Before the
FEDERAL COMMUNICATIONS COMMISSION
Washington, D.C. 20554

In the Matters of	)	
	)	
Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions	)	GN Docket No. 12-268
	)	
Amendment of the Commission's Rules with Regard to Commercial Operations in the 1695-1710 MHz, 1755-1780 MHz, and 2155-2180 MHz Bands	)	GN Docket No. 13-185

REPLY COMMENTS OF AT&T

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Reply Comments of AT&T

AT&T Services, Inc., on behalf of the subsidiaries and affiliates of AT&T Inc. (collectively, "AT&T"), respectfully submits these Reply Comments in response to the Public Notice entitled "*Wireless Telecommunications Bureau Seeks Comment on a Proposal to License the 600 MHz Band Using 'Partial Economic Areas'*".[1]

I. Introduction and Summary

In deciding what geographic license size and bidding procedures to adopt, the Commission should focus sharply on the central factor that distinguishes this auction proceeding from all others. In a typical auction, the Commission first defines the frequency blocks it commits to clear and simply asks carriers to bid for those blocks. If the auction rules are suboptimal, less money is deposited into the Treasury, but consumers nonetheless reap the benefits of greater bandwidth for mobile broadband applications. In *this* auction, by contrast, the Commission must persuade a variety of auction participants to satisfy the statutory auction-closing criteria for any target level of spectrum: namely, forward-auction revenues must exceed winning reverse-auction bids plus administrative and estimated repacking costs. If revenues fall short of that benchmark, the Commission will have to settle for less cleared spectrum, and in the worst-case scenario, the auction could fail altogether.

That unique fact has profound consequences for the decisions the Commission makes in establishing a geographic license size area and bidding procedures. To a significant degree, those decisions (among others) will determine not only how much money changes hands, and not only whether spectrum goes promptly to providers able to extract the most value from it, but also *how*

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[1] *Wireless Telecommunications Bureau Seeks Comment on a Proposal to License the 600 MHz Band Using "Partial Economic Areas"*, Public Notice, DA 13-2351, GN Docket Nos. 12-268, 13-185 (rel. Dec. 11, 2013) ("*Public Notice*").

much spectrum is available to such providers and their customers in the first place. Suboptimal decisions would not only reduce revenues, but deprive consumers of the primary benefit that Congress sought to achieve in the Spectrum Act: reallocating as much spectrum as possible for commercial mobile broadband services. The Commission should thus take all steps needed to make this auction succeed, in the sense that the auction will meet the statutory closing conditions for the maximum possible amount of freed-up spectrum.

Towards that end, the Commission should adopt AT&T's package bidding proposal, which AT&T has called "Clock Package Auction" or "CPA".[2] Such package bidding is necessary to maximize efficiency, revenue, and the amount of repurposed spectrum by capturing the large complementarities that regional and national carriers will derive from offering service on the same 600 MHz bands across multiple geographic areas, without handicapping smaller carriers' ability to obtain individual or relatively few licenses. Indeed, in the *absence* of such package bidding rules, bidders might exit the forward auction early to avoid the classic exposure risk of "winning" a hodgepodge of scattered spectrum assets that lack much of the value they would have presented had they been part of a seamless geographic package. That exposure risk would thus suppress forward-auction participation, reduce the amount of repurposed spectrum, and increase the risk of auction failure.

Tellingly, most opponents of package bidding in this proceeding simply ignore the exposure problem and, if unaddressed, the threat that it poses to auction efficiency, revenues, and the amount of cleared spectrum. At the same time, these commenters raise a host of theoretical

[2] *See, e.g.*, Comments of AT&T Inc., GN Docket No. 12-268 (filed Jan. 25, 2013) ("AT&T Comments") at pp. 51-58 and Ex. B, Design of the FCC Incentive Auction, Yeon-Koo Che, Phil Haile, Michael Kearns (Attachment 2 hereto); Reply Comments of AT&T Inc., GN Docket No. 12-268 (filed Mar. 12, 2013) ("AT&T Reply Comments") at pp. 53-60 and Ex. C, Reply Analysis of Yeon-Koo Che and Phil Haile (Attachment 3 hereto).

objections to package bidding that simply do not apply to the specific CPA package bidding approach proposed by AT&T.

As demonstrated in the attached analysis by economist Phil Haile, AT&T's CPA design proposal does *not* materially increase auction complexity, introduce bias against small/local bidders, enhance opportunities for gaming, or create any additional exposure risk for small/local bidders.[3] As Dr. Haile cautions: "When evaluating the package bidding criticisms of T-Mobile, U.S. Cellular and others, it is important to be precise about which package bidding approach is being considered. Limitations of particular package auction designs considered in the past should not be mistaken for limitations inherent to package bidding generally."[4] AT&T's CPA package bidding proposal can be incorporated in the 600 MHz auction design to address the exposure problem and enhance efficiency and revenues without triggering any of the harms that its opponents claim.

T-Mobile complains that any type of package bidding is likely to increase excess supply (*i.e.*, the risk of "undersell") due to a package bidder's dropping its demand for a package when only some of its components are in excess demand.[5] But what matters is how package bidding would affect auction revenue and efficiency, and any "extra" undersell that arises under AT&T's CPA proposal would be the result of allowing bidders to express complementarities in order to attain *more* efficient license allocations. In addition, it is well understood among auction experts that undersell is an implication of (*i.e.*, a necessary condition for) revenue maximization. In any

[3] Philip A. Haile, Reply Comments on Package Bidding, GN Docket Nos. 12-268, 13-185 (dated Jan. 23, 2014) ("Haile Analysis") (Attachment 1 hereto) at 3**.**

[4] Haile Analysis at 2.

[5] Comments of T-Mobile USA, Inc., GN Docket Nos. 12-268, 13-185 (filed Jan. 9, 2014) ("T-Mobile Comments") at 3-4.

event, as Dr. Haile explains, the available evidence suggests that the incremental undersell risk of package bidding is modest and that the benefits of package bidding vastly outweigh the costs.[6]

T-Mobile's proposal to substitute participation restrictions (*i.e.*, spectrum caps on AT&T and Verizon) for package bidding is equally misguided and appears to reflect an agenda that has nothing to do with the wisdom of introducing package bidding. T-Mobile proposes to reduce exposure risk by *limiting competition* for licenses.[7] As Dr. Haile notes: "one cannot take seriously the argument that harming efficiency and revenue by limiting competition is the best remedy (or any remedy at all) for the harms to efficiency and revenue that would arise from failing to address the auction design flaws [i.e., failing to provide for package bidding] that create exposure risk."[8]

Finally, AT&T's CPA proposal would work best with the Economic Area ("EA") licenses proposed by the Commission.[9] But if necessary, AT&T's CPA proposal could work with smaller license areas, such as the so-called Partial Economic Areas ("PEAs") proposed by the Competitive Carriers Association ("CCA"), as long as they fully nest within EAs.[10] In all events, the Commission should not employ Cellular Market Area ("CMA") licenses, as they engender the greatest exposure risk and thus possess the least potential to generate revenue and repurpose spectrum.

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[6] Haile Analysis at 6-8.

[7] T-Mobile Comments at 6.

[8] Haile Analysis at 10-11.

[9] *See, e.g.*, *In the Matter of Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, Notice of Proposed Rulemaking, 27 FCC Rcd 12357 (2012) ("*Incentive Auction NPRM*") at ¶ 148.

[10] Haile Analysis at 2.

II. Discussion

A. AT&T's CPA Proposal Does Not Disadvantage Smaller Carriers or Rural Consumers.

Package bidding opponents assert that package bidding violates section 309(j) the Act[11] and sound public policy by disadvantaging smaller carriers and rural customers in various ways.[12] Many of these assertions have already been fully debated earlier in this proceeding; moreover, many of these assertions simply do not apply to the CPA auction design proposed by AT&T.[13] Thus, AT&T will address them in relatively short order here.

Package bidding opponents argue that package bidding will inevitably steer rural license areas towards package bidders and away from smaller carriers who purportedly have a greater interest in serving those areas.[14] This argument fails for several reasons.

