of any location system. Both the analysis submitted by SigmaOne in June, and the attached document, demonstrate that the CR lower bound for MNLS is on the order of 900-1400 meters 67%.

All Referenced Network Based System Trials Were Audited By Independent Third Parties

Unlike AT&T's trial of MNLS, SigmaOne's field trials were audited by an independent third party. Since AT&T has now also put itself into the role of a location technology provider (MNLS), it too could be described as a "self-interested" party seeking to promote its own Phase II solution. There is one major difference. Unlike the AT&T trials, SigmaOne conducted three separate field trials under the auspices of an independent wireless carrier.

AT&T suggests that "only self-interest can explain SigmaOne's opposition to AT&T's proposed "use of a technology that would accelerate the implementation of Phase II E911 for TDMA customers."[3] It is not SigmaOne but the public safety community of the United States represented by APCO and NENA that challenges the value of AT&T's MNLS solution. SigmaOne merely questions the technical assumptions behind AT&T's accuracy and deployment schedule claims.

Rapid Deployment Of MNLS Is Very Unlikely

AT&T's claim that it can deploy a nationwide MNLS system by the first quarter of 2002 is highly suspect. It remains unclear which of AT&T's infrastructure vendors have committed to the timeline required for a nationwide rollout. Nortel and Lucent, representing a significant portion of AT&T's wireless infrastructure footprint have not even begun the development of the necessary modifications to their TDMA infrastructure in support of MNLS, not to mention the integration of the MNLS PDE within their respective network infrastructures. Nortel has stated that "generally 18 to 24 months are needed between standard adoption (April 2001 for MNLS) and development of compatible technology."[4] Furthermore, Nortel is still "evaluating development of Mobile Assisted Network Location System software ('MNLS') for the DMS-MTX switch...Nortel Networks has not yet committed to the development of the MNLS feature..."[5] Nor has Lucent committed to development of MNLS, stating that this "optional feature is being considered as a candidate for a future TDMA Software Release with a targeted General Availability (GA) of August 2002."[6]

The deployment of any location system first requires the development of a complete end to end real time system. The development of such an end to end MNLS system does not appear to be even close to completion. A closer look at the published test results by AT&T show that none of these tests included the demonstration of an end-to-end real time solution, providing location data in real time during a call (in contrast to all

[3] Id., Page 8 of 13.
[4] Cingular Wireless, Petition For Limited Waiver of Sections 20.18(e)-(h), July 6, 2001, Attachment I(Nortel Networks, Letter to Cingular Wireless, July 3, 2001, page 3).
[5] Id., Page 2.
[6] Id., Attachment I (Lucent Technologies, Letter to Cingular Wireless, July 6, 2001).

the network based trials conducted by AT&T and the trials performed by SigmaOne). The tests described by AT&T included collection of RSSI measurements and the "offline" analysis of these results in order to process location. Given the embryonic stage of development of MNLS, it remains unclear how AT&T can guarantee the deployment of a complete system in the time frame suggested.

All location technology and cellular infrastructure providers understand that there are no short-cuts in the development, testing and commercializing of any new, complex wireless system. As the FCC has itself witnessed over the past five years, the development of any location system is a difficult and time consuming proposition. AT&T has stated that it began the development process for MNLS in the first quarter of 2001, will begin deployment by the fourth quarter of 2001 and will complete its nationwide deployment by the first quarter of 2002. To put this effort into perspective, companies such as Cambridge Positioning have been developing their location system since 1994, True Position since 1992, SnapTrack (Qualcomm) since 1995, U.S. Wireless since 1996, Grayson since 1997 and SigmaOne since 1997. While not wishing to challenge the skills of its engineering organization the development and optimization of any location system must by definition take significant time. Despite its promises, rapid development and deployment of MNLS is highly unlikely and will be far slower than the deployment of many network based location systems.

SigmaOne Stands By Its Prediction That MNLS Accuracy Will Be 900-1400 meters 67%

As a result of the AT&T supplemental filing, SigmaOne has had the opportunity to gain greater insight into the AT&T approach for its MNLS solution. AT&T's accuracy claims depend on comparing the RSSI measurements to a reference database obtained through RF propagation tools and performance of calibration drive tests throughout the area. Following this clarification we updated our technical and theoretical analysis of the expected performance of MNLS (attached as Exhibit A). As a result of this analysis and simulation of MNLS with mapping data, SigmaOne continues to believe that real world

MNLS performance will be 900-1400 meters. This is far worse than the 250 meter 67% suggested by AT&T or the 300-400 meters achieved by AT&T in their recent tests.

AT&T's Recent Tests Do Not Reflect Their Proposed Approach for the Deployment Of MNLS

AT&T has conducted its tests under optimal conditions which it has admitted will be very different from its proposed "nation-wide deployment" of MNLS. AT&T has stated that in order to rapidly deploy Phase II MNLS service to its TDMA customers it will "initially" use predictive data in the creation of its MNLS database. Yet in all of the tests conducted to date AT&T has utilized "measured" data in the creation of its reference database. Had AT&T conducted its MNLS tests with predictive data, along highways and roads that were not repeatedly measured prior to each test run, their results would have undoubtedly been many times worse. It is quite clear, that MNLS will not function and will not be reliable until AT&T collects a 50x50 meter grid of measured data over its entire foot-print. It is safe to assume that this process will require meticulous and never ending database updates that will prove impractical and unreliable.

