Pivotal Buyers and Bargaining Position

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Abstract. Securing sales to a large buyer can be pivotal to a supplier's decision to produce. Conventional wisdom holds that being pivotal improves a buyer's bargaining position vis-a-vis the supplier. This paper finds otherwise. In a model in which a supplier bargains bilaterally with multiple buyers, becoming pivotal through merger tends to worsen the merging buyers' bargaining position.

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1. Introduction

Where scale economies or network effects are important, securing a purchasing agreement with a large buyer can be pivotal to a supplier's decision to produce. Conventional wisdom holds that a pivotal buyer has the upper hand in bargaining with the supplier, because the supplier's threat point is poor. But this insight is only half the story. A pivotal buyer also is on the hook to ensure that the supplier's costs are covered, or else will lose the opportunity to consume. Being pivotal means that payments by other buyers fall short of the supplier's costs. This shortfall must be covered in bargaining between the supplier and pivotal buyer, detracting from the joint surplus of their trade. On net, becoming pivotal tends to increase a buyer's payment to the supplier in the Nash bargaining outcome. Put differently, production has a public good aspect when sales are made through bilateral bargaining and there are network effects or fixed costs common to serving multiple buyers. A large, pivotal buyer internalizes the effect of hard bargaining on the supplier's decision to sink costs. The richer contributions of pivotal buyers can then be said to cross-subsidize consumption by non-pivotal buyers.

A numerical example can illustrate the point. Consider a good with fixed costs of 40 and zero marginal cost. There are initially five buyers, each of whose total value of consuming the good is 20. The supplier engages in separate bilateral bargaining with each buyer, sinking fixed costs only after all bargaining has been completed, and only if aggregate payments would cover the fixed costs. In the two-player Nash bargaining outcome, the supplier and any given buyer evenly split their joint surplus of 20. Each buyer pays 10 and retains net surplus of 10. After covering fixed costs, the supplier also retains net surplus of 10. Note that no buyer is pivotal in this example. If negotiations with any buyer were to break down, aggregate payments of 40 by the remaining buyers could still cover fixed costs. This verifies that the joint surplus from the supplier and a given buyer reaching a deal is indeed 20. No portion of fixed costs enters into this two-player joint surplus, because the buyer is non-pivotal.

Now suppose that two of the five buyers merge. The merging buyers, who jointly value the good at 40, become pivotal post-merger. If negotiations with the merged buyer were to break down, aggregate payments of 30 from the remaining three buyers could not cover fixed costs. The fact that the merged buyer is pivotal affects the joint surplus from the supplier and merged buyer reaching a deal. This surplus is 30, calculated as follows. The surplus is the merged buyer's valuation of the good, which is 40, plus payments from other buyers of 30 (which the supplier would not receive absent a deal with the pivotal buyer), minus fixed costs of 40 (which the supplier would not sink absent a deal with the pivotal buyer). Again, the joint surplus of 30 is evenly split between the supplier and merged buyer in the two-player Nash bargaining outcome. The merged buyer pays 25, retaining net surplus of 15. Recall that without the merger the merging buyers would together have paid 20, by bargaining separately with the supplier as non-pivotal buyers. Thus becoming pivotal worsens the merging buyers' bargaining position.

The bargaining position of pivotal buyers is relevant to antitrust and regulatory policy in a variety of industries characterized by scale economies or network effects in which sales are individually negotiated. One such industry is cable network programming. Most costs of such programming are fixed production costs. The marginal cost of program delivery to cable systems (typically via satellite) is low. Cable networks negotiate carriage agreements with cable system operators. For a start-up cable network to succeed, it must attain minimum viable scale by securing carriage agreements with cable systems representing a sufficient number of subscribers. The number of subscribers reached is related both to the carriage fees that cable systems pay to a cable network and to the advertising revenues the network can command.

The Federal Communications Commission (FCC) enforces horizontal ownership limits on cable multiple system operators (MSOs) with the intention of precluding the creation of a pivotal buyer of cable network programming. A rationale for this policy is that allowing a cable MSO to grow to pivotal size would tip the advantage in bargaining for program carriage decisively to the MSO, and that this could unfairly impede the supply of programming. The analysis below raises some questions in this regard.

The issues treated here differ from analyses of opportunism *ex post* of specific investment, such as Klein *et al*. (1978) and Williamson (1979). The focus here is on whether a buyer is pivotal to the production decision, an issue which arises *ex ante* of industry specific investment by the supplier. The formal analysis abstracts from issues of contractual incompleteness and imperfect enforceability to focus on the nature of pre-contractual bargaining. Section 3 builds on the model of Chipty and Snyder (1999), in which a supplier engages in simultaneous, bilateral Nash bargaining with multiple buyers. The result of bargaining between the supplier and each buyer is assumed to be the two-person Nash bargaining outcome. The bargaining outcomes are interdependent, in that the supplier's disagreement outcome in bargaining with one buyer will generally depend on bargaining outcomes with other buyers. Buyers' valuations and the supplier's costs are assumed to be common knowledge. Chipty and Snyder (1999) adopt the assumption, which is maintained here, that each buyer believes the supplier will reach efficient agreements with all other buyers. Thus each negotiated deal is "on the margin," in that the supplier's deals with other buyers, which can be inferred from the structure of the game, are taken as given.

The model of Section 3 below diverges from that of Chipty and Snyder (1999) in relaxing their assumption that the Nash bargaining outcome is continuous in buyer size. This assumption holds only if no buyer is large in the absolute sense of being pivotal to the supplier's production decision. If the FCC's horizontal ownership limits on cable MSOs are effective, permissible mergers of cable MSOs can be analyzed under Chipty and Snyder's (1999) assumption that no buyer of programming is pivotal either pre- or post-merger. In relaxing this assumption, the present analysis may shed some light on the effects of the regulatory regime itself.

The remainder of the paper is organized as follows. Section 2 reviews several strands of the literature on public goods provision. Section 2 draws some links between this literature and the bargaining literature, and shows that being pivotal (suitably defined) tends to depress the net surplus of a public good consumer in a variety of settings in which no self-interested supplier is present. Section 3 then introduces a self-interested supplier and analyzes the effects of a buyer merger on bargaining outcomes. Section 4 discusses the implications of the models and concludes.

2. Public Goods Provision

2.1 The One-Streetlight Problem

Bagnoli and Lipman (1989) treat a full information game in which a discrete public good is privately provided (the "one-streetlight problem"). Each player $i = 1,\dots,n$ places a value $v_i \geq 0$ on consuming the good and chooses a contribution $T_i \geq 0$ toward covering the good's fixed cost of *K*. If aggregate contributions are sufficient to cover the good's cost, $\sum_{i=1}^{n} T_i \geq K$, then the good is provided. Otherwise all contributions are refunded. For concreteness, one may think of this process as being carried out by a passive "collector" of contributions.