First, AT&T's CPA proposal does *not* favor bidders for packages over bidders for individual EAs (or PEAs). AT&T's CPA proposal *does* favor maximization of consumer welfare, instead. Specifically, AT&T's CPA proposal does not set aside any licenses for package bidders; and smaller bidders continue to have the opportunity to acquire individual licenses needed to complement existing holdings. Furthermore, a package bidder could win all licenses in all EAs within the relevant geographic package only if the total price for that package exceeds the sum of the bids that would otherwise prevail, including all EA (or PEA)-specific

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[11] 47 U.S.C. § 309(j) (directing the Commission to design competitive bidding systems that, *inter alia*, avoid excessive concentration of licenses and disseminate licenses among a wide variety of applicants).

[12] *See, e.g.*, Comments of United States Cellular Corporation, GN Docket Nos. 12-268, 13-185 (filed Jan. 9, 2014) ("USCC Comments") at 32-48.

[13] As stated previously, when evaluating the package bidding criticisms in this record, it is important to keep in mind precisely which package bidding approach is being challenged. Limitations of particular package auction designs considered in the past should not be mistaken for limitations inherent in package bidding generally. Many of the concerns expressed in this record simply do not apply to the CPA auction design proposed by AT&T. *See, e.g.*, Haile Analysis at 2-3.

[14] *See, e.g.*, USCC Comments at 32-33, 36, 44-47.

bids. That mechanism will pick winners solely on the basis of which combination of bids expresses – and can be presumed to produce – the greatest economic value for consumers.[15]

Second, AT&T's CPA proposal would not risk leaving 600 MHz licenses in rural areas neglected by larger carriers when there may have been smaller bidders allegedly more interested in actively serving those areas. The Commission undoubtedly will adopt build-out requirements for 600 MHz licenses, and those requirements will surely apply in every license area, regardless of whether the licensee won the license through a package bid or a bid on an individual license. Accordingly, the suggestion that package bidding would lead to reduced deployment in rural areas is baseless.

Third, AT&T already serves rural areas as well as urban/suburban areas.[16] Indeed, AT&T competes vigorously on the basis of scope and quality of coverage. As a result, there is no credible ground to believe that AT&T will shortchange rural license areas in packages that it may win.

Package bidding opponents also raise theoretical concerns about a "threshold" or "free rider" problem to justify rejection of package bidding.[17] This concern is not determinative under the circumstances here. Restricting packages to a hierarchical structure helps bidders overcome the threshold problem by eliminating ambiguity about which component prices must rise to displace the package bidder. In any event, a threshold problem cannot be a basis for opposing a

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[15] *See, e.g.*, Haile Analysis at 4-6; Attachment 3 (AT&T Reply Comments, Ex. C) at 1-10. USCC's claim that "[p]ackage bidding could also permit large carriers to obtain a package of licenses for a sum lower than what individual licensees are willing to pay on a per-license basis" (USCC Comments at 45) is incorrect in the case of AT&T's CPA proposal. *See, e.g.*, Haile Analysis at 4-5.

[16] *See, e.g.*, http://www.att.com/network/ (then click on "Coverage" to see a map of AT&T's wireless network coverage nationwide); http://news.cnet.com/8301-1035_3-57616700-94/at-t-covers-more-than-270-million-with-4g-lte/ (stating that, as of January 6, 2014, AT&T's LTE service covered 270 million Americans, and AT&T plans to raise that number to 300 million by the end of 2014).

[17] *See, e.g.*, USCC Comments at 33-36, 45.

well-designed mechanism for package bidding (such as AT&T's CPA proposal), because it *neither arises from nor is exacerbated by package bidding*. Instead, the threshold problem arises from the market reality that – in *any* spectrum auction, with or without package bidding – some bidders will perceive complementarities in holding licenses in geographically adjacent regions, and they will place bids designed to capture those complementarities.[18]

Package bidding opponents contend, in addition, that package bidding increases auction complexity to such a degree that smaller carriers will lack the resources to competitively participate.[19] This contention is erroneous as applied to AT&T's CPA proposal. To be sure, an auction without AT&T's CPA proposal would have fewer bidding objects. But given the small number of package tiers proposed here (especially if the Commission sticks with its proposal to license on the basis of EAs),[20] the difference would be trivial, and far from enough to deter auction participation. And even more importantly, by eliminating exposure risk for package bidders, AT&T's CPA proposal can actually simplify bidding for those seeking individual licenses by simultaneously eliminating the incentives to manipulate such individual bids in attempts to exploit the exposure risk of package bidders, which has been a complex and highly demanding strategy prevalent in past spectrum auctions.[21]

Package bidding opponents raise two additional purported sources of undue complexity, neither of which applies to AT&T's CPA proposal. T-Mobile argues that package bidding would introduce several complications due to the ability of a package bidder to quit demanding a

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[18] *See, e.g.*, Attachment 3 (AT&T Reply Comments, Ex. C) at 4-8; Haile Analysis at 5-6.

[19] *See, e.g.*, USCC Comments at 40-43.

[20] Indeed, the CCA's proposal to employ PEAs for geographic license areas would add more auction objects to the 176 originally proposed by the Commission than AT&T's CPA proposal would add.

[21] *See, e.g.*, Attachment 3 (AT&T Reply Comments, Ex. C) at 1-3; Haile Analysis at 3-4.

package when its price rises.[22] However, AT&T's CPA proposal does not permit any bidder to withdraw any bid. Thus, perhaps T-Mobile's real concern involves the circumstances under which a bidder is permitted to reduce the quantity of an object it demands. But AT&T's CPA proposal treats all objects (packages and individual licenses) symmetrically in this regard, thus favoring neither package bidders nor individual license bidders.[23] USCC argues that package bidding would introduce a new exposure risk for bidders on individual licenses, due to potential reinstatement of previously losing bids.[24] Such reinstatement risks might exist in some kinds of package bidding, but not in AT&T's CPA proposal, which does not permit reinstatement of previously losing bids.[25]

Package bidding opponents further claim that prepackaging groups of licenses does not allow efficient aggregation. In their view, setting predetermined packages of licenses improperly presumes that each bidder has the same aggregation strategy and would value the packages equally; in actuality, each bidder will have different packaging needs and strategies, depending on its unique business model and existing portfolio. Thus, opponents assert that prepackaging groups of licenses harmfully interferes with bidders' ability to tailor packages as they wish, thereby reducing auction participation and revenues.[26] This assertion fatally ignores two key points. First, incorporating AT&T's CPA proposal would not hinder any carrier's ability to attempt to tailor unique license packages on an ad hoc EA-by-EA (or PEA-by-PEA) basis; it

[22] T-Mobile Comments at 1-5. T-Mobile refers to this ability as an option to "withdraw" a bid. *See, e.g.*, T-Mobile Comments at 2.

[23] Haile Analysis at 12. Specifically, any bidder may reduce the quantity demanded of an object only when the price of that object increases. *Id*.

[24] *See, e.g.*, USCC Comments at 36-40.

[25] Haile Analysis at 9.

[26] *See, e.g.*, USCC Comments at 45-48.

would simply provide additional options for carriers interested in attempting to obtain relatively larger, contiguous groupings of EAs (or PEAs). And second, those additional, pre-packaged options are essential to minimizing the exposure risk that would otherwise substantially suppress bidding by such carriers. Thus, the reality is that AT&T's CPA proposal would *promote* efficiency and revenues, not diminish them.

B. Under AT&T's CPA Proposal, the Efficiency and Revenue Benefits Vastly Outweigh Any "Undersell" Costs.

T-Mobile raises a concern about the potential for excess supply or "undersell" – the failure of some licenses to sell at the incentive auction, potentially due to a package bidder's dropping its demand for a package when only some of its components are in excess demand.[27] Such potential for undersell likely does exist to some degree under AT&T's CPA proposal. But this observation is meaningless in isolation, without attempting to quantify the elevation of undersell risk or the auction benefits that would inherently accompany any increase in such risk. Any "extra" undersell that arises under AT&T's CPA proposal would be the direct result of the proposal's ability to minimize exposure risk, allowing bidders to express complementarities in order to attain more efficient license allocation and thereby maximize auction revenue. All available evidence suggests that, under AT&T's CPA proposal, the increased risk of undersell would be relatively tiny, whereas the increase in efficiency and revenues would be huge. In other words, any undersell costs would be vastly outweighed by efficiency and revenue benefits.[28]

[27] T-Mobile Comments at 3-4.