AT&T's Tests Do Not Reflect a "Real World" Environment

SigmaOne believes that the test scenarios used by AT&T do not reflect a "real world' environment and therefore do not reflect the accuracy that could be expected from this technology. The complexities of the MNLS approach and its tremendous dependency upon accurate mapping data, reliance on power measurements performed by the handset, and the dramatic variations on these measurements due to external and nearby factors (well within a 50x50 grid), were not thoroughly considered, nor tested in these scenarios. As a result, the trial scenarios reflect a very controlled and unrealistic environment for location testing.

In real-world scenarios handset RSSI measurements (even the relative measurements) could vary significantly within the 50x50 grid. Examples of such scenarios include those in which the handset is located in different locations within a vehicle (different windows) or located immediately outside of a vehicle. If the vehicle

moves along a different portion of the road, the relative signal strength could be severely impacted by proximity to landmarks, buildings and other physical obstructions. Relative signal strength mapping can vary significantly within a 50m by 50m grid if the handset is located outside a specific structure versus inside the structure. RSSI measurements could also vary significantly from a vehicle located on the road to a scenario in which the vehicle is located, for example, in a ditch (accident). The location of the handset is not the only factor. The specific orientation of the handset at the time of the call, the handset specific model, including other factors such as antenna radiation pattern or placement can cause tremendous variations in the relative RSSI measurements reported by each of these various handsets differing from those obtained through a drive run.

AT&T Does Not Understand SigmaOne's Analysis

In its analysis of RSSI-based location performance, SigmaOne never used absolute measurements. The analysis only used relative measurements and the variations in these relative measurements. It is the variations in signal strength that have a severe impact on location estimation. Whether moving inside a vehicle, or whether impacted by nearby vehicles; whether walking around a building, between rooms or among different floors, the variations in directional path loss cause different biases in received signal strength from different sites. Since the phone does not have diversity reception, these differences are more significant that on the reverse channel. These significant variations are definitely the result of multipath, shadowing and other effects AT&T claims not to be significant.

It is obvious that AT&T does not understand the claims made by SigmaOne. The accuracy of the RF propagation tools is irrelevant to the analysis performed by SigmaOne. For the purpose of its analysis, SigmaOne assumed that the initial database used by MNLS is fundamentally accurate, with no initial errors attributed to the prediction tool or to the data obtained through the calibration drive tests. SigmaOne's analysis of MNLS accuracy is only attributed to RSSI measurement fluctuations.

There is no mystery in how MAHO works. The issue at hand is location. For example, there are additional problem areas that AT&T has not fully considered. MAHO

may report receptions from unexpected sites, due to channel reuse, rather than those expected. In our analysis we refrained from considering this as an additional source of error and we assumed all RSSI reports to be relevant in order to determine the theoretical bound of achievable accuracy.

The accuracy prediction analysis provided by SigmaOne takes into account the averaging of four successive measurements. This however, does not solve the problem of poor accuracy results. However, as AT&T well knows, in stationary scenarios, which probably constitute the vast majority of emergency situations, RSSI measurements are repetitive, rendering the averaging process useless.

AT&T claims that "the standard deviation used by TDMA carriers like AWS is actually 8 dB or less,"[7] versus 11.8 dB. - Even though our field tests indicate that such a standard deviation is too low, the performance illustration in Exhibit A, uses an 8 dB standard deviation. As can be seen, the Cramer-Rao bound applicable to these conditions for MNLS continues to be in the order of 900-1400 m 67%.

The Benefits of Network Based Location Solutions Far Outweigh MNLS

Key Factors	MNLS	TDOA/AOA
Development stage	Preliminary alpha testing of the concept– No end to end system	Ready for deployment
Technical analysis (Cramer-Rao Lower Bound)	900-1400 meters	80-120 meters
Field Accuracy (67%)	300-400 meters [8]	110 meters
Field Accuracy (95%)	600-700 meters	300 meters
Indoor location	No	Yes
AMPS	Not supported	Supported
Audited trials	None	Multiple vendors with multiple carriers
Public safety benefits	Low	High
Deployment time	After completion of development (2003?)	6 months from equipment purchase

[7] AT&T Wireless, Ex Parte Presentation, July 13, 2001, Page 3 of 13.
[8] Based upon controlled, un-audited test results submitted by AT&T – "Supplemental Response of AT&T wireless to order of the Wireless Telecom Bureau – June 12th, 2001

Accordingly, SigmaOne requests that the FCC deny AT&T's waiver request for its AMPS and TDMA networks and require that it move expeditiously to deploy location solutions that can meet both the letter and spirit of the Commission's mandate. Over the past five years the FCC, the public safety community, wireless carriers, and location technology providers have devoted a great deal of time, dollars and energy towards creating a regulatory framework by which the Phase II E911 location requirement will become a reality. Many years of effort have gone into the formulation of accuracy requirements for Phase II E911 implementation and the development and testing of location technologies. Notwithstanding the assertions made by AT&T, the FCC's long-standing assumption that there are viable and acceptable location solutions in the marketplace that are deployable and acceptable within the FCC guidelines are in fact correct. After five hard years of development and testing and only months before the mandate deadline, it is time for the industry to move forward.

If you have any questions concerning this filing, please do not hesitate to contact the undersigned.