Let $k \in \{0,1\}$ denote a project outcome, where $k = 1$ means the project is undertaken, and let $T = (T_1,...,T_n)$ be a profile of non-negative contributions, so that (k,T) is an outcome of the game. The payoff to player i in outcome (k,T) is $u_i(k,T) = k(v_i - T_i)$. If production is efficient, $\sum_{i=1}^n v_i \geq K$, then $(1,T)$ is a Nash equilibrium if $0 \leq T_i \leq v_i$ for all i and $\sum_{i=1}^n T_i = K$. Every player earns non-negative surplus in such an outcome. Any player would be worse off contributing more, and no player could gain by contributing less since this would result in the good not being provided.

There can also be inefficient Nash equilibria in which the good is not provided: $\sum_{i=1}^n T_i < K$, and for all i, $T_i \geq 0$ and $v_i + \sum_{j \neq i} T_j \leq K$. The focus of Bagnoli and Lipman's (1989) analysis is to show that such inefficient equilibria can be eliminated with suitable refinements to the Nash equilibrium concept. In particular, Bagnoli and Lipman (1989) show that the set of undominated perfect equilibrium outcomes of the game is exactly the core of the economy.

The issue of present interest is the relationship between the size distribution of players and the distribution of surplus in Nash equilibria in which the good is provided. Note that in every Nash equilibrium in which the good is provided, every player is pivotal to the good's provision in the outcome of the game (hereafter called *outcome-pivotal*). Also if production is efficient, any distribution of surplus can be supported as a Nash equilibrium outcome of the game. Nonetheless, useful insights about the distribution of surplus can be gained by considering the *mean* of the Nash equilibium outcomes. For a given fixed cost K and profile of gross surpluses v such that $\sum_{i=1}^n v_i \geq K$, let $ET_i(v,K)$ be the mean contribution of player i within the set of Nash equilibria for the game. Consider the simplest case of $n = 2$, $v_1 + v_2 \geq K$, for which the mean Nash equilibrium contributions of each player are given by

$$ET_i(v,K) = \tfrac{1}{2}\left(\max\{0,\, K - v_j\} + \min\{K, v_i\}\right), \quad i = 1,2;\ \ j \neq i. \tag{1}$$

The mean of the Nash equilibrium outcomes is not intended to be a refinement of the Nash equilibrium concept, but simply a summary statistic applied to the set of Nash equilibria. If production is efficient when all players are served, but

$$\sum_{j \neq i} v_j \leq K, \tag{2}$$

player *i* is pivotal to the efficiency of production (hereafter called *efficiency-pivotal*). By equation (1), the players' mean contributions depend on their status as efficiency-pivotal or non-pivotal.

Figure 1 illustrates the case of $v_1 > v_2 > 0$. According to equation (1), the players' mean contributions ET_1 and ET_2 will depend on the magnitude of fixed costs *K*. For $K < v_2 < v_1$ (depicted in Figure 1A), each player could sponsor the project independently of contributions by the other, so neither player is efficiency-pivotal. In this case, the cost burden of the project is borne symmetrically in terms of mean contributions, $ET_1 = ET_2 = \frac{1}{2}K$. This cost sharing is represented in Figure 1D by the 45 degree line segment between the origin and first kink, at which point $K = v_2$.

For $v_2 < K < v_1$ (Figure 1B), player 1 could sponsor the project but player 2 could not. Thus player 1 is efficiency-pivotal but player 2 is not. In this case, there is a floor of $K - v_2$ on player 1's contribution. Player 1 then tends to bear more of the project's cost: $ET_1 = K - \frac{1}{2}v_2 > \frac{1}{2}v_2 = ET_2$. This corresponds to the horizontal line segment in Figure 1D. As *K* rises above v_2, so long as player 1 remains capable of sponsoring the project (and so remains the only efficiency-pivotal player), player 2's expected contribution holds steady at $\frac{1}{2}v_2$, while player 1 covers the full increment to *K*. Player 2's expected surplus in this range of fixed costs is $\frac{1}{2}v_2$. Note that this is also what the two-player Nash bargaining outcome would be for player 2 in bargaining with a supplier, if fixed costs did not figure in the surplus from trade between them (*i.e.*, if buyer 2 were not outcome-pivotal to the supplier's production decision). The comparison of mean contributions between an *efficiency*-pivotal player and a non-pivotal player is similar to the comparison of equilibrium payments for *outcome*-pivotal and non-pivotal buyers in the multilateral bargaining game studied in Section 3.

Finally, Figure 1C depicts the case of $v_2 < v_1 < K$. In this case, neither buyer could sponsor the project independently, so both players are efficiency-pivotal. There is a floor on the equilibrium contribution of each player *i* of $K - v_j$, $i = 1, 2$, $j \neq i$. The larger player still bears more of the cost burden in mean terms, $ET_1 = \frac{1}{2}[K + (v_1 - v_2)] > \frac{1}{2}[K - (v_1 - v_2)] = ET_2$. This corresponds to the 45 degree line segment past the rightmost kink in Figure 1D. As *K* rises above v_1, the buyers once again share the burden of increments to *K* equally.

The foregoing comparative statics holds fixed the project's gross surpluses *v* while varying the project's cost *K*. Now consider a merger that combines the gross surpluses of the merging players while keeping *K* fixed. Suppose that there are three symmetric players pre-merger, each having the common gross surplus v_2. Production is efficient if $v_2 > \frac{1}{3}K$, in which case by symmetry the pre-merger mean contribution of each firm is $ET = \frac{1}{3}K$, so the pre-merger mean contribution of the merging players is $\frac{2}{3}K$ altogether. Label the two merging players as player 1 post-merger; the non-merging player is player 2. For simplicity, assume that no efficiencies arise from the merger: the project's cost *K* is unchanged and $v_1 = 2v_2$.

Consider three cases in turn. First, for $\frac{1}{3}K < v_2 < \frac{1}{2}K$, the merging players' post-merger expected contribution is $ET_1 = \frac{1}{2}(K - v_2 + v_1) = \frac{1}{2}(K + v_2) > \frac{1}{2}(K + \frac{1}{3}K) = \frac{2}{3}K$ (recalling Figure 1C). In this case the merger increases the merging players' expected contribution. Second, for $\frac{1}{2}K \le v_2 < \frac{2}{3}K$, the post-merger expected contribution is $ET_1 = K - \frac{1}{2}v_2 > K - \frac{1}{2}(\frac{2}{3}K) = \frac{2}{3}K$ (recalling Figure 1B), so again the merger increases the merging players' expected contribution. Finally, for $v_2 > \frac{2}{3}K$ the merger decreases the merging players' expected contribution (and, recalling Figure 1A, $ET_1 = \frac{1}{2}K$ for $v_2 \ge K$).

To sum up, if production is efficient but the project's cost is sufficiently large ($K > \frac{3}{2}v_2$), then in a three-to-two merger involving *ex ante* identical players, the merging players' mean contributions increase post-merger. Intuitively, the merging players internalize the benefits between them of the project being undertaken, and so become jointly more willing to fund the project after their merger. Thus post-merger a greater proportion of Nash equilibria involve low contributions by the non-merging player.