[28] Haile Analysis at 6-8; Attachment 3 (AT&T Reply Comments, Ex C) at 12-13. As Dr. Haile explains, "although failing to sell a license may sound like a source of inefficiency (and it would be if all else could be held fixed), this is not true in general, since bidder behavior depends on the auction rules. Any 'extra' undersell that arises under the CPA would be the result of allowing bidders to safely express complementarities in order to attain more efficient license allocations." Haile Analysis at 6-7.

C. AT&T's CPA Proposal Does Not Encourage Gaming.

T-Mobile suggests that package bidding would introduce opportunities for a particular type of gaming called "parking", whereby a bidder can, potentially to its advantage, harm price discovery by hiding its intentions early in the auction, maintaining eligibility by bidding for a package it does not want but whose price is sure to keep rising.[29] In AT&T's CPA proposal, however, such behavior would involve substantial risk for the bidder, because "[b]ids in the CPA are binding offers to purchase" and "[o]nly if a bidder were certain that others will drive up the price of an object can he bid for it without risk of winning."[30] Moreover, T-Mobile's concern seems to reflect a presumption that the traditional MHz-Pop based activity rule would be used. But AT&T's CPA proposal recommends use of a "revealed preference" activity rule, instead – consistent with recent auction theory designed precisely to minimize manipulative hindrances to price discovery such as parking.[31]

D. Spectrum Caps Are No Substitute For Package Bidding.

T-Mobile argues that, given the purported problems with AT&T's CPA proposal, the Commission should minimize exposure risk by adopting spectrum caps, instead.[32] T-Mobile is correct that exposure risk could be reduced by limiting competition itself. However, as AT&T has already argued at length in this proceeding and elsewhere before the Commission, such limitations on competition would likely result in dramatic reductions in auction efficiency and revenue, contrary to the fundamental purposes of the incentive auction. As a result, AT&T's CPA proposal is the far superior choice for reducing exposure risk and thereby maximizing the

[29] T-Mobile Comments at 4-5.

[30] Haile Analysis at 8 n.15**.**

[31] Haile Analysis at 8. *See generally* Attachment 3 (AT&T Reply Comments, Ex. C) at 10-13.

[32] T-Mobile Comments at 6.

incentive auction's revenue generation and spectrum reallocation.[33] Indeed, T-Mobile seems to have shoehorned its spectrum cap proposal into this pleading cycle for purposes other than to truly address the relative merits of package bidding, such as the purpose of potentially reducing the cost to T-Mobile of obtaining 600 MHz spectrum at auction.[34]

E. The Commission Should Not Employ CMAs.

Some commenters continue to press the Commission to adopt CMAs as the geographic license area for the incentive auction of 600 MHz spectrum.[35] As even the economists[36] commissioned by the Rural Wireless Association, Inc. and NTCA-The Rural Broadband Association (collectively, "RWA/NTCA") admit, however, as license size shrinks, geographic exposure risk enlarges.[37] Thus, employing 734 CMAs rather than the 176 EAs proposed by the Commission would make the exposure risk for carriers seeking regional or national footprints skyrocket, which could substantially suppress bids, reduce the amount of repurposed spectrum, and perhaps scuttle the incentive auction altogether. Moreover, because CMAs do not nest within EAs or any other larger existing geographic areas,[38] AT&T's CPA proposal could not readily be used to minimize exposure risk. Furthermore, adopting CMAs would entail auction implementation risks for the Commission and bidders, and also substantial deployment

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[33] *See, e.g.*, Haile Analysis at 9-11.

[34] *See, e.g.*, Haile Analysis at 9-11.

[35] *See, e.g.*, USCC Comments at 9-32.

[36] NERA Economic Consulting, *Local and Regional Licensing for the US 600 MHz Band (Incentive Auction)* (January 2014) ("NERA Report"), attached to Ex Parte Letter from Richard Marsden, NERA, to Marlene Dortch, FCC, GN Docket Nos. 12-268, 13-185 (undated).

[37] NERA Report at 21-23.

[38] *See, e.g.*, NERA Report at 5, 34.

complications for winning bidders.[39] Consequently, along with AT&T's CPA proposal regarding package bidding, the Commission should adopt its own EA proposal regarding geographic license areas. The Commission could, instead of EAs, adopt some form of the PEAs proposed by the CCA; but for the reasons explained above and in AT&T's Comments, such a choice would be defensible only if accompanied by AT&T's CPA proposal or equally effective package bidding rules.[40]

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[39] *See, e.g.*, *Incentive Auction NPRM* at ¶¶ 147-148; Comments of Verizon and Verizon Wireless, GN Docket Nos. 12-268, 13-185 (filed Jan. 9, 2014) at 3. Even the economists commissioned by RWA/NTCA concede that using CMAs rather than EAs "would not be ideal from a perspective of managing implementation risks" (although the complexities allegedly would not be "insurmountable"). NERA Report at 11, 25-26, 31.

[40] Comments of AT&T, GN Docket Nos. 12-268, 13-185 (filed Jan. 9, 2014). The bifurcated auction proposal proffered by RWA/NTCA (*see, e.g.,* NERA Report) is a flawed, Rube Goldberg contraption that has already drawn considerable cogent criticism in the record. AT&T will not pile on here except to observe that the proposal assumes – quite incorrectly – that AT&T has immaterial interest in serving rural areas. *See, e.g.*, NERA Report at 44. As previously stated, AT&T competes vigorously regarding breadth of wireless service coverage.

III. Conclusion

For the foregoing reasons, the Commission should adopt its proposal to license 600 MHz spectrum on an EA basis, and should also adopt AT&T's CPA package bidding proposal. However, it would not be arbitrary or capricious for the Commission to choose to license 600 MHz spectrum on a PEA basis, but only if the Commission were also to adopt AT&T's CPA proposal or equally effective package bidding rules.[41]

Respectfully submitted,

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[41] AT&T's positions here apply to the AWS-3 proceeding, as well. *See, e.g.*, *Public Notice* at 3.

ATTACHMENT
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Reply Comments on Package Bidding

FCC GN Docket Nos. 12-268 & 13-185

January 23, 2014

Philip A. Haile[1]

1 Introduction

I have been asked by counsel for AT&T to review and comment on recent FCC filings on the topic of package bidding in the 600 MHz spectrum auctions.[2] Package bidding is motivated by the exposure problem and its adverse effects on auction revenue and efficiency, issues already discussed extensively in this proceeding and in prior FCC auction proceedings.[3] In the most recent comment round, some commenters, including T-Mobile and U.S. Cellular, contend that allowing package bidding in the 600 MHz auctions would necessarily introduce excess complexity, bias against smaller bidders, gaming opportunities, harm to revenue and efficiency due to excess supply (or "undersell"), and other undesirable effects. T-Mobile further contends that the exposure problem can be mitigated without these problems if auction participation limits (*i.e.*, spectrum caps) are substituted for package bidding. As I explain below, these claims are misguided: package bidding can be incorporated in the 600 MHz auction design in ways

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[1] Haile is the Ford Foundation Professor of Economics at Yale University, Research Associate of the National Bureau of Economic Research, and member of the research staff of the Cowles Foundation for Research in Economics.
[2] "Comments of T-Mobile USA, Inc.," GN Docket No. 12-268, January 9, 2014 (hereinafter "T-Mobile Package Bidding Comments"); "Comments of United States Cellular Corporation," GN Docket No. 12-268, January 9, 2014 (hereinafter "U.S. Cellular Package Bidding Comments").
[3] See, *e.g.*, Yeon-Koo Che, Philip Haile and Michael Kearns, "Design of the FCC Incentive Auctions," attachment to Comments of AT&T Inc., GN Docket No. 12-268, January 25, 2013 (hereinafter "CHK"); Gregory L. Rosston, "Implementing Package Bidding in the 700 MHz Band to Improve Consumer Welfare," WT Docket No. 06-150, February 5, 2007; Declaration of Dr. Gregory L. Rosston and Dr. Scott Wallsten, Attachment A to Comments of Access Spectrum, LLC, Columbia Capital III, LLC, Pegasus Communications Corporation and Telecom Ventures , LLC, WT Docket No. 06-150, September 29, 2006; Comments of Paul Milgrom and Karen Wrege, WT Docket No. 06-150, September 20, 2006; Reply Comments of Paul Milgrom and Karen Wrege, WT Docket No. 06-150, October 20, 2006.

that would enhance efficiency and revenues. T-Mobile's alternative proposal to reduce competition for 600 MHz licenses, in contrast, could "address" the exposure problem only at the cost of auction revenues and efficiency.