Respectfully submitted,

/s/ Mark Licht
Mark Licht
PRESIDENT
SigmaOne Communications Corporation

Exhibit A

Fundamental Lower Bound on the Attainable Accuracy of RSSI Based Location Systems

Abstract

This analysis derives the Cramer-Rao Bound (CRB) on the attainable accuracy of an RSSI based location system using the "Contour Matching" technique described by AT&T in its July 13th filing. It is shown that regardless of the algorithm used for location (including pre-processing, post-processing, etc.) the attainable accuracy remains very poor compared to other well known methods such as GPS, AOA, TDOA, TOA etc.

From the analysis forthwith and the numerical illustrations provided at the end of this attachment, it is clear that the Cramer Rao lower accuracy bound for RSSI "Contour Matching" methods will be within the range of 900-1400 meters 67%.

Introduction

Recently, AT&T proposed MNLS (Mobile Assisted Network Location System) as its phase II wireless 911 solution for TDMA networks. This method is based on performing Radio Signal Strength Indication (RSSI) measurements by the IS-136 handset on the forward control channels transmitted by the base stations in the vicinity of the handset. These measurements are then compared to a RSSI database for location determination. The RSSI database is generated initially by utilizing RF propagation tools and subsequently by the performance of calibration drive tests throughout the relevant area.

An alternative approach is based upon using the RSSI measurements for distance assessment (the distance between the handset and the cellular base stations) and then using the distance estimation for location determination via triangulation. SigmaOne provided an analysis of the triangulation method in Exhibit C of its June 5th FCC filing.

The RSSI measurements are location dependent and therefore can be a (rough) location indicator. The aim of this analysis is to characterize the accuracy of this method.

Mathematical Discussion

Assume that there is a one to one mapping from location to RSSI measurements. In other words, each location, $\mathbf{z}=(x, y)$, is associated with a unique RSSI vector $\mathbf{a}(\mathbf{z})$. However, due to the specific situation of the handset (horizontal or vertical, within a vehicle or outside, etc.) the actual measurements may be attenuated by a factor α. Thus, the measured vector is given by

$$\hat{\mathbf{m}} = \alpha \mathbf{a}(\mathbf{z}) + \mathbf{e} \tag{1}$$

where **e** stands for the measurement error vector and **m** stands for an error free measurement vector.

Cramer-Rao Lower Bound

The Cramer-Rao Lower bound (CRB) is a theoretical lower bound on the covariance matrix of <u>*any*</u> unbiased estimator. In other words, the difference between the covariance of the estimator and the CRB is a positive semidefinite matrix.

We assume that the measurement error ector, **e**, is normally distributed multivariate random vector having zero mean, and a positive definite covariance matrix, $\mathbf{C}_m$. The conditional probability density function of the measurements is given by

$$f(\mathbf{m}|\alpha,\mathbf{z}) = \frac{1}{(2\pi)^{M/2}|\mathbf{C}_m|^{1/2}} \exp\left\{-(1/2)[\mathbf{m}-\alpha\mathbf{a}(\mathbf{z})]^T \mathbf{C}_m^{-1}[\mathbf{m}-\alpha\mathbf{a}(\mathbf{z})]\right\} \tag{2}$$

where M represents the length of the measurement vector.

The CRB is given by the inverse of the Fisher Information Matrix. The elements of the Fisher Information Matrix are given by,

$$F_{ij} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_i \partial\phi_j}\right\} \tag{3}$$

where $\boldsymbol{\varphi} = [x, y, \alpha]$. Thus we get,

$$F_{11} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_1^2}\right\} = \alpha^2 \mathbf{a}_x^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}_x(\mathbf{z}) \tag{4}$$

$$F_{22} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_2^2}\right\} = \alpha^2 \mathbf{a}_y^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}_y(\mathbf{z}) \tag{5}$$

$$F_{33} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_3^2}\right\} = \mathbf{a}^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}(\mathbf{z}) \tag{6}$$

$$F_{12} = F_{21} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_1 \partial\phi_2}\right\} = \alpha^2 \mathbf{a}_y^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}_x(\mathbf{z}) \tag{7}$$

$$F_{13} = F_{31} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_1 \partial\phi_3}\right\} = \alpha \mathbf{a}^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}_x(\mathbf{z}) \tag{8}$$

$$F_{23} = F_{32} = -E\left\{\frac{\partial^2 \log f(\mathbf{m}|\boldsymbol{\varphi})}{\partial\phi_2 \partial\phi_3}\right\} = \alpha \mathbf{a}^T(\mathbf{z})\mathbf{C}_m^{-1}\mathbf{a}_y(\mathbf{z}) \tag{9}$$

The CRB matrix is given by

$$\text{CRB} = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}^{-1} \tag{10}$$

An estimator whose performance is close to the CRB is the Least Squares Estimator. It is useful to derive this estimator and verify that its performance is near the CRB in order to verify that no error exists either in the analysis or the coding of the bound.