2.2 Unknown Project Cost

Now suppose that the game of Section 2.1 is modified so that players know only the distribution of the project's cost, $F(\cdot)$. Abusing notation a bit, let *T* denote both the vector of contributions as well as their sum. The expected payoff to player *i* is $u_i(T) = (v_i - T_i)F(T)$. This game can be interpreted in one of two ways. It can be viewed as a contribution game in which project costs are privately known to the "collector." Alternatively, the game can be viewed as one in which buyers make simultaneous take-it-or-leave-it offers to a supplier whose fixed costs are private information. The first order conditions for maximum are

$$v_i - T_i = \frac{F(T)}{f(T)}, \quad i = 1, \ldots, n, \tag{3}$$

where $f(T)$ is the first derivative of $F(T)$. Assume that $F(T)/f(T)$ is increasing in the sum of payments T over the relevant range. Equations (3) then have a unique solution.

Because net surplus conditional on the project being undertaken, $v_i - T_i$, is the same for all players i in equilibrium, the ratio T_i/v_i decreases with player size v_i in equilibrium. If v_i is interpreted as the gross surplus from a given programming project to cable MSO *i*, and v_i is proportional to the MSO's subscriber base, then T_i/v_i is proportional to the MSO's per-subscriber fee. This equilibrium feature is consistent with the belief that per-subscriber fees are lower for large cable MSOs. However, it would be a mistake to infer from this that a merger of players would reduce their per-subscriber fees, as presently shown.

Summing together equations (3) and rearranging terms yields the equilibrium condition

$$T + n\frac{F(T)}{f(T)} = \sum_{i=1}^{n} v_i \ . \tag{4}$$

Now consider a merger of two players, labeled *A* and *B*, that yields no programming efficiencies in that $F(\cdot)$ is unchanged, $v_{AB} = v_A + v_B$, and v_i is unchanged for every non-merging player *i*. The right hand side of equation (4) is then unchanged by the merger, while the number of players on the left hand side of (4) is reduced by one. Aggregate payments *T* thus increase in the post-merger equilibrium. Moreover, payments by non-merging players decrease post-merger, according to equations (3). Therefore payments by the merging players increase post-merger, both in total and in per-subscriber terms.

The intuition for this result is that payments have a positive externality, increasing the likelihood that the project will be undertaken and so increasing the expected surplus of other players. This externality is internalized post-merger as between the merging players. A merger in this setting is always privately profitable, and also increases expected total surplus.

2.3 The Subscription Game

Admati and Perry (1991) explore a "subscription game" that is related to both Bagnoli and Lipman's (1989) one-streetlight problem and to Rubinstein's (1982) alternating-offer bargaining game. Two players take turns making subscriptions (conditional commitments to contribute in the future) toward the provision of a discrete public good whose cost is K. In every period of play, each player can only add to the amount the player has previously subscribed (or stand pat). The game ends when total subscriptions reach K, at which point each player i contributes the committed amount T_i and the good is provided. The payoff to player i is then $\delta^{\tau}(v - T_i)$, $i = 1,2$, where δ is the common discount factor, τ is the earliest time at which $T_1 + T_2 \geq K$ ($\tau = 0$ at the end of the first period), and v is the players' common valuation of the public good.

Let f denote the buyer subscribing first in each period, and let l denote the buyer subscribing last in the period. Assume $v(1-\delta) < K < 2v$. In the unique subgame perfect equilibrium, buyer f immediately subscribes $T_f^* = [K - v(1-\delta)]/(1+\delta)$, after which buyer l completes the project by subscribing $T_l^* = K - T_f^* = [\delta K + v(1-\delta)]/(1+\delta)$.[1] As Admati and Perry (1991) note, for the special case $v = K = 1$ this outcome is exactly the outcome of Rubinstein's (1982) alternating-offer bargaining game. As in Rubinstein's (1982) bargaining game, the first-mover in the subscription game has the advantage, $T_f^* < T_l^*$. Firm f chooses the subscription level T_f^* that is just low enough to render firm l indifferent between completing the project by subscribing T_l^* and deferring the project by subscribing zero. If firm l were to subscribe zero at $\tau = 0$, firm f would then complete the project at $\tau = 1$. Firm l is thus outcome-pivotal, in that the project's completion would be delayed one period if firm l were to fail to complete the project. The more impatient the firms (the lower is δ), the greater the disadvantage to moving second and thereby being outcome-pivotal.

2.4 The Pivotal Mechanism

Now suppose that the project's valuation v_i by each player i is known only to i. A social planner wishes to induce truth revelation in order to implement the efficient project choice k. This

[1] Admati and Perry (1991), Proposition 5.1, case (ii).

can be accomplished through a *pivotal mechanism* (Clarke (1971); see also Groves (1973)), which renders a player outcome-pivotal if and only if the player is efficiency-pivotal Suppose that every player is assigned a baseline contribution (or rather tax) of K/n if the project is undertaken. By assumption, each player has a sufficient endowment that the tax schemes described here are feasible. Let k^* be the efficient project choice, where

$$k^* \sum_{i=1}^{n} \left(v_i - K/n\right) \geq k \sum_{i=1}^{n} \left(v_i - K/n\right), \text{ for } k \in \{0,1\}. \tag{5}$$

In the pivotal mechanism, each player *i* receive a (possibly negative) transfer t_i, given by

$$t_i = \left(\hat{k}^* - \hat{k}^*_{-i}\right) \sum_{j \neq i} \left(\hat{v}_j - K/n\right), \quad i = 1, \ldots, n, \tag{6}$$

where $\hat{v}_j$ is player *j*'s announced valuation, $\hat{k}^*$ is the optimal project choice taking every player's announced valuation as truthful, and $\hat{k}^*_{-i}$ is what the optimal project choice would be ignoring $\hat{v}_i$. That is, $\hat{k}^*_{-i}$ satisfies

$$\hat{k}^*_{-i} \sum_{j \neq i} \left(\hat{v}_j - K/n\right) \geq k \sum_{j \neq i} \left(\hat{v}_j - K/n\right), \text{ for } k \in \{0,1\}. \tag{7}$$

The baseline tax is collected and the transfer payments in equations (6) are carried out only if the project is undertaken ($\hat{k}^* = 1$), in which case player *i*'s total contribution is $T_i = (K/n) + t_i$. As in Section 2.1, the payoff to player *i* in outcome (k, T) is $u_i(k, T) = k\left(v_i - T_i\right)$.

Truthful announcements are a Nash equilibrium given the pivotal mechanism. From equations (6), $t_i = 0$ if player *i* is non-pivotal ($\hat{k}^*_{-i} = \hat{k}^* = 1$) and $t_i < 0$ if player *i* is efficiency-pivotal ($\hat{k}^*_{-i} = 0, \hat{k}^* = 1$). Note that a non-pivotal player cannot gain by making a false announcement. An efficiency-pivotal player could avoid the negative transfer implied by (6) by falsely announcing a low valuation. But in this case the project would not be undertaken. By construction of the pivotal mechanism, truthful announcements imply $v_i \geq T_i$ for any efficiency-pivotal player *i*. Thus an efficiency-pivotal player cannot gain by announcing falsely.

Starting from a situation in which production is efficient and two firms *A* and *B* are each non-pivotal, the merger of these firms would tend to lower their joint payoffs if *AB* were to become efficiency-pivotal post-merger. In this case, $t_A = t_B = 0 > t_{AB}$ by equations (6).