Previously, AT&T provided a detailed proposal for incorporating package bidding in the 600 MHz auction.[4] Relative to the clock auction design of Milgrom, Ausubel, Levin and Segal ("MALS"),[5] this proposal involves adding a small number[6] of pre-defined packages to the auction and allowing competition to determine which licenses are allocated as parts of packages and which are awarded on a standalone basis. CHK called this auction design the Clock Package Auction ("CPA"). CHK also suggested a particular package structure with licenses for each Economic Area ("EA"), Major Economic Area ("MEA"), Regional Economic Area Group ("REAG"), and for the nation as a whole. That suggestion yields a hierarchical structure for packages, something shared by the "Hierarchical Package Bidding" ("HBP") auction design explored previously by the FCC.[7] However, because the CPA is based on the MALS clock auction design rather than the traditional Simultaneous Multi-Round (SMR) auction, the CPA would inherit the substantial advantages of the MALS design and would operate quite differently from the HPB auction. The CPA could be adapted to other package designs as well—for example, adding smaller licenses that nest into EAs (such as the "PEAs" favored by some smaller carriers) or dropping the nationwide package.

When evaluating the package bidding criticisms of T-Mobile, U.S. Cellular and others, it is important to be precise about which package bidding approach is being considered. Limitations of particular package auction designs considered in the past should not be mistaken for limitations inherent to package bidding generally. Many of the concerns expressed by T-Mobile and U.S. Cellular simply do not apply to the CPA auction design. For example, the comments

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[4] See CHK and "Reply Comments of Yeon-Koo Che and Phil Haile," attachment to Comments of AT&T Inc., GN Docket No 12-268, March 12, 2013 (hereinafter "CH").

[5] P. Milgrom, L. Ausubel, J. Levin, and I. Segal, "Incentive Auction Rules Options and Discussion," appendix to Notice of Proposed Rulemaking, FCC 12-118, September 28, 2012.

[6] With the specific design proposed in CHK, the number of objects (license types) would increase from 172 to 229.

[7] See, *e.g.*, Jacob K. Goeree and Charles A. Holt, "Hierarchical Package Bidding: A Paper and Pencil Combinatorial Auction," *Games and Economic Behavior*, 70, 2010 and "Auction of H Block License in the 1915-1920 MHz and 1995-2000 MHZ Bands," Comments Sought on Competitive Bidding Procedures for Auction 96,'' Federal Communications Commission, AU Docket No. 13-178, July 15, 2013.

regarding excess complexity, bias against small/local bidders, new opportunities for gaming, and exposure risk for local bidders all appear to reflect either a misunderstanding of the CPA design or an assumption that a different type of package bidding would be used. As previously demonstrated in CHK and CH and discussed further below, the CPA design does not materially increase complexity, introduce bias against small/local bidders, enhance opportunities for gaming, or create any additional exposure risk for small/local bidders.

Two of T-Mobile's comments warrant further consideration. T-Mobile contends that any type of package bidding is likely to increase excess supply (the risk of undersell) due to a package bidder's dropping its demand for a package when only some of its components are in excess demand. That is likely true, but it is not a useful observation on its own because it amounts to forgetting about the benefits component of cost-benefit analysis. Any "extra" undersell that arises under the CPA would be the result of allowing bidders to express complementarities in order to attain more efficient license allocations. The available evidence suggests that the incremental undersell risk is modest and that the benefits of package bidding vastly outweigh the costs.

T-Mobile's proposal to substitute participation restrictions (on AT&T and Verizon) for package bidding appears to reflect an agenda that has nothing to do with the wisdom of introducing package bidding. T-Mobile's proposal does reveal a great deal about the risks to revenue and efficiency that would result from the artificial limits on competition favored by T-Mobile. I elaborate on each of these points below.

2 Excessive Complexity

Both T-Mobile and U.S. Cellular assert that package bidding would introduce excessive complexity to the auction.[8] One can certainly imagine package auction designs that would introduce substantial, even prohibitive, complexity. However, such complexity is not inherent

[8] See *e.g.*, T-Mobile Package Bidding Comments at 1, 3, 5, 6; U.S. Cellular Package Bidding Comments at 32, 40-43, 45.

to package bidding in general. And the CPA would, in particular, be substantially *less* complex than prior FCC spectrum auctions.

The CPA is based on the new clock auction design proposed by MALS. As discussed extensively elsewhere, the MALS auction design offers a number of substantial simplifications to the SMR auction design used previously by the FCC. These design improvements substantially simplify bidding decisions, aid price discovery, and eliminate many opportunities for gaming. The CPA builds on these major design improvements, altering the MALS design in only three ways: (i) expanding the set of objects offered to include a small number of pre-specified packages; (ii) specifying how excess demand can be properly calculated; and (iii) specifying a rule to govern price clocks, ensuring that package prices are additive in the prices of the package components when possible (and otherwise superadditive).

Consequently, the CPA would not add significant complexity to the MALS auction design, particularly from the perspective of bidders. This has already been explained in detail in CHK and CH. In fact, the CPA would substantially *simplify* the auction relative to the MALS design (and, *a fortiori,* relative to the SMR auction) by substantially eliminating the exposure problem, thereby avoiding the need for bidders to pursue bidding schemes aimed at reducing their own exposure risk or at exploiting the exposure risk of their opponents (see CH).

3 Intrinsic Bias Against Small/Local Bidders

Both T-Mobile and U.S. Cellular assert that package bidding is inherently biased against small bidders or local bidders—those seeking smaller coverage footprints.[9] This is incorrect. As discussed already in CHK and CH, the CPA would create no bias in favor of or against any class of bidder. No licenses would be set aside for package bidders, and a package would be awarded only when a package bidder offers a price that exceeds the sum of prices offered for the component licenses. U.S. Cellular's claim[10] that "[p]ackage bidding could also permit large carriers to obtain a package of licenses for a sum lower than what individual licensees are

[9] T-Mobile Package Bidding Comments at 3; U.S. Cellular Package Bidding Comments at 32-36.
[10] U.S. Cellular Package Bidding Comments at 45.

willing to pay on a per-license basis" is incorrect in the case of the CPA. In fact, as discussed at length by CHK and CH, the CPA would *eliminate* an important bias against local bidders—that arising from the "overflow problem" under the MALS auction design.

It is true that the introduction of package bidding would also remove (or at least reduce) a bias against package bidders: that arising from exposure risk. As a result, package bidders would be more likely to win when their value for the spectrum was highest. Bidders seeking smaller footprints understandably would not favor this. But one should not confuse removal of bias against package bidders with introduction of bias against local bidders.[11] Removal of biases is one of the purposes of improving auction design: to obtain more efficient allocations and to allow bidders to better express their full willingness to pay, thereby enhancing auction revenue.

U.S. Cellular makes specific reference to the "threshold problem'' as a source of bias against local bidders under package bidding. This claim reflects a common but incorrect belief that the threshold problem is introduced by package bidding. This has been discussed at length in CH. As explained there, the threshold problem arises not from package bidding but from the presence of package *valuations* (complementarities between licenses). As CH showed, the threshold problem is present in both the traditional SMR auction design and in the clock auction design proposed by MALS.

Where it exists, the threshold problem can be a potentially significant source of inefficiency and suppression of revenue. However, I am not aware of even an example of a threshold problem that would arise under the CPA auction design but not the MALS design, much less an argument that the threshold problem would be systematically more severe under the CPA. Thus, whereas there has been a tendency to think that optimal spectrum auction design must make tradeoffs between the exposure problem and the threshold problem, I am not aware of evidence that such a tradeoff even exists in comparing the MALS (without package bidding) vs. CPA (with package bidding) designs. On the other hand, concerns about the threshold problem offer one motivation for abandoning the SMR auction design. As CH pointed out, the MALS auction and CPA may substantially mitigate the threshold problem relative to that which would arise under

[11] See U.S. Cellular Package Bidding Comments at 34.

the traditional SMR auction design, due to the proposed information disclosure rules, the use of price clocks, and the use of generic licenses.