The Least Squares Estimator

The Least Squares Estimator of $\mathbf{z}$ is based on minimizing the following cost function,

$$Q(\mathbf{z},\alpha) \quad \|\hat{\mathbf{m}} - \mathbf{a}(\mathbf{z})\alpha\|^2 \tag{11}$$

The scalar $\hat{\alpha}$ that minimizes the cost function is given by

$$\hat{\alpha} = (\mathbf{a}^T(\mathbf{z})\mathbf{a}(\mathbf{z}))^{-1}\mathbf{a}^T(\mathbf{z})\hat{\mathbf{m}} \tag{12}$$

Substituting (12) in (11) yields

$$Q(\mathbf{z}) = \left\|\hat{\mathbf{m}} - \|\mathbf{a}(\mathbf{z})\|^{-2}\,\mathbf{a}(\mathbf{z})\mathbf{a}^T(\mathbf{z})\hat{\mathbf{m}}\right\|^2 \tag{13}$$

The simplest and perhaps the most reliable minimization scheme is a simple search over the set of vectors $\mathbf{a}(\mathbf{z})$ associated with a grid of locations determined by the desired precision. Actual field measurements or RF prediction tools can be used to obtain the vectors $\mathbf{a}(\mathbf{z})$ associated with the grid.

We now turn to evaluate the accuracy of this method.

First define the normalized vectors

$$\mathbf{b}(\mathbf{z}) \quad \mathbf{a}(\mathbf{z})/\|\mathbf{a}(\mathbf{z})\| \tag{14}$$

Using (14) in (13) yields,

$$Q(\mathbf{z}) = \left\|\hat{\mathbf{m}} - \mathbf{b}(\mathbf{z})\mathbf{b}^T(\mathbf{z})\hat{\mathbf{m}}\right\|^2 \tag{15}$$

Instead of minimizing (15) we can maximize

$$P(\mathbf{z},\hat{\mathbf{m}}) = \hat{\mathbf{m}}^T\mathbf{b}(\mathbf{z})\mathbf{b}^T(\mathbf{z})\hat{\mathbf{m}} = \left|\mathbf{b}^T(\mathbf{z})\hat{\mathbf{m}}\right|^2 \tag{16}$$

At the maximum point, the derivative of $P(\mathbf{z},\hat{\mathbf{m}})$ with respect to $\mathbf{z}$ must be zero. The derivative is given by

$$\mathbf{f}(\mathbf{z},\hat{\mathbf{m}}) \quad \frac{\partial P}{\partial \mathbf{z}} = \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial y} \end{bmatrix} = 2\begin{bmatrix} \hat{\mathbf{m}}^T \dot{\mathbf{b}}_x(\mathbf{z})\mathbf{b}^T(\mathbf{z})\hat{\mathbf{m}} \\ \hat{\mathbf{m}}^T \dot{\mathbf{b}}_y(\mathbf{z})\mathbf{b}^T(\mathbf{z})\hat{\mathbf{m}} \end{bmatrix} \tag{17}$$

where $\dot{\mathbf{b}}_x(\mathbf{z})$ and $\dot{\mathbf{b}}_y(\mathbf{z})$ are the derivatives of $\mathbf{b}(\mathbf{z})$ with respect to x and y, respectively. In the case where the measurement is error free, the estimate is precisely $\mathbf{z}_0$, the exact location. For all other measurements we can write a first order Taylor series approximation,

$$\mathbf{f}(\hat{\mathbf{z}},\hat{\mathbf{m}}) \cong \mathbf{f}(\mathbf{z}_0,\mathbf{m}) + \frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{z}}\Delta\mathbf{z} + \frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{m}}\Delta\mathbf{m} \tag{18}$$

where

$$\Delta\mathbf{z} \quad \hat{\mathbf{z}} - \mathbf{z}_0 \tag{19}$$

$$\Delta\mathbf{m} \quad \hat{\mathbf{m}} - \mathbf{m} \tag{20}$$

and $\hat{\mathbf{z}}$ denotes the value of $\mathbf{z}$ that maximizes the cost function (16) for a given measurement $\hat{\mathbf{m}}$. Obviously we have

$$\mathbf{f}(\hat{\mathbf{z}},\hat{\mathbf{m}}) = \mathbf{f}(\mathbf{z}_0,\mathbf{m}) = \mathbf{0} \tag{21}$$

so that (18) becomes

$$\frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{z}}\Delta\mathbf{z} \cong -\frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{m}}\Delta\mathbf{m} \tag{22}$$

Define the following matrices

$$\mathbf{F} \quad \frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{z}} \tag{23}$$

$$\mathbf{G} \quad \frac{\partial \mathbf{f}(\mathbf{z}_0,\mathbf{m})}{\partial \mathbf{m}} \tag{24}$$

Using (23) and (24) in (22) and right multiplying by the transpose of the equation we get

$$\mathbf{F}\Delta\mathbf{z}\Delta\mathbf{z}^T\mathbf{F}^T \cong \mathbf{G}\Delta\mathbf{m}\Delta\mathbf{m}^T\mathbf{G}^T \tag{25}$$

Taking expectations of (25) and multiplying on the left by the inverse of **F** and on the right by the inverse transpose of **F** yields

$$\mathbf{C} \quad \mathrm{cov}(\mathbf{z}) = \mathrm{E}\left\{\Delta\mathbf{z}\Delta\mathbf{z}^T\right\} \cong \mathbf{F}^{-1}\mathbf{G}\mathrm{E}\left\{\Delta\mathbf{m}\Delta\mathbf{m}^T\right\}\mathbf{G}^T\mathbf{F}^{-T} = \mathbf{F}^{-1}\mathbf{G}\mathbf{C}_{\mathbf{m}}\mathbf{G}^T\mathbf{F}^{-T} \tag{26}$$

$$\mathbf{C}_{\mathbf{m}} \quad \mathrm{cov}(\hat{\mathbf{m}}) = \mathrm{E}\left\{\Delta\mathbf{m}\Delta\mathbf{m}^T\right\} \tag{27}$$