3. Multilateral Bargaining

A self-interested supplier seeks to sell to n potential buyers, indexed $i = 1,...,n$. Buyer i's gross surplus from consuming q_i units is $v_i(q_i, q_{-i})$, where $q_{-i} = (q_1,...,q_{i-1}, q_{i+1},...,q_n)$ is the profile of consumption by other buyers.[2,3] The supplier's gross surplus (excluding buyer payments) is $V(Q)$, where $Q = \sum_{i=1}^{n} q_i$. Both $v_i(\cdot)$ and $V(\cdot)$ are twice continuously differentiable, and these functions are common knowledge. The supplier's gross surplus can be decomposed as $V(Q) = A(Q) - C(Q)$, where $A(Q)$ is ancillary revenue and $C(Q)$ is total cost. This formulation allows for network effects, scale economies in production, or both. For example, a cable programming network earns ancillary revenue through the sale of advertising time. The network's advertising revenue depends on the number of cable subscribers Q the network reaches.

The supplier will produce if and only if aggregate buyer payments and ancillary revenues allow costs to be recovered:[4]

$$V(Q) + \sum_{i=1}^{n} T_i \geq 0, \tag{8}$$

where T_i is buyer i's total payment to the supplier.

Bargaining between the supplier and buyer i is over the terms of trade (q_i, T_i), where T_i is

[2] Cable MSOs are typically franchise monopolists with geographically disjoint service territories, so their demands for programming are likely to be independent, as Chipty and Snyder (1999) assume in their model. The more general formulation here allows for buyers to be producers of substitutible or complementary goods.

[3] In 1999, there were 83.1 million multichannel video program distribution (MVPD) subscribers in the U.S., of which 81% were served by cable systems with the remainder primarily served by direct broadcast satellite (DBS) (Kagan, 2000, at 16. The FCC (1999, at &14) states that cable systems typically have greater than 80% share of MVPD subscribers within their service territories. DBS subscribership is highest (22% to 34%) in the rural states of Montana, Vermont and Wyoming (GAO, 1999, at 12).

[4] Start-up cable networks typically obtain carriage commitments from a number of cable MSOs prior to sinking substantial costs in program production. See Higgins (1997).

buyer i's total payment to the supplier. Each of the n bargaining outcomes is assumed to be the two-person Nash (1950, 1953) bargaining solution.[5] Bargaining outcomes are interdependent in two senses. First, consumption by buyers $j \neq i$ enters $V(\cdot)$, and so affects the joint surplus to the supplier and buyer i from reaching a deal. Given aggregate consumption by other buyers of $Q_{-i} = \sum_{j \neq i} q_j$, the ith buyer's consumption maximizes the joint surplus of the supplier and buyer i:

$$q_i^* = \arg\max_x \left\{ v_i(x, q_{-i}) + V(Q_{-i} + x) \right\} \tag{9}$$

Bargaining outcomes are also linked through the supplier's participation constraint (8).

3.1 *Pivotal Buyers*

A large buyer can have effective veto power over the supplier's production decision. Given payments T_j and aggregate consumption Q_{-i} by buyers $j \neq i$, buyer i is *pivotal* to the supplier's production decision if and only if both of the following conditions hold:

$$V(Q_{-i}) + \sum_{j \neq i} T_j < 0, \tag{10.1}$$

$$\max_x \left\{ v_i(x, q_{-i}) + V(Q_{-i} + x) \right\} + \sum_{j \neq i} T_j \geq 0. \tag{10.2}$$

Condition (10.1) states that production is unprofitable without a sufficiently attractive deal with buyer i. Condition (10.2) states that there are joint gains to reaching such a deal.

3.2 *Disagreement Outcomes*

If the supplier and buyer i fail to agree, i's disagreement outcome is $v_i(0, q_{-i}) = 0$. Characterizing the supplier's disagreement outcome is more involved, given the multilateral nature of bargaining. A key issue is how to model the responses of buyers $j \neq i$ to a breakdown in the supplier's negotiations with a given buyer i. One possibility, following Stole and Zweibel (1999a,

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[5] Binmore *et al*. (1986) establish a connection between Nash's (1950, 1953) axiomatic approach and sequential strategic approaches to bargaining. As Chipty and Snyder (1999) note, the perfect equilibrium to the alternating-offer bargaining game with exogenous probability of breakdown that Binmore *et al*. (1986) analyze approaches the Nash bargaining outcome in the limit as the breakdown probability goes to zero.

1996b), would be to assume that once negotiations with buyer *i* have broken down, *i* exits the game, and thereafter quantities consumed and payments made by remaining buyers are renegotiated in subsequent rounds of bargaining with the supplier.[6] However, this approach relies on a rather strong assumption that players know when negotiations with a given player have ended irrevocably. In practice, negotiations that appear to break down can frequently be resumed. Moreover, there often are strategic reasons for players to wish to manipulate public perceptions about the status of negotiations.[7]

Following Chipty and Snyder (1999), the assumption maintained here is that in every negotiation the supplier and buyer believe that their failure to reach agreement would not affect the supplier's bargaining outcomes with other buyers. Given this assumption, the *n* bilateral negotiations are modeled as occurring simultaneously. Negotiated contract terms (q_i, T_i) are binding if the supplier chooses to produce,[8] but an escape clause or bankruptcy constraint is assumed to allow the supplier to nullify all contracts if buyer payments plus ancillary revenues would not cover costs.

The supplier's disagreement outcome in bargaining with buyer *i* depends on whether *i* is pivotal. If buyer *i* is pivotal, the supplier's payoff is zero absent an agreement with *i*. If buyer *i* is

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[6] Chae and Yang (1994) and Krishna and Serrano (1996) also develop models of multilateral, sequential bargaining in which exit by one player is known to remaining players.

[7] The supplier could gain by convincing a buyer *j* that *i* has exited, if this would imply a smaller increment to the supplier's gross surplus from reaching a deal with *j*, hence higher payment by *j* in the Nash bargaining outcome. Also, a pivotal buyer could gain by purporting to exit, if this would convince other buyers to increase their payments to ensure that the supplier still produces. This would render *i* non-pivotal and lower *i*'s payment in subsequent bargaining.

[8] By contrast, Stole and Zweibel (1996a, 1996b) analyze contracts that are terminable at-will.

not pivotal, the supplier's disagreement outcome is $V(Q_{-i}) + \sum_{j \neq i} T_j \geq 0$.