4 Bid "Withdrawals"

T-Mobile argues that package bidding would introduce several problems due to the ability of a package bidder to quit demanding a package when its price rises. T-Mobile refers to this ability as an option to "withdraw" a bid, although neither the MALS nor CPA auction rules permit any bidder to withdraw any expression of demand: any expression of demand (*i.e.*, any bid) would be a binding offer. Thus T-Mobile's concern here involves the circumstances under which a bidder is permitted to reduce the quantity of an object it demands. The CPA rules treat all objects (packages and individual licenses) symmetrically on this issue: a bidder may reduce the quantity demanded of an object only when the price of that object increases.[12]

4.1 Undersell

T-Mobile raises a concern about the potential for excess supply or "undersell"—the failure of some licenses to sell at the auction, potentially due to a package bidder's dropping its demand for a package when only some of its components are in excess demand. Although undersell can arise in the MALS auction design as well, I agree that undersell is likely to be somewhat greater under the CPA rules. However, this is not a useful observation on its own. What matters is how package bidding would affect auction revenue and efficiency.

Perhaps surprisingly (although not to auction experts) it is not true in general that auctions with less undersell generate greater revenue or greater efficiency. In fact, even in the simplest auction settings, undersell is an *implication* (*i.e.*, a necessary condition for) revenue maximization.[13] Likewise, although failing to sell a license may sound like a source of

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[12] This is a more restrictive rule than that used in the "clock phase" of the "Combinatorial Clock Auction," where a bidder may drop its demand on an object whose price has not risen as long as the price rises for some object the bidder demanded in the preceding round. Thus, in the CPA, only a bidder for a nationwide license would have the same flexibility in the CPA as in the Combinatorial Clock Auction. As suggested previously in CH, if this flexibility is a concern, nationwide licenses could be excluded from the CPA. Including packages only at the MEA and REAG levels would still yield a multitree structure, as the CPA requires for unambiguous determination of excess demand.

[13] See, *e.g.*, Roger B. Myerson, "Optimal Auction Design," *Mathematics of Operations Research*, 6, 1981.

inefficiency (and it would be if all else could be held fixed), this is not true in general, since bidder behavior depends on the auction rules. Any “extra” undersell that arises under the CPA would be the result of allowing bidders to safely express complementarities in order to attain more efficient license allocations.

Thus, an evaluation of undersell alone is irrelevant: it amounts to forgetting about the benefits component of cost-benefit analysis. The relevant questions about revenue and efficiency concern (a) whether, on net, allowing package bidding is likely to enhance or diminish revenue and efficiency, and (b) the potential for correcting any adverse outcomes that arise.

The only attempt I am aware of to evaluate (a) is the simulation analysis in CHK. There we found, consistent with T-Mobile's concern, that package bidding leads to a larger rate of undersell, although the undersell rate was fairly small in both cases (about 3 percent undersell with package bidding, 0.3 percent without). However, the net effects of package bidding on revenues and efficiency were positive and large. For example, in the baseline simulation, the median percentage gain in revenue from the CPA was 67.1 percent, and the 75th percentile gain was more than 100 percent. In terms of efficiency, the CPA achieved at least 90 percent of the (infeasible) first-best in over 91 percent of the replications. By contrast, the MALS auction (without packages) achieved this level of efficiency in only 37 percent of the replications.[14]

No simulation study can answer with certainty what the net effects of package bidding will turn out to be. But these findings provide the only evidence available thus far regarding the likely effects of package bidding (following the CPA) on efficiency and revenue. At a minimum, they reinforce what is already a simple matter of logic: one cannot merely argue that package bidding might lead to greater undersell, since greater undersell can arise from improvements in auction design that lead to vast improvements in efficiency and revenue.

Finally, regarding question (b) above, it seems worth pointing out that revenue losses due to a failure to account for complementarities in the auction design would be irreversible. We might hope that resale markets would undo at least some of the inefficiency resulting from an

[14] See CHK for additional details and simulations.

absence of package bidding, but the Federal Government will capture no additional revenue. Undersell, on the other hand, can be at least partially mitigated by offering residual licenses at some future date.

4.2 Gaming

T-Mobile suggests that package bidding would introduce opportunities for a particular type of gaming, whereby a bidder hides its intentions early in the auction, maintaining eligibility by bidding for a package it does not want but whose price is sure to keep rising. T-Mobile does not elaborate on the motives for such "parking" behavior, but it is well understood that, should it arise, parking can harm price discovery, potentially to the advantage of the bidder.

In the CPA, such behavior would involve substantial risk for the bidder.[15] But, more important, the concern seems to reflect a presumption that the traditional MHz-Pop based activity rule would be used. The possibility of parking under that rule is well recognized and has led to the recent development of alternative "revealed preference" activity rules, which minimize such hindrances to price discovery.[16] CHK recommended the use of a revealed-preference activity rule in the CPA.

More broadly, although opportunities for gaming must be considered very carefully, initial intuitions that the CPA would introduce substantial new opportunities for gaming or "manipulative bidding" have so far failed to stand up to careful scrutiny. Many of the notions experts in the field have about package bidding and gaming appear to be tied tightly to the opportunities to game the rules when package bids are added to the SMR auction design. This is understandable, given the long history of the SMR auction at the FCC. However, when comparing the CPA to the MALS auction design, the most important differences in likely gaming appear to be the CPA's elimination of incentives for package bidders to pursue manipulative bidding schemes designed to minimize their own exposure risk or to exploit that of their competitors. Thus, adoption of the CPA design for package bidding would simultaneously serve

[15] Bids in the CPA are binding offers to purchase. Only if a bidder were certain that others will drive up the price of an object can he bid for it without risk of winning.
[16] See, *e.g.*, Peter Cramton, "Spectrum Auction Design," *Review of Industrial Organization*, 42, 2013.

the goals of limiting the harmful effects of exposure risk and limiting the opportunities for manipulative bidding. These issues have been discussed previously by CH.

5 Reinstatement of Losing Bids

U.S. Cellular argues that package bidding would introduce a new exposure risk for local bidders, due to potential reinstatement of previously losing bids.[17] I agree that any auction design that includes reinstatement of losing bids could create substantial risks (for any bidder). Such a risk would exist in some types of package auctions, but not the CPA. In the CPA there is no reinstatement of previously losing bids.[18] Under the CPA rules, a bidder would be allocated a license only if this bidder is demanding that license when the price clock stops.

6 Participation Restrictions

T-Mobile argues that limits on bidder participation offer a better approach to limiting exposure risk that arises from complementarities between licenses. This is an argument the FCC should examine closely, as it reveals a great deal about the dangers that participation restrictions would pose to the auction.

Exposure risk arises from the interaction between two auction features:

> ***competition***: bidders must compete for scarce licenses; thus, a package bidder may fail to win the package it desires because others value the licenses more highly;
>
> ***noncontingent offers***: without package bidding, offers to buy a license are not contingent on whether other licenses are also acquired.

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[17] U.S. Cellular Package Bidding Comments at 36-40. Reinstatement of losing bids refers to the phenomenon, permitted by some auction rules, that a bidder not offering to buy a given license at the end of the auction is forced to buy that license based on an offer to buy made earlier in the auction. This practice can limit undersell, but at the cost of introducing substantial risk to bidders, inhibiting straightforward bidding and price discovery.

[18] U.S. Cellular may be confusing AT&T's CPA proposal with other package auction designs. This is also suggested by their use of the moniker HPB to refer to the AT&T proposal. The specific package design proposed in CHK is hierarchical but, as explained above, the CPA rules differ considerably from those of the SMR-based HPB auction previously considered by the FCC.

Exposure risk is the risk that competition and noncontingent offers interact—that a package bidder unwilling to outbid others for the licenses in a package is stuck making good on his offer to buy a component license, one that may have little standalone value to him. Because this risk involves the interaction between competition and noncontingent offers, by eliminating/reducing either feature, one could eliminate/reduce exposure risk.