We now turn to obtain mathematical expressions for the matrices **F** and **G**.

$$\mathbf{F} = 2\begin{bmatrix} \mathbf{m}^T\left(\ddot{\mathbf{b}}_{xx}\mathbf{b}^T + \dot{\mathbf{b}}_x\dot{\mathbf{b}}_x^T\right)\mathbf{m} & \mathbf{m}^T\left(\ddot{\mathbf{b}}_{xy}\mathbf{b}^T + \dot{\mathbf{b}}_x\dot{\mathbf{b}}_y^T\right)\mathbf{m} \\ \mathbf{m}^T\left(\ddot{\mathbf{b}}_{xy}\mathbf{b}^T + \dot{\mathbf{b}}_x\dot{\mathbf{b}}_y^T\right)\mathbf{m} & \mathbf{m}^T\left(\ddot{\mathbf{b}}_{yy}\mathbf{b}^T + \dot{\mathbf{b}}_y\dot{\mathbf{b}}_y^T\right)\mathbf{m} \end{bmatrix} \tag{28}$$

$$\mathbf{G} = 2\begin{bmatrix} \mathbf{m}^T\left(\mathbf{b}\dot{\mathbf{b}}_x^T + \dot{\mathbf{b}}_x\mathbf{b}^T\right) \\ \mathbf{m}^T\left(\mathbf{b}\dot{\mathbf{b}}_y^T + \dot{\mathbf{b}}_y\mathbf{b}^T\right) \end{bmatrix} \tag{29}$$

Since the norm of **b** is unity (see equation (14)), its first derivative is orthogonal to $\mathbf{m} = \alpha\mathbf{a}(\mathbf{z})$. Thus,

$$\mathbf{F} = 2\mathbf{b}^T\mathbf{m}\begin{bmatrix} \mathbf{m}^T\ddot{\mathbf{b}}_{xx} & \mathbf{m}^T\ddot{\mathbf{b}}_{xy} \\ \mathbf{m}^T\ddot{\mathbf{b}}_{xy} & \mathbf{m}^T\ddot{\mathbf{b}}_{yy} \end{bmatrix} \tag{30}$$

$$\mathbf{G} = 2\mathbf{b}^T\mathbf{m}\begin{bmatrix} \dot{\mathbf{b}}_x^T \\ \dot{\mathbf{b}}_y^T \end{bmatrix} \tag{31}$$

where the derivative of **b** w.r.t. x is

$$\dot{\mathbf{b}}_x = \dot{\mathbf{a}}_x(\mathbf{a}^T\mathbf{a})^{-1/2} - \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_x^T\mathbf{a}) \tag{32}$$

where the derivative of **b** w.r.t. x is

$$\dot{\mathbf{b}}_y = \dot{\mathbf{a}}_y(\mathbf{a}^T\mathbf{a})^{-1/2} - \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_y^T\mathbf{a}) \tag{33}$$

the second derivative of **b** w.r.t. to x is

$$\ddot{\mathbf{b}}_{xx} = \ddot{\mathbf{a}}_{xx}(\mathbf{a}^T\mathbf{a})^{-1/2} - 2\dot{\mathbf{a}}_x(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_x^T\mathbf{a}) - \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\ddot{\mathbf{a}}_{xx}^T\mathbf{a}) \\ -\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_x^T\mathbf{a}_x) + 3\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-5/2}(\dot{\mathbf{a}}_x^T\mathbf{a})^2 \tag{34}$$

the second derivative of **b** w.r.t. to y is

$$\ddot{\mathbf{b}}_{yy} = \ddot{\mathbf{a}}_{yy}(\mathbf{a}^T\mathbf{a})^{-1/2} - 2\dot{\mathbf{a}}_y(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_y^T\mathbf{a}) - \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\ddot{\mathbf{a}}_{yy}^T\mathbf{a}) \\ -\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_y^T\mathbf{a}_y) + 3\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-5/2}(\dot{\mathbf{a}}_y^T\mathbf{a})^2 \tag{35}$$

where the derivative of **b** w.r.t. x and y is

$$\ddot{\mathbf{b}}_{xy} = \ddot{\mathbf{a}}_{xy}(\mathbf{a}^T\mathbf{a})^{-1/2} - \dot{\mathbf{a}}_x(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_y^T\mathbf{a}) - \dot{\mathbf{a}}_y(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_x^T\mathbf{a}) - \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\ddot{\mathbf{a}}_{xy}^T\mathbf{a}) \\ -\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-3/2}(\dot{\mathbf{a}}_x^T\dot{\mathbf{a}}_y) + 3\mathbf{a}(\mathbf{a}^T\mathbf{a})^{-5/2}(\dot{\mathbf{a}}_y^T\mathbf{a})(\dot{\mathbf{a}}_x^T\mathbf{a}) \tag{36}$$