3.3 Consistent Candidate Outcomes

An equilibrium is of the form (q^*, T^*), where q^* is given by equations (9). It remains to determine the equilibrium payments vector T^*. A first set of restrictions on T^* is implied by the two-person Nash bargaining solution, according to which the joint surplus from trade between the supplier and any given buyer *i* is split evenly between them, over their disagreement outcomes. Simplifying notation, hereafter let $v_i = v_i(q_i^*, q_{-i}^*)$, $V = V(Q^*)$, and $V_{-i} = V(Q_{-i}^*)$. Given payment T_i, buyer *i*'s net surplus is $v_i - T_i$. The supplier's "net" surplus (inclusive of payment T_i) depends on whether buyer *i* is pivotal, as discussed above. If buyer *i* is non-pivotal, the supplier's net surplus from trade with *i* is $V - V_i + T_i$. Equating the net surpluses of the supplier and non-pivotal buyer *i* then yields payment by *i* of $T_i = \frac{1}{2}\left(v_i + V_{-i} - V\right)$ as the Nash bargaining solution to the two-person game. If buyer *i* is pivotal, the supplier's disagreement outcome is zero. In this case the supplier's net surplus from reaching agreement with buyer *i* is $V + \sum_{j=1}^{n} T_j$. As before, the Nash bargaining solution sets buyer *i*'s payment to equate the net surpluses of the supplier and buyer *i*, yielding $T_i = \frac{1}{2}\left(v_i - V - \sum_{j \neq i} T_j\right)$. More generally, let θ_i be an indicator variable equal to one if condition (10.1) holds for buyer *i* within a given set of contracts, and zero otherwise. Then the Nash bargaining solution implies

$$T_i = \frac{1}{2}\left[v_i + V_{-i} - V - \theta_i\left(V_{-i} + \sum_{j \neq i} T_j\right)\right]. \tag{11}$$

Note that for $\theta_i = 1$ the expression $\left(V_{-i} + \sum_{j \neq i} T_j\right)$ on the right-hand side of equation (11) is negative, by condition (10.1). Equation (11) indicates that, "all else equal," a buyer tends to pay the supplier more if pivotal to the supplier's production decision. This point will be elaborated in the discussion of buyer merger in below.

At this point, it is useful to define concepts of *candidate outcome* and *consistent candidate outcome*. Let $\Theta = \left(\theta_1, \ldots, \theta_n\right)$ be a vector of zeros and ones, and let the corresponding buyer payments that jointly satisfy equations (11) be $T(\Theta) = (T_1(\Theta),...,T_n(\Theta))$. The set of contracts $(q^*, T(\Theta))$ is then a *candidate outcome*. Further, a candidate outcome is *consistent* if, for every buyer *i*, $\theta_i = 1$ if and only if condition (10.1) holds for *i*, and otherwise $\theta_i = 0$. Equilibrium is defined to be a consistent candidate outcome that satisfies the supplier's participation constraint (8).

The supplier's participation constraint is a central feature of the analysis. While there always exists a consistent candidate outcome $(q^*, T(\Theta))$ in which $\Theta = (0,...,0)$, it may not be an equilibrium as defined here for no buyers to be pivotal. For example, suppose there are three identical buyers each of whose gross surplus is $v(q^*) = 100$, and the supplier's gross surplus function implies $V(2q^*) = -300$ and $V(3q^*) = -260$. A negative gross surplus for the supplier means that ancillary (*e.g.*, advertising) revenues are insufficient to cover the supplier's fixed costs. The supplier must rely on aggregate payments from buyers to cover this shortfall. Production is efficient in this example, since $3v(q^*) + V(3q^*) = 40 > 0$. For the consistent candidate outcome in which $\Theta = (0,0,0)$, equations (11) yield payments of $T = \frac{1}{2}(100 - 300 - (-260)) = 30$ by each non-pivotal buyer. However, this outcome fails the supplier's participation constraint (8), since $V(3q^*) + 3T = -170 < 0$. Using equations (11), one can verify that there is a unique equilibrium for this numerical example, in which all three buyers are pivotal, each buyer pays 90, and the supplier and each buyer captures net surplus of 10.

3.4 Equilibrium

Let buyers be indexed so that $v_1 \geq v_2 \geq ... \geq v_n$. The following assumption is maintained hereafter:

$$V_{-1} \leq V_{-2} \leq ... \leq V_{-n} \leq V . \tag{12}$$

Assumption (12) holds trivially for $V(Q) = -K$, the case of a discrete, excludable public good treated in Section 2 above. More generally, this assumption states that the larger a buyer's gross surplus from reaching agreement with the supplier, the greater would be the adverse impact on the supplier's gross surplus if negotiations between them were to break down. This assumption fits the stylized facts of the cable television industry. The gross surplus of a cable MSO obtaining carriage of a popular cable network will typically increase with the number of homes the cable MSO passes and ultimately with the number of subscribers the cable MSO reaches. Most cable network revenue flows from advertising, whose value likewise depends on the number of subscribers the cable network reaches.[9] Thus the larger a cable MSO's subscriber base, the greater a cable network's potential loss of advertising revenue if negotiations between them were to break down.

Lemma 1. If buyer *i* satisfies condition (10.1) for being pivotal in a given candidate outcome, then every buyer $h < i$ also satisfies condition (10.1) in that candidate outcome.

Proof: See the Appendix.

Lemma 1 follows from assumption (12). Let Θ^p, $p \in \{0,1,...,n\}$, denote the $1 \times n$ vector whose first *p* elements are ones and remaining $n - p$ elements are zeros. To avoid notational clutter, let Θ^p also signify the corresponding candidate outcome $(q^*, T(\Theta))$. That is, Θ^p is the candidate outcome in which buyer *p* is the smallest (highest-numbered) buyer taken to be pivotal (and for *p* = 0, no buyer is taken to be pivotal in the candidate outcome). Lemma 1 narrows the set of candidate outcomes to be considered as possible equilibria to $\{\Theta^0, \Theta^1, ..., \Theta^n\}$, since only these $n+1$ candidates are potentially consistent.

For a given candidate outcome Θ^p, equations (11) represent a system of *n* equations in the *n* unknowns $T_i(\Theta^p)$. The solution is given by

$$T_i(\Theta^p) = \begin{cases} v_i - \frac{1}{p+1}\left(V + \sum_{j=1}^{p} v_j + \frac{1}{2}\sum_{j=p+1}^{n}\left(v_j + V_{-j} - V\right)\right), & \text{for } i \le p, \quad (13.1) \\ \frac{1}{2}\left(v_i + V_{-i} - V\right), & \text{for } i > p. \quad (13.2) \end{cases}$$

Equations (13) have a straightforward interpretation. First, buyers $i > p$ are treated as non-pivotal in candidate outcome Θ^p. For each such buyer, the joint from trade with the supplier is split evenly with the supplier through *i*'s payment $T_i(\Theta^p)$, as shown in equation (13.2). Second, buyers *i* # *p*

[9] In 1999, basic cable network net advertising revenues were \$7.3 billion, affiliate license fees were \$5.5 billion, and other revenues were \$0.6 billion. See Kagan (2000) at 16.

are treated as pivotal in Θ^p. For these buyers, the bracketed expression on the right-hand side of equation (13.1) represents the joint surplus to the supplier and *all* buyers $j \le p$ collectively from their trade. This collective surplus is the sum of the supplier's gross surplus V, the aggregate of gross surpluses v_j for buyers $j \le p$, and aggregate payments to the supplier by buyers $j > p$. Since this joint surplus would be lost if negotiations were to break down between the supplier and *any* of the p buyers, the supplier and each of the p buyers would end up capturing an equal $1/(p+1)$ share of their joint surplus in the candidate outcome.

Lemma 2. If production is efficient,

$$V + \sum_{i=1}^{n} v_i \ge 0\,, \tag{14}$$

then the candidate outcome Θ^n satisfies the supplier's participation constraint (8).