Auction theorists have given a great deal of attention to how best to relax the restriction to noncontingent offers in order to reduce the exposure problem and, thereby, preserve package bidders' willingness to compete. Presumably this focus reflects the view that competition is essential to an auction. But T-Mobile is correct in pointing out that exposure risk could also be limited by limiting competition itself. By limiting competition, especially that of bidders likely to value licenses highly, the chance that a package bidder is displaced by higher-value competitors is diminished. Of course this would be achieved at the cost of auction revenues and efficiency, by preventing bidders who place higher value on a license from expressing this demand. In an extreme case, limiting the level of competition relative to the number of licenses available could lead to disastrous results.[19] This dire scenario seems stunningly close to the hope expressed by T-Mobile:

"*With reasonable spectrum-aggregation limits, the threat of any bidder losing a substantial number of key market areas (i.e., the exposure risk) is greatly reduced. For instance, should multiple paired blocks of spectrum come to market with reasonable aggregation limits, every carrier should be able to acquire licenses over all or substantially all of their desired footprint*." (Comments of T-Mobile at 6).

The point of holding an auction is, of course, that not all carriers will be able to acquire all the licenses that they would like to have. An auction is meant to determine how best to allocate the scarce spectrum resource and to ensure that the Federal Government obtains a substantial share of the spectrum value. Limiting competition works directly against both purposes.

Unrestricted bidders seeking packages would be helped in many ways by the restrictions on competition proposed by T-Mobile. But one cannot take seriously the argument that harming

[19] See Paul Klemperer, ``How (Not) to Run Auctions: The European 3G Telecom Auctions," *European Economic Review*, 46, 2002.

efficiency and revenue by limiting competition is the best remedy (or any remedy at all) for the harms to efficiency and revenue that would arise from failing to address the auction design flaws (restrictions to noncontingent offers) that create exposure risk.

ATTACHMENT

2

Design of the FCC Incentive Auctions*

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Part I

Executive Summary

1 Overview

The FCC's September 2012 Notice of Proposed Rulemaking ("NPRM") proposes an ambitious plan to create a marketplace in which spectrum currently licensed for broadcast television can be repurposed for more highly valued mobile wireless use. This "Incentive Auctions" proposal involves using auctions to determine which users value the spectrum most highly, license spectrum to these users, and compensate current licensees who relinquish spectrum rights. The FCC has conducted many spectrum auctions in the past. But this is the first time an auction to sell spectrum rights is to be combined with an auction to purchase spectrum rights from existing licensees. Designing such a marketplace is complicated not only by incumbent claims on the spectrum, but also by technological constraints for mobile wireless and broadcast television uses.

Broadly speaking, there are two components of the proposed auction design: (1) a *reverse auction* in which current broadcast television licensees can offer to sell their licenses; and (2) a *forward auction* in which mobile wireless carriers may offer to purchase licenses. The quantities of spectrum transferred from broadcast television licensees to wireless carriers in each geographic area will be determined through a sequence of trials, starting with high *target* quantities that subsequently decline as necessary. Payments and spectrum reallocations take place only when a target quantity is identified at which wireless carriers are willing to buy at prices generating revenues that exceed the total cost of purchasing from broadcast television licensees. Indeed, because remaining broadcast television licensees must be "repacked" into another region of the frequency spectrum in order to create a viable wireless spectrum band plan, buyers will be required to pay prices sufficient not only to compensate sellers, but also to cover repacking costs and costs of administering the auction itself.

The need to repack TV licensees without creating interference introduces significant complications to the design of a reverse auction. Repacking constraints are "global," in the sense that placing a broadcaster in one market in a particular frequency can affect the feasibility of placing broadcasters in the same or nearby frequencies in other markets. For example, Washington, D.C. and Philadelphia, PA are sufficiently proximate that use of a UHF channel in one may preclude use of the same channel in the other market. For the same reason, decisions regarding channel usage in Philadelphia are also likely to affect New York City, which then may affect Bridgeport, CT, and then Boston, MA. Similarly, a Washington D.C. assignment may affect feasible assignments in Baltimore, MD and possibly Richmond, VA. Thus, channel usage can have a "daisy-chain" effect that can link major metropolitan areas—as well as smaller communities—that are hundreds of miles apart.

There is considerable variation across markets in the density of broadcast television licensees, as well as within-market variation in stations' "contours"—geographic coverage areas, which must be substantially maintained after repacking. Further, cross-market variation in the willingness of TV licensees to go off the air creates challenges to designing a viable nationwide band plan, and introduces the likelihood that the prices required to clear a given quantity of spectrum will vary substantially across markets.

The main auction design proposal was developed in Milgrom, Ausubel, Levin, and Segal (2012) ("MALS"). The authors are prominent economists with extensive experience in market design, including the design of prior FCC spectrum auctions. They propose the use of "clock" auctions in both the forward and reverse auctions. In a clock auction, prices are offered by the FCC. Bidders respond by indicating the trades they would like to make at those prices. Prices are then adjusted (e.g., upward in the forward auction), and the process continues until the trades desired by all bidders match the trades that are available.

The term "clock" refers to the fact that price offers are made by the "auctioneer" (here, the FCC), not by bidders. Bidders' choices in the auction are limited to expressions of quantities that would be demanded or supplied at the current price offers. Relative to other types of auction designs, including the Simultaneous Multi-Round (SMR) auction used in prior FCC spectrum auctions, a clock auction offers a number of advantages, including simplification of bidding and assurance that the auction keeps moving toward a final allocation.

The reverse auction would begin with provisional target quantities of spectrum to be cleared in each market. Although there would be a single nationwide reverse auction, it will utilize different clocks for each market and each *relinquishment option*—e.g., go off the air, share with another station, or move to a lower band. The option to "remain in the current band" (i.e., to be repacked) is always available as well. Bidders who do not participate in the auction are automatically assigned to this option. Participation in the auction is strictly voluntary. Clock prices may differ across bidders within a market and may move at different speeds, due to differences in coverage areas, in the feasibility of each option, in the costs of repacking, or in the effect of repacking one station on the ease of repacking others.

The auction clocks begin with high initial ("reserve") prices chosen by the FCC for each feasible relinquishment option in each market. Each bidder selects the option he[1] finds most attractive at these prices. Roughly speaking, prices are then lowered, bidders again express their preferences, and the process iterates until prices reach a level just low enough that the quantity of spectrum offered in each market equals the target quantity.

Complicating this simple description is the fact that feasibility considerations will limit repacking options. Each clearing target determines the amount of spectrum available for repacking bidders who choose to stay on the air. Before prices are adjusted during the auction, the feasibility of repacking each remaining bidder (i.e., the feasibility of adding that bidder to the set to be repacked) must first be checked. If it becomes infeasible for a given

[1] For clarity we use "he" to refer to bidders despite the fact that bidders are firms.

bidder to be repacked, his current relinquishment offer will be provisionally accepted at the current clock price for that option. The feasibility checking iterates until it is the case that each remaining bidder could feasibly be repacked. Only at this point do clock prices decline for these remaining bidders.

In the proposed forward clock auction, the FCC would simultaneously offer licenses in every market. Here the proposed notion of a market is an Economic Area ("EA") as defined by the Bureau of Economic Analysis. There will typically be more than one class of license available in each market; e.g., licenses for 10MHz of "paired" spectrum (5 MHz for wireless uplink, 5MHz for downlink), and licenses for 5MHz of unpaired (downlink only) spectrum. Licenses for different combinations of EA and class are treated as distinct *objects* and will have different clocks. Multiple *units* of each object, corresponding to specific frequencies, will typically be available. However, all units of the same object are treated as generic. At the beginning of the auction, all objects are given low initial (reserve) prices by the FCC. Potential buyers respond to these prices by specifying the number of units of each object that they would wish to purchase at these prices. Prices for objects with excess demand (demand that exceeds the available supply) then rise, and bidders indicate their demands at these new prices. This process continues until the quantity of spectrum demanded in each market is no larger than the associated target quantity. Thus, the forward auction stops at prices just high enough to eliminate excess demand relative to the target quantities.