The vector **a**(**z**) is given by

$$[\mathbf{a}(\mathbf{z})]_i = \frac{K}{R_i^{\gamma}} = \frac{K}{\left[(x_i - x)^2 + (y_i - y)^2\right]^{\gamma/2}} \tag{37}$$

where (x_i, y_i) are the i-th base-station's coordinates, (x,y) are the MS coordinates, K is a constant and R_i is the distance between the MS and the base station. Equation (37) leads to the following derivatives

$$[\dot{\mathbf{a}}_x]_i = \frac{K\gamma(x_i - x)}{R_i^{2+\gamma}} \tag{38}$$

$$[\dot{\mathbf{a}}_y]_i = \frac{K\gamma(y_i - y)}{R_i^{2+\gamma}} \tag{39}$$

$$[\ddot{\mathbf{a}}_{xx}]_i = \frac{K\gamma\left\{-R_i^2 + (x_i - x)^2(2+\gamma)\right\}}{R_i^{4+\gamma}} \tag{40}$$

$$[\ddot{\mathbf{a}}_{yy}]_i = \frac{K\gamma\left\{-R_i^2 + (y_i - y)^2(2+\gamma)\right\}}{R_i^{4+\gamma}} \tag{41}$$

$$[\ddot{\mathbf{a}}_{xy}]_i = \frac{K\gamma(x_i - x)(x_i - x)(2+\gamma)}{R_i^{4+\gamma}} \tag{42}$$

It is not difficult to verify that the accuracy of the Least Squares Estimator of the problem at hand is very close to the CRB.

Conversion of Error Covariance to CEP

We assume that the location estimate errors, ξ, are normally distributed with zero mean and covariance **C**. In mathematical notation,

$$f(\xi) = \frac{1}{\sqrt{(2\pi)^2 |\mathbf{C}|}} \exp\left\{-\frac{1}{2}\xi^T \mathbf{C}^{-1}\xi\right\} \tag{43}$$

The CEP is defined as the radius of the circle, centered at the true location, that contains the location estimate with a specified probability P_{in}. Thus,

$$\begin{aligned} P_{in} &= \iint_S f(\xi)\, d\xi \\ S &= \{\xi : \|\xi\| \le r\} \end{aligned} \tag{44}$$

where r stands for the CEP. The Singular Value Decomposition (SVD) representation of **C** is given by

$$\begin{aligned} \mathbf{C} &= \mathbf{U}\mathbf{S}\mathbf{U}^T \\ \mathbf{C}^{-1} &= \mathbf{U}\mathbf{S}^{-1}\mathbf{U}^T \end{aligned} \tag{45}$$

where **U** is a unitary matrix and **S** is a diagonal matrix whose non-zero entries are the eigen-values of **C**,

$$\mathbf{S} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{46}$$

Define

$$\zeta = \mathbf{U}^T \xi \tag{47}$$

Substituting (47) and (45) in (43) and (44) we get

$$\begin{aligned} P_{in} &= \frac{1}{2\pi\sqrt{|\mathbf{C}|}} \iint_S \exp\left\{-\frac{1}{2}\xi^T \mathbf{U}\mathbf{S}^{-1}\mathbf{U}^T \xi\right\} d\xi = \frac{1}{2\pi\sqrt{|\mathbf{C}|}} \iint_{S_1} \exp\left\{-\frac{1}{2}\zeta^T \mathbf{S}^{-1}\zeta\right\} d\zeta \\ S &= \{\xi : \|\xi\| \le r\} \Rightarrow S_1 = \{\zeta : \|\mathbf{U}\zeta\| = \|\zeta\| \le r\} \end{aligned} \tag{48}$$

Equation (48) can be further simplified as follows,

$$P_{in} = \frac{1}{2\pi\sqrt{\lambda_1\lambda_2}} \iint_{S_1} \exp\left\{-(\zeta_1^2/\lambda_1 + \zeta_2^2/\lambda_2)/2\right\} d\zeta_1 d\zeta_2$$
$$S_1 = \left\{\zeta : \sqrt{\zeta_1^2 + \zeta_2^2} \le r\right\} \tag{49}$$

Changing to polar coordinates by substituting $\zeta_1 = \rho\cos\theta,\ \zeta_2 = \rho\sin\theta$, we get

$$P_{in} 2\pi\sqrt{\lambda_1\lambda_2} = \int_0^{2\pi}\int_0^{r} \rho \exp\left\{-\frac{\rho^2}{2}\left(\frac{\cos^2\theta}{\lambda_1} + \frac{\sin^2\theta}{\lambda_2}\right)\right\} d\rho\, d\theta \tag{50}$$

Using the trigonometric identity,

$$\frac{\cos^2\theta}{\lambda_1} + \frac{\sin^2\theta}{\lambda_2} = \frac{1}{2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) + \frac{1}{2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\cos 2\theta \tag{51}$$

we get

$$P_{in}\sqrt{\lambda_1\lambda_2} = \int_0^{r} \rho \exp(-c\rho^2) I_0(e\rho^2)\, d\rho$$
$$c \quad \frac{1}{4}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) \tag{52}$$
$$e \quad \frac{1}{4}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

Hence, given $P_{in}, \lambda_1, \lambda_2$ it is possible to find the CEP, r, via numerical integration.