<u>Proof</u>: For $p = n$, summing together the n equations (13) and adding V to both sides yields

$$V + \sum_{i=1}^{n} T_i(\Theta^n) = \frac{1}{n+1}\left(V + \sum_{i=1}^{n} v_i\right),$$

which is nonnegative by condition (14) and therefore satisfies condition (8). *Q.E.D.*

Lemma 2 follows from the efficiency of Nash bargaining. It states that if there are gains from trade overall, and if every buyer is pivotal, then the set of surplus-splitting payments would allow the supplier to recover costs.

Lemma 3. For $p \in \{1, 2, ..., n\}$, if buyer p fails to satisfy condition (10.1) for being pivotal in candidate outcome Θ^p, then p also fails to satisfy condition (10.1) in candidate outcome Θ^{p-1}.

<u>Proof</u>: See the Appendix.

Lemma 3 describes an ordering property which assures that a buyer's status as pivotal or non-pivotal need only be checked once. If buyer p is non-pivotal in candidate outcome Θ^p, Lemma 3 implies that p is also non-pivotal in every candidate outcome Θ^i, $i < p$. Note that if buyer p fails to satisfy the pivotal condition (10.1) in candidate outcome Θ^p, then Θ^p must satisfy the supplier's participation constraint (8). Moreover, as an immediate corollary to Lemma 3, Θ^{p-1} must also satisfy condition (8). These points underlie the algorithm for finding equilibrium in the proof of Proposition 1 below.

Proposition 1. If production is efficient, then there exists a $p \in \{0,1,\ldots,n\}$ such that Θ^p is an equilibrium of the game.

<u>Proof</u>: The proof follows from Lemmas 1-3, as shown by the following algorithm. If production is efficient and buyer n satisfies condition (10.1) in candidate outcome Θ^n, then Θ^n is consistent by Lemma 1 and satisfies the supplier's participation constraint by Lemma 2. In this case Θ^n is an equilibrium. If buyer n does not satisfy condition (10.1) in Θ^n, then consider candidate outcome Θ^{n-1}. By Lemma 3, buyer n will still fail to satisfy condition (10.1) in Θ^{n-1}, and Θ^{n-1} will also satisfy the supplier's participation constraint. If buyer *n* ! 1 satisfies condition (10.1) in Θ^{n-1}, then by Lemmas 1 and 3, Θ^{n-1} is consistent and therefore is an equilibrium. If not, then consider candidate outcome Θ^{n-2}, and repeat the foregoing procedure in search of a candidate outcome Θ^p in which buyer p satisfies condition (10.1), in which case Θ^p will be an equilibrium. If the candidate outcomes Θ^1, Θ^2, ... , Θ^n are all eliminated by this procedure, then by Lemma 3 Θ^0 is consistent, satisfies the supplier's participation constraint, and therefore is an equilibrium. *Q.E.D.*

Equilibrium is frequently C but not invariably C unique.[10] Typically, comparative statics analyses are undermined by the possibility of multiple equilibria. That is not the case here, however, for reasons discussed in the following section.

[10] For example, consider ten buyers with $v_i = 21 - i$, $V_{-i} = -71 + i$, and V = ! 60. This game has two equilibria: Θ^2, with $T_1(\Theta^2) = 13\frac{2}{3}$, $T_2(\Theta^2) = 12\frac{2}{3}$, and $T_i(\Theta^2) = 5$ for $i \geq 3$; and also Θ^3, with

3.5 Buyer Merger

Suppose that two buyers merge, labeled A and B pre-merger and AB post-merger. Assume that production is efficient both pre- and post-merger, so that an equilibrium exists by Proposition 1. By the nature of the comparative statics exercise, the merger is treated as occurring prior to any bargaining and prior to the supplier sinking any production costs. In the discussion below, the s-equilibrium refers to the pre-merger equilibrium, in which the supplier bargains bilaterally with n separate buyers. The m-equilibrium refers to the post-merger equilibrium in which the supplier faces $n-1$ buyers, the merged buyer AB and the remaining $n-2$ buyers. Equilibrium values are distinguished by m or s superscripts.

From equations (11), the net surplus captured by buyer i in a given equilibrium is

$$\tfrac{1}{2}\left[v_i^* - V_i^* + \theta_i\left(V_{-i}^* + \sum_{j\neq i} T_j^*\right)\right], \tag{15}$$

where the superscripted asterisks stand in for s or m, according to whether the net surplus is calculated in a pre- or post-merger equilibrium. Focusing on the effect of a buyer merger that creates a pivotal buyer, the assumption maintained hereafter is that neither A nor B is pivotal pre-merger ($\theta_A^s = \theta_B^s = 0$). In this case, the net surplus of the merged buyer AB in the m-equilibrium is larger than the sum of net surpluses of buyers A and B in the s-equilibrium if and only if

$$v_{AB}^m + V^m - V_{-AB}^m + \theta_{AB}^m\left(V_{-AB}^m + \sum_{j\neq i} T_j^m\right) \quad > \quad v_A^s + v_B^s + 2V^s - V_{-A}^s - V_{-B}^s. \tag{16}$$

Multiple equilibria might exist both pre- and post-merger. However, within the set of pre-merger equilibria (likewise, within the set of post-merger equilibria), the only differences across equilibria would be in the vector of buyer payments. Thus only two terms in condition (16) may vary across multiple equilibria: the indicator variable θ_{AB}^m, which equals one if AB is pivotal post-merger and is zero otherwise, and post-merger aggregate payments by other buyers $\sum_{j\neq i} T_j^m$. All other terms in condition (16) are functions solely of quantities traded, which under efficient bargaining do

$T_1(\Theta^3) = 12$, $T_2(\Theta^3) = 11$, $T_3(\Theta^3) = 10$, and $T_i(\Theta^3) = 5$ for $i \geq 4$.

not depend on how rents are shared in equilibrium. Moreover, note that while the precise value of $\sum_{j \neq i} T_j^m$ may depend on the particular post-merger equilibrium in which *AB* find themselves, the *sign* of the expression

$$\theta_{AB}^{m}\left(V_{-AB}^{m}+\sum_{j \neq i} T_j^m\right) \tag{17}$$

on the left-hand side of condition (16) will invariably be negative, by condition (10.1), for any post-merger equilibrium in which *AB* are pivotal. This point is central to the comparative statics analysis that follows.

Following Chipty and Snyder (1999), condition (14) can be written as

$$DE + UE + BP > 0, \tag{18}$$

where

$$DE = v_{AB}^{m} - v_{A}^{s} - v_{B}^{s}, \tag{19.1}$$

$$UE = \left(V^{m} - V_{-AB}^{m}\right) - \left(V^{s} - V_{-AB}^{s}\right), \tag{19.2}$$

$$BP = \left(V_{-B}^{s} - V_{-AB}^{m}\right) - \left(V^{s} - V_{-A}^{s}\right) + \theta_{AB}^{m}\left(V_{-AB}^{m}+\sum_{j \neq i} T_j^m\right) \tag{19.3}$$

and V_{-AB}^{s} denotes what the supplier's gross surplus would be in the *s*-equilibrium absent trade with both (non-merged) buyers *A* and *B*. *DE*, *UE* and *BP* have natural interpretations. *DE* (downstream efficiency) captures the merger's effect on the merging buyers' gross surplus.[11] *UE* (upstream efficiency) captures the merger's effect on the supplier's gross surplus. Finally, *BP* captures the merger's effect on the merging buyers' bargaining position. The merging buyers' bargaining position improves if $BP > 0$, and worsens if $BP < 0$. Chipty and Snyder's (1999) equations (4) are a special case of equations (19) above, in which $\theta_{AB}^{m} = 0$.