At this point, the total revenue that would be generated nationwide by selling the target quantities at the forward auction prices are compared to the total cost of purchasing the same quantities at the provisional reverse auction prices. The difference between the total nationwide revenue and total nationwide cost is the *net revenue* that would be generated at the current clearing target. If this net revenue is sufficient to cover the expected nationwide repacking costs, the "grand auction" ends, licenses are reallocated, and all payments are made. If the net revenue is insufficient, there is a provision for continuing the forward auction to determine whether there are higher prices that forward auction bidders are willing to accept and which would close the gap. If this *closing trial* fails, the clearing targets are reduced. The reverse auction then resumes, pushing prices lower until supply is equated to the new target. Symmetrically, the forward auction resumes, with prices rising until demand is equated to the new targets. Net revenue is checked and the grand auction either terminates or resumes with still lower clearing targets.

2 Strengths of the Proposal

The proposed design has a number of significant virtues.

Simplicity A clock auction keeps the decisions for a bidder as simple as possible: a bidder need only choose the trades he would like to make at the prices currently offered. In the reverse auction, this means choosing one from a short list of relinquishment options, including the option to remain on the air in one's current band. For the forward auction, this means choosing the types and quantities of licenses the bidder would wish to purchase if the current clock prices turn out to be the final prices. The use of generic licenses provides additional simplicity by avoiding artificial distinctions between substantially identical licenses.

Simplicity is one of the dimensions in which the proposed design appears to be a significant improvement on the SMR design used in the previous ("forward") FCC auctions. We discuss the SMR auction in more detail below. Simplicity is also a substantial advantage of the clock design relative to the theoretically attractive but highly opaque Vickrey-Clark-Groves (VCG) auction. In a VCG auction it can be difficult for bidders to understand how their actions translate to outcomes during the auction.

Success The clock auction design ensures that the auctions will proceed to termination in reasonable time and will terminate at a feasible allocation.[2] Some alternative auction designs can stall or proceed only very slowly.

Transparency Identical licenses will trade at identical prices. No bidder will discover that he was excluded despite offering a price more favorable than that at which a similarly situated bidder was allowed to trade. These properties need not hold in a VCG or SMR auction.

Price Discovery In the forward auction, many licenses are offered simultaneously. A bidder's valuation of a particular license may depend on which complementary licenses he is also able to acquire. Further, the desirability of one combination of licenses will depend on the prices at which alternative combinations are available. In a multi-object clock auction, clock prices provide valuable feedback along the way about where all prices are likely to end up. This facilitation of "price discovery" can enable bidders to focus on the most relevant sets of licenses and to reoptimize their spectrum aggregation strategies as the auction proceeds.

Competition The auction design allows bidders on both sides (forward and reverse) to compete on even footing for the most valuable use of the spectrum. No particular type of bidder is treated more favorably than another. And no spectrum is set aside for either use: bids determine the quantity of spectrum allocated to mobile wireless vs. broadcast television. Combined with the use of generic licenses in the forward auction, which avoids artificial thinning of competition, these features will encourage efficiency of the final spectrum allocation.

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[2] This assertion is subject to potential computational challenges, disccussed below, that are associated with determining the feasibility of repacking each television licensee that has not already dropped out of the auction.

Limited Scope for Manipulation We see little scope for manipulation that could undermine the efficiency of the allocation or limit revenues generated. This contrasts, for example, with the SMR design, which is susceptible to collusion, signaling, and entry by speculators who can harm both revenues and efficiency.

3 Concerns

We see many favorable features of the MALS proposal and believe it is the best starting point from which to design the details of the incentive auctions. Nonetheless, there are several features that give us concern. Below we discuss a few of the most important issues. In the subsequent section we summarize several proposed modifications that we believe can address these concerns.

3.1 Forward Auction

Exposure Risk Our most serious concern with the proposed forward auction design is its treatment of complementarities between licenses. Such complementarities will exist for many (perhaps the vast majority of) forward auction bidders: any bidder who will wish to assemble coverage over regions larger than a single EA. The clock auction design allows bidders to pursue packages of licenses, but only by offering to buy each individual license that is part of the desired package. Such a strategy becomes problematic when the value of one license is highly dependent on whether other licenses in the package are also acquired.

Consider, for example, a bidder seeking to obtain a coverage footprint in the Philadelphia Major Economic Area ("MEA"), which is comprised of three EAs. Under the MALS auction design, such a bidder would have two options. One is to aggressively pursue all three EA licenses, offering to buy all of them as long as the total cost stays below the value these licenses have to him as a whole. This can be a very risky strategy. It is possible that the clock stops for one of the EA licenses while prices for the other two rise to levels that make the package unprofitable. A bidder following the aggressive strategy would be stuck: he would be forced to make good on his offer to purchase the first license. This license alone might not allow a viable service offering, but the bidder would be forced pay a price that he agreed to under the assumption that he would be able to offer coverage throughout the Philadelphia MEA. The risk of this highly unprofitable outcome is known as "exposure risk" or the "exposure problem."[3] It is a serious problem for firms seeking to offer service in any area larger than a single EA.

The other option for such a bidder is to bid conservatively, avoiding the exposure risk. This is also undesirable. It can lead to inefficient allocations (if the most valuable use of the spectrum is as part of a package) and to reduced revenues (even if the most valuable use is not in packages). Revenues are of particular importance here due to the interaction between the forward and reverse auctions. Unlike prior FCC auctions, here revenues and efficiency cannot be separated.

Frequency Contiguity A second major concern is that the NPRM and MALS proposals do not make adequate provisions for bidders to obtain contiguous blocks of spectrum. Two types of frequency contiguity are important. First, within a geographic area, wireless service providers seeking to offer LTE or LTE-Advanced services desire significant blocks of spectrum in adjacent frequencies. We understand that such *vertical contiguity* is essential to optimal performance of these technologies. Second, bidders seeking coverage areas encompassing multiple EAs desire licenses in identical frequencies in each EA. We understand that such *horizontal contiguity* minimizes interference at boundaries and allows use of more efficient network deployment.

The current proposals for assigning individual licenses to winners of generic licenses in

[3]See, for example, the extensive discussion of the exposure problem in the edited volume of Cramton, Shoham, and Steinberg (2010).

each EA may address the need for vertical contiguity within a single EA. But they would not allow bidders to express their broader needs for frequency contiguity. For example, the proposed random assignment of specific frequencies to winners in each EA virtually guarantees a lack of horizontal contiguity. The result would be a reduction in bidders' willingness to pay, inefficiently low quantities of spectrum cleared, and inefficient allocation of the spectrum that is cleared.

Other Sources of Misallocation: The Overflow Problem We have already discussed potential misallocation arising from the exposure problem and the needs for frequency contiguity. In some cases misallocation may take the form of *undersell*—failure to sell some units. Misallocation can also arise from other sources. Some, like the possibility that market clearing prices do not exist—are inherent to any allocation mechanism with anonymous prices. However, at least one additional source of misallocation is particular to the MALS design, which provides no language for bidders to express whether their demand for a particular license is driven by interest in a larger bundle of licenses. Without such language, bidders seeking small groups of licenses can face rising prices even when their own demands are not a source of scarcity.

An example will illustrate the problem. Consider the Boston MEA, which includes the Bangor, Portland, and Boston EAs. Suppose that six 10MHz paired licenses are available in Bangor and Portland, but only five in the Boston EA. This implies that at most five bidders could obtain MEA-wide coverage. Suppose that at some point in the auction "regional bidders" seeking MEA-wide coverage (or coverage in a larger area that includes the MEA) are demanding seven licenses, while there is only one Portland EA license demanded by a firm seeking coverage in Portland alone. Suppose the regional bidders have an all-or-nothing strategy in this MEA: they are not interested in any strict subset of the 3 EAs. In such a situation, we would like the auction to "hold on" to the offer of the Portland-only bidder: we know that only five units of demand from the regional bidders can be satisfied. So selling all licenses requires allocating to the Portland-only bidder. However, under the MALS design, the auction will count eight units of demand for the Portland license, conclude that this license is in excess demand, and raise its price. This will force the Portland-only bidder to either pay a higher price or exit, even though his demand is not a source of scarcity. This will tend to force such a bidder out of the market unnecessarily.