Substituting $t = c\rho^2$ we get from (52)

$$P_{in} = \frac{1}{2c\sqrt{\lambda_1\lambda_2}} \int_0^{cr^2} \exp(-t) I_0(te/c)\, dt \tag{53}$$

A closed form expression for this integral can be found in [A. Nuttal]. The result is

$$
\begin{aligned}
P_{in} &= \left[1+\exp(C_1)I_0(C_2)-2Q(C_3,C_4)\right] \\
\eta^2 &= \frac{\lambda_2}{\lambda_1} \\
C_1 &= -\frac{r^2(1+\eta^2)}{4\lambda_2} \\
C_2 &= \frac{r^2(1-\eta^2)}{4\lambda_2} \\
C_3 &= \frac{r(1-\eta)}{2\sqrt{\lambda_2}} \\
C_4 &= \frac{r(1+\eta)}{2\sqrt{\lambda_2}}
\end{aligned}
\tag{54}
$$

Thus, using both of the covariance matrix eigen values it is possible to find the CEP.

References

[1] Harry L. Van Trees, *Detection, Estimation, and Modulation Theory, Part I,* John Wiley & Sons,1968.

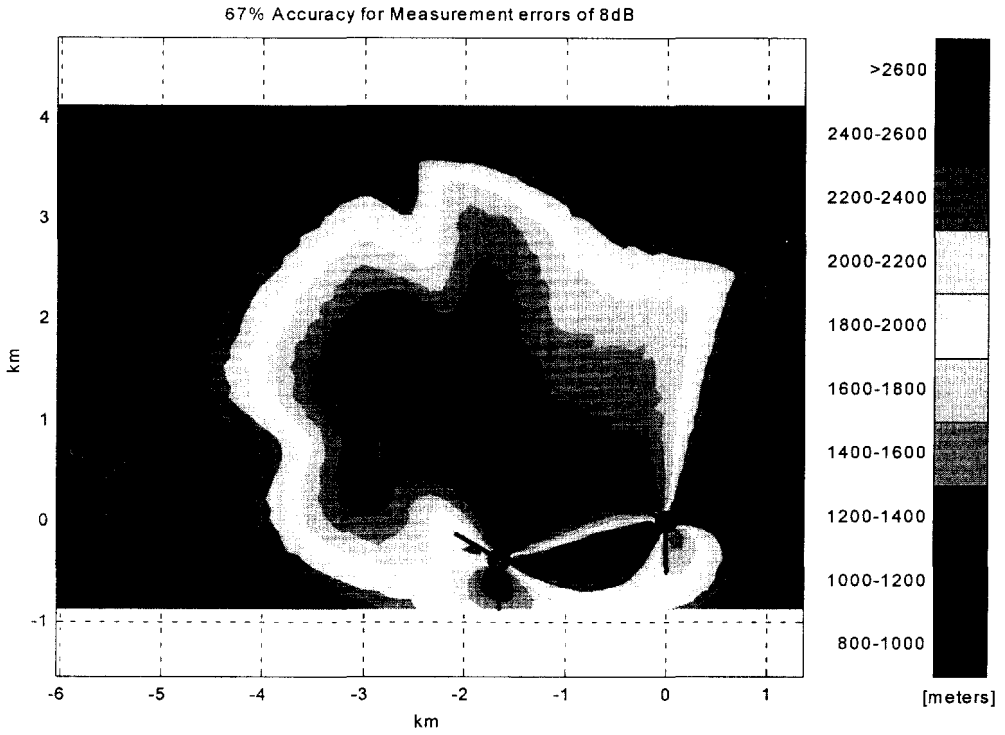
[2] Don J. Torrieri, "Statistical Theory of Passive Location System," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-20, No. 2, March 1984.

[3] Albert H. Nuttall, "Some Integrals Involving the Q-Function," *NUSC Technical Report 4297*, 17 April 1972.

[4] J. T. Gillis, "Computation of the Circular Error Probability Integral" *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-27, No. 6, November 1991.