The present focus of interest is on how becoming pivotal would affect *AB*'s bargaining position. The fact that expression (17) in the *BP* equation (19.3) is negative by condition (10.1) when *AB* are pivotal post-merger ($\theta_{AB}^{m} = 1$) leads immediately to:

---

[11] Cable MSOs and DBS providers compete (GAO, 2000). In a hypothetical merger between a cable MSO and DBS provider, *DE* would include an effect of reduced competition tending to raise the merging parties' gross surplus, which is not an efficiency.

Proposition 2. If neither merging buyer is pivotal pre-merger, the merging buyers' bargaining position tends to worsen if the buyers become pivotal post-merger, in the sense that expression (17) in equation (19.3) is negative in this case.

Proof: The proof follows immediately from the definition of being pivotal: that the merging buyers *AB* satisfy condition (10.1) in the post-merger equilibrium. *Q.E.D*

Expression (17) in equation (19.3) could be called an *absolute size* effect on the merging buyers' bargaining position. The remaining terms in equation (19.3) correspond to the *relative size* effect that Chipty and Snyder (1999) identify.[12] An absolute size effect is always adverse to the merging buyers' bargaining position. This result may seem paradoxical, given that the supplier's disagreement outcome in bargaining with the merging buyers falls to *zero* if they become pivotal post-merger. However, the joint surplus from reaching a deal with a newly pivotal buyer falls by an even greater amount, because the supplier's net surplus would turn *negative* if the supplier were hypothetically to produce without reaching a deal with the pivotal buyer. Net revenue from sales to other buyers, which pre-merger had been a private concern of the supplier's, becomes a (negative) component of joint surplus in bargaining with the newly pivotal buyer post-merger.

A merger that creates a pivotal buyer may still improve the merging buyers' bargaining position, if the suppliers' gross surplus is sufficiently concave and thus the countervailing relative size effect is sufficiently strong. In this case the supplier's gross surplus (*e.g.*, ancillary advertising revenue) is diminishing at the margin (in number of cable subscribers reached), so the merging buyers tend to benefit from bargaining jointly and thereby making their purchases more "inframarginal." If the supplier's gross surplus is convex, the relative size effect reinforces the

[12] As Chipty and Snyder (1999) show, the sign of the relative size effect depends on the shape of the supplier's gross surplus function. If the supplier's gross surplus function is concave, incremental surplus is low at the margin, so buyers tend to gain by merging and bargaining jointly, thereby making their purchases more inframarginal. Conversely, if the supplier's gross surplus is convex, incremental surplus is high at the margin, so a buyer merger tends to worsen the merging buyers' bargaining position through the relative size effect.

absolute size effect in worsening the merging buyers' bargaining position.[13] Note also from equation (16) that even if $BP < 0$, a buyer merger is still profitable if the countervailing efficiencies are great enough (*i.e.*, if $DE + UE > -BP$).

4. Discussion and Conclusions

Two lessons can be drawn from the analyses in this paper. First, bigger is not always better in a bargaining context. If a buyer grows so large as to become pivotal to the supplier's production decision, the buyer loses the ability to credibly abdicate responsibility for ensuring that the supplier's costs are covered. Second, cross-sectional variation in buyer payments is not a reliable guide to the effects of a buyer merger. In the model of Chipty and Snyder (1999), in which all buyers are small in the absolute sense of being non-pivotal, the relationship between buyer size and per-subscriber fees depends on the shape of the supplier's gross surplus function. However, in the extension of their model devloped in Section 3 above, becoming pivotal *always* tends to worsen the merging buyers' bargaining position vis-a-vis the supplier. The FCC's horizontal ownership limits on cable MSOs may have been effective in precluding the formation of a pivotal buyer of cable network programming. If so, then the current relationship between cable MSO size and per-subscriber fees may tell little about the effects of the FCC's policy. The model of Section 3 is a full information bargaining game. But similar results hold in other settings. In the model of Section 2.2, for example, buyers make simultaneous take-it-or-leave-it offers to the supplier, knowing only the distribution of the supplier's costs. Here each buyer is pivotal in a probabilistic sense. The equilibrium can be interpreted as exhibiting smaller per-subscriber fees for larger buyers. However, a merger of buyers in this model increases their per-subscriber fees. The expected profit of the merging buyers increases, as does expected total surplus, because efficient programming projects are more likely to be undertaken.

[13] Empirically, Chipty and Snyder (1999) find that cable network advertising revenues are convex in subscribers reached in the relevant range. This would suggest that larger cable MSOs tend to pay higher per-subscriber fees for programming. Chipty (1995) and Ford and Jackson (1997) find evidence that larger cable MSOs face lower marginal costs. However Chipty and Snyder (1999) note that such a finding could be attributable to efficiencies of larger cable MSOs rather than to a bargaining effect.

Appendix

Proof of Lemma 1

For proof by contradiction, assume buyer i satisfies condition (10.1) but that buyer $h < i$ does not. This means that $V_{-i} + \sum_{j \neq i} T_j < 0$ but $V_{-h} + \sum_{j \neq h} T_j \geq 0$. By equation (11), buyer payments are $T_i = \frac{1}{2}\left(v_i - V - \sum_{j \neq i} T_j\right)$ and $T_h = \frac{1}{2}\left(v_h - V + V_{-h}\right)$. We may write the difference in payments as $T_h - T_i = \frac{1}{2}\left(v_h - v_i + V_{-h} + T_h - \sum_{j \neq i,h} T_j\right)$. Collecting T_h terms on the right-hand side of this equation, adding $\frac{1}{2}T_h$ to both sides and then multiplying the resulting equation through by two yields $T_h - T_i = v_h - v_i + V_{-h} + \sum_{j \neq h} T_j$. The right-hand side of this equation is nonnegative, since $v_h \geq v_i$ by convention and $V_{-h} + \sum_{j \neq h} T_j \geq 0$ by assumption. Thus $T_h \geq T_i$, but this together with $V_{-h} \leq V_{-i}$ by (12) leads to the contradiction $V_{-h} + \sum_{j \neq h} T_j \leq V_{-i} + \sum_{j \neq i} T_j$. *Q.E.D.*

Proof of Lemma 3

Begin by assuming

$$V + \sum T_i(\Theta^p) \geq 0. \tag{A1}$$

The proof is complete upon showing $V + \sum_{i \neq p} T_i(\Theta^p) \geq 0$. To reduce clutter in what follows, it is helpful to rewrite equation (13.2) using (13.1), as

$$T_i(\Theta^p) = v_i - \left(\tfrac{1}{p+1}\right)\left(V + \sum_{j=1}^{p} v_j + \sum_{j=p+1}^{n} T_j(\Theta^p)\right), \quad \textit{for } i \leq p. \tag{A2}$$