Some Numerical Examples

The Lower Bound in a 'Real World' Suburban area



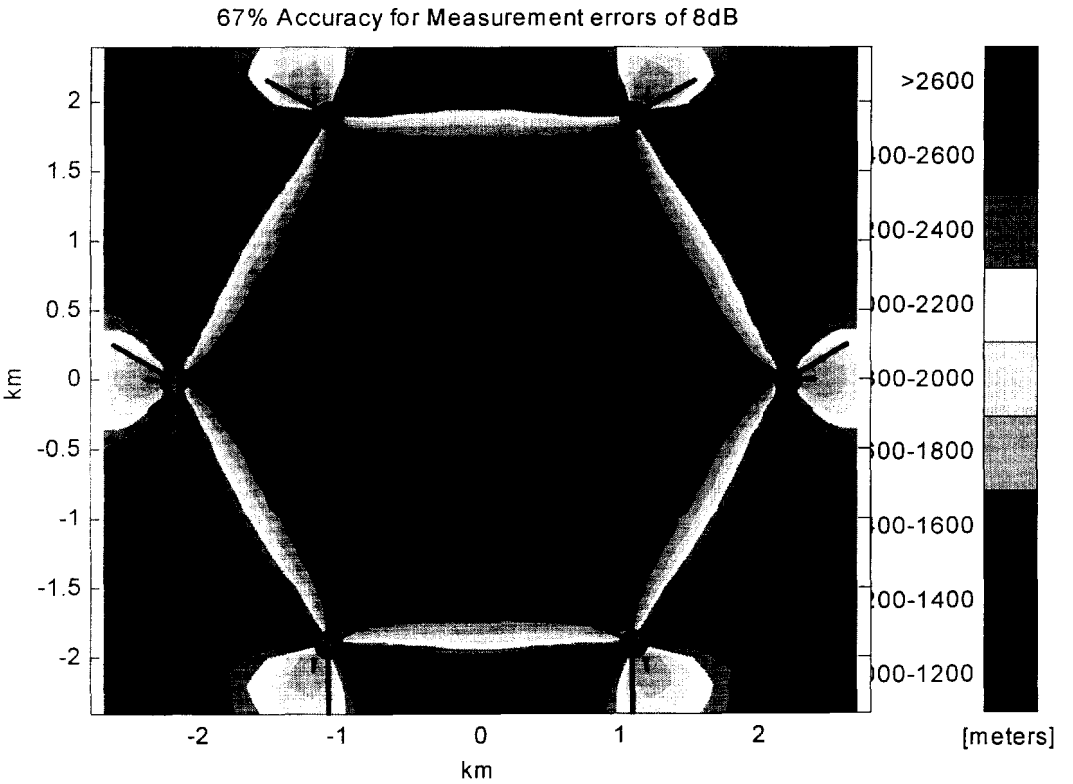

67% Accuracy for Measurement errors of 8dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
km
-2
-1
0
1
2
km
>2600
00-2600
00-2400
00-2200
00-2000
00-1800
00-1600
00-1400
00-1200
[meters]

Exhibit B

Response to U.S. Wireless Ex Parte Comments On Performance, Viability and Application of the Mobile-Assisted Network Location System (MNLS)[9]

While the U.S. Wireless filing was directed at AT&T, comments made regarding SigmaOne's analysis are without merit:

- U.S. Wireless supported the MNLS approach of AT&T, but stopped short of supporting its accuracy claims. U.S. Wireless only restated AT&T's MNLS accuracy performance goals
- U.S. Wireless tried very hard to refute SigmaOne's conclusion that the accuracies claimed by AT&T do not have any theoretical basis. However, U.S. Wireless ***did not*** supply ***any*** technical analysis of its own to support AT&T's accuracy claims.
- U.S. Wireless did not criticize nor comment on the SigmaOne's theoretical analysis in Exhibit C of our June filing.
- By stating that they have technology that is compliant with the FCC requirements, U.S. Wireless has indirectly challenged AT&T's waiver request (which stated that no supplier network solution (including radio camera) could satisfy the FCC requirements, therefore justifying their waiver for MNLS).
- U.S. Wireless maintains that it has relevant MNLS experience, gained through the development of its Radio Camera (RC) techniques. However, there are fundamental differences between RC and MNLS.
    - Radio camera utilizes the uplink channels while MNLS utilizes the downlink channels. With RC, all receive antennas process the same transmitted signal (Single transmission source). In MNLS, several transmission sources (channels) are involved. Even when only one of these channels is received abnormally, MNLS will produce erroneous

[9] U.S. Wireless Corporation, Ex Parte Comment, July 11, 2001

results. Abnormal reception can be due to shading by a passing truck, different shading aspect to the different Base Station Antennas, new Base Station Antenna, higher transmit power, etc.

  - RC systems utilizes **6 additional receive antennas** at each base station. That means that the total number of receive channels is 6 times the number of base stations, while MNLS utilizes **1 control channel** per base station. As a result, these additional receive channels provide RC with 6 times as many measurements from which to process location.
  - RC techniques utilize both amplitude and phase. MNLS uses amplitude only. The phase data provides RC with additional information from which to process location.

- U.S. Wireless admits that its "Power Calibration techniques are still in development". In other words, the claimed relevant experience by U.S. Wireless is only a future hope.
- U.S. Wireless admits that the test region for the "Power Calibration Technique" was approximately two square miles[10]. This is a very small region that automatically precludes large errors. It certainly does not include rural and suburban environments.
- Contrary to U.S. Wireless' claims, the tables on page 6 and 7 of its filing, provide evidence that MNLS is **by far inferior to RC "Power Calibration" techniques**. This is because RC utilizes a technique similar to the "Relative Power Matching" described in the filing (6 antennas per site used) and MNLS is similar to the "Total Site Power" matching technique (one channel per site used).
- Note that, unlike SigmaOne, U.S. Wireless never tested RSSI measurement location techniques. They argue in support of MNLS, based on the performance achieved with their "power calibration" technique. However, due to the differences outlined above their conclusions cannot be accepted.

[10] U.S. Wireless – Ex Parte comments from July 11th – Page 5

- Contrary to the claims made by U.S. Wireless in paragraph 3.1 SigmaOne has evaluated the Cramer-Rao lower bound on the location accuracy that can be achieved from RSSI measurements. The Cramer-Rao lower bound is **a theoretical lower bound** that no estimation procedure can overcome. **Thus, no data pre-processing, system modeling, optimization and/or post-processing can improve the accuracy beyond the errors predicted by the bound.**
- Finally, U.S. Wireless has been developing its RC techniques for the past 5 years. Since AT&T and U.S. Wireless have almost no experience with MNLS, it is fair to assume that the development and optimization of this technique would require a substantial amount of development effort that could extend easily to many years.