Summing together the p equations (A2), then adding $V + \sum_{i>p} T_i(\Theta^p)$ to both sides yields

$$V + \sum_{i=1}^{n} T_i(\Theta^p) = \left(\tfrac{1}{p+1}\right)\left(V + \sum_{i=1}^{p} v_i + \sum_{i=p+1}^{n} T_i(\Theta^p)\right). \tag{A3}$$

Subtracting equation (A2), evaluated at $i = p$, from equation (A3) then yields

$$V + \sum_{i \neq p} T_i(\Theta^p) = \left(\tfrac{2}{p+1}\right)\left(V + \sum_{i=1}^{p} v_i + \sum_{i=p+1}^{n} T_i(\Theta^p)\right) - v_p. \tag{A4}$$

Exactly similar to the derivation of (A3), with regard to candidate outcome Θ^{p-1} we may write

$$V+\sum_{i=1}^{n}T_i(\Theta^{p-1})=\left(\tfrac{1}{p}\right)\left(V+\sum_{i=1}^{p-1}v_i+\sum_{i=p}^{n}T_i(\Theta^{p-1})\right). \tag{A5}$$

Note that $T_i(\Theta^{p-1})=T_i(\Theta^{p})$, for $i\geq p+1$, by equations (13). This together with some algebra allows (A5) to be rewritten as

$$V+\sum_{i=1}^{n}T_i(\Theta^{p-1})=\left(\tfrac{1}{p}\right)\left(V+\sum_{i=1}^{p}v_i+\sum_{i=p+1}^{n}T_i(\Theta^{p})-v_p+T_p(\Theta^{p-1})\right). \tag{A6}$$

Subtracting $T_p(\Theta^{p-1})$ from both sides of (A6) yields

$$V+\sum_{i\neq p}T_i(\Theta^{p-1})=\left(\tfrac{1}{p}\right)\left(V+\sum_{i=1}^{p}v_i+\sum_{i=p+1}^{n}T_i(\Theta^{p})-v_p-(p-1)T_p(\Theta^{p-1})\right). \tag{A7}$$

The proof is complete once the right-hand side of (A7) is shown to be nonnegative. (A1) and (A4) together imply

$$V+\sum_{i=1}^{p}v_i+\sum_{i=p+1}^{n}T_i(\Theta^{p})\geq\left(\tfrac{p+1}{2}\right)v_p. \tag{A8}$$

Subtracting $v_p+(p-1)T_p(\Theta^{p-1})$ from both sides of (A8) yields

$$V+\sum_{i=1}^{p}v_i+\sum_{i=p+1}^{n}T_i(\Theta^{p})-v_p-(p-1)T_p(\Theta^{p-1})\geq\left(\tfrac{p-1}{2}\right)v_p-(p-1)T_p(\Theta^{p-1}). \tag{A9}$$

By equation (11.2), $T_p(\Theta^{p-1})=\frac{1}{2}(v_p+V_{-p}-V)$, so (A9) can be rewritten as

$$V+\sum_{i=1}^{p}v_i+\sum_{i=p+1}^{n}T_i(\Theta^{p})-v_p-(p-1)T_p(\Theta^{p-1})\geq\left(\tfrac{p-1}{2}\right)(V-V_{-p}) . \tag{A10}$$

For $p\geq 1$, the right-hand side of (A10) is nonnegative since $V_{-p}\leq V$ by assumption (12), and therefore the right-hand side of (A7) is also nonnegative. *Q.E.D.*

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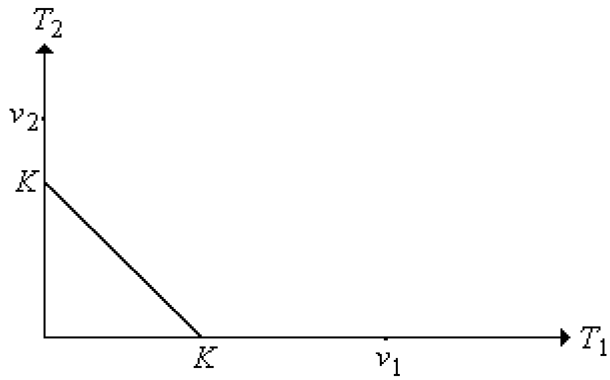

Figure 1A

$ET_1 = ET_2 = ½\,K$

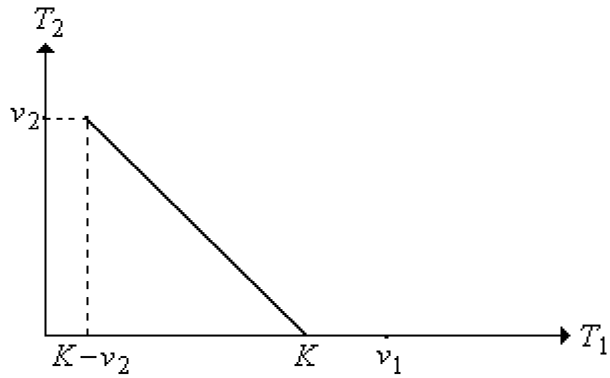

Figure 1B

$ET_1 = K - ½\,v_2$

$ET_2 = ½\,v_2$

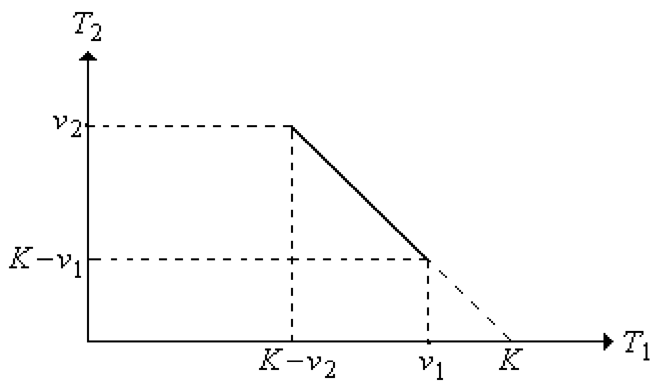

Figure 1C

$ET_1 = ½[K + (v_1 - v_2)]$

$ET_2 = ½[K - (v_1 - v_2)]$

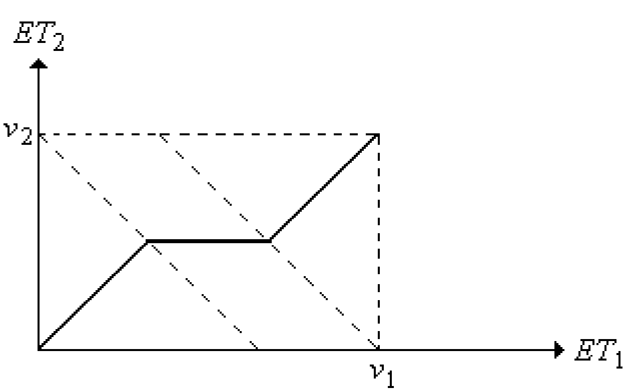

Figure 1D