We term this phenomenon the *overflow problem*. With a richer bidding language, the auction design could account for feasibility constraints and limit the extent to which demand for large packages "flows down" to be counted as demand for the smaller packages and individual licenses nested in the larger package. Without such a language, too much demand flows down. This creates artificial competition for smaller bidders. This bias against smaller bidders will lead to undersell and/or inefficient allocations of licenses.

3.2 Reverse Auction

Computational Complexity Fully optimal clearing of a given target quantity of spectrum would minimize the total costs of clearing and of repacking television broadcasters who remain on the air. Reaching this fully optimal solution is certainly infeasible. An immediate problem is that the reservation values of broadcast TV licensees are unknown. Licensees must volunteer to sell and will not oblige if asked to reveal their true reservation values so that the efficient quantity of spectrum could be cleared at the lowest possible cost. This *incentive compatibility* constraint alone makes it impossible to pay each cleared television licensee the lowest price he would accept for his license. A second problem is the computational complexity involved in finding a cost-minimizing allocation. This complexity arises from the effect of repacking one station on the feasibility (or cost) of repacking others. Even if bidders' reservation values were known to the FCC, the cost minimization problem is an integer programming problem of a type known to be computationally hard in general.

The unique challenge in the reverse auction design is to handle adequately both incentive compatibility and computational feasibility while also keeping the rules sufficiently simple and transparent. Here, issues of auction design and computational considerations are intertwined in a way that has not previously been well explored in the theory or practice of auction design.

The clock auction proposed by MALS offers one possible approach. Taking bidding strategies as given, it replaces minimization of total cost with minimization of payments to cleared licensees, subject to sequential verification of repacking feasibility. As long as more bidders than necessary are volunteering to be cleared, prices fall, leading to repacking of bidders with the highest reservation values. In the simplest form of the proposed design, repacking constraints would enter only through verification that it is feasible to repack a bidder before he is offered a new set of prices. This approach puts priority on the cost of clearing relative to the cost of repacking. Only when it is infeasible for a bidder to be repacked would the auction allow repacking costs to affect the choice of which broadcasters are to be cleared. Possible variations include using repacking constraints to adjust relative clock speeds or to score bidders' offers. This would allow repacking costs to play a larger role, but repacking would still be done sequentially, possibly leading to suboptimal choices. The degree of suboptimality may depend heavily on the nature of the repacking constraints, about which the FCC has thus far released little information.

Further, the MALS clock auction does not avoid significant computational challenge. Formally, feasibility checking takes the form of a "graph coloring problem," which is known to be computationally hard in the worst case. Without knowing the fine details of repacking constraints, it is impossible to determine whether the feasibility checking required by the MALS proposal is itself computationally feasible.

The NPRM also entertains other types of reverse auction designs, where full optimization over bids and repacking costs might play a more prominent role. Few details are provided, making a careful analysis of such options difficult. However the discussion in the NPRM suggests the possibility of a tradeoff between computational complexity and optimality of alternative auction designs. Whether there is such a tradeoff—and where particular proposals lie on (or inside) the frontier—is unclear however, again because the NPRM is vague about the nature of the repacking constraints and imprecise about the amenability of this type of constrained optimization problem to solution using standard algorithms or specialized algorithms adapted to the features of the repacking problem.

Without details of the repacking constraints, it is impossible to evaluate the computational complexity of alternative auction designs, or to evaluate their performance in terms of expenditure, quantity of spectrum cleared, and efficiency.

Real-Time Feasibility Checking A potentially significant drawback of the proposed reverse clock auction is its requirement of real-time feasibility checking. As already noted, feasibility checking is likely be computationally challenging. Requiring that this be done in real time as the auction proceeds may be especially demanding. Because the feasibility of repacking must be checked for each bidder every time clock prices are to be decremented, there will be many opportunities for the auction to stall or even fail altogether, due to computational delay or failure. This may lead to the use of approximate solutions, harming efficiency.

Delay and Uncertainty Due to Target Failures. Because the FCC seeks to find the maximum quantity of spectrum that should be reallocated, the initial clearing targets will almost certainly be too high. It is likely that several clearing targets will need to be tried before reaching one that satisfies the closing conditions for the grand auction.

Under the current rules, each time the target is reduced, the forward auction must pause while reverse auction bidders are reconvened in order to obtain new "supply" prices for the reduced target quantities. In addition to the burden this creates for bidders, this feature of the auction design will delay completion of the grand auction. It may also introduce uncertainty for reverse auction bidders, since truthful bidding would require some broadcasters to take actions that commit them to assignments days or weeks ahead of other bidders.

3.3 Closing Rules

The MALS proposal recognizes that the forward and reverse auctions could stop at prices that lead to failure of the net revenue requirement even when some bidders would be willing to improve their offers enough for the revenue requirement to be met. Some inefficiency of this type is unavoidable, due to incentive compatibility constraints (Myerson and Satterthwaite (1983)). However, separation of the forward and reverse auctions leads to greater inefficiency than necessary. Although the purpose of the incentive auctions is to allocate scarce spectrum competitively, a clock auction in the forward market induces competition only between mobile wireless carriers. Forward auction bidders compete to be among the provisional winners, but do not compete directly with sellers for final ownership of spectrum. Likewise, a reverse clock auction creates competition only between current broadcast television licensees. Reverse auction bidders compete to be provisional sellers, but the two sides never compete directly to determine how much spectrum should be transferred from potential sellers to potential buyers. The result is that both the forward and reverse auction will tend to stop too soon, i.e., at prices that are unnecessarily low in the forward auction and unnecessarily high in the reverse auction.

This is recognized in the MALS proposal and underlies their suggestion that, when net revenue falls short, the forward auction would continue—prices would continue rising until either the revenue target is reached or total demand falls. This *closing trial* would allow provisional winners in the forward auction to express their willingness to offer better prices when necessary to ensure that they are allocated the licenses they seek. This is an important addition to the design. However there is no such provision for the supply side: reverse auction bidders lack the corresponding opportunity to improve their price offers to ensure that profitable sales take place. Even if there are reverse auction bidders who would like to offer prices substantially below those set by the reverse auction (recall that these prices are the maximum prices that clear the fixed target), those bidders will have no opportunity to express this. The result will be unnecessary inefficiency. Some trades that both buyers and sellers would like to make will not take place. The total quantity of spectrum shifted from broadcast television to mobile wireless will be inefficiently (and unnecessarily) low.

4 Key Proposals

We offer a number of specific proposals aimed at addressing the concerns described above, as well as others discussed in more detail below. Here we describe the most important of these proposals.

4.1 Forward Auction

Clock Package Auction To address several concerns, including those relating to exposure, horizontal contiguity, and the overflow problem, we propose a variation of the MALS forward auction in which prices are offered not only for EA licenses, but also for a pre-specified set of packages. For example, the "objects" offered in the auction might be licenses for each EA, for each Major Economic Area (MEA), for each Regional Economic Area Group (REAG), and for the entire United States. Within the fifty states there are 172 EAs, which lie in 48 MEAs, and 8 REAGs. Thus, relative to the original design, the number of objects for sale would expand only marginally.

As with the MALS proposal, clock prices would start low, with bidders expressing their demands at each set of prices. Prices rise on objects with excess demand until reaching a level at which demand no longer exceeds supply. When possible, prices of packages will equal the sum of the prices of the package components. However, when feasibility constraints and strong demand for packages necessitate, the package price may exceed the sum of component prices. This ensures that package bidders have no advantage or disadvantage relative to bidders seeking individual licenses and encourages straightforward bidding.

We call this proposed design the Clock Package Action ("CPA").[4] It departs only modestly from the original MALS design, but addresses several of our most important concerns. Important to our proposal is that packages are specified as consisting of horizontally frequency-contiguous spectrum.[5] By allowing bidders to make offers for horizontally contiguous packages, the CPA provides a way for bidders to express the substantial complementarities between individual licenses that may be dependent on horizontal frequency contiguity.

To the extent that the offered packages cover the packages of interest to bidders, the CPA also addresses the problem of bidder exposure arising from geographic complementarities. A bidder interested in a package can bid on the package without risk: if the price climbs to a level that exceeds his valuation of the package, he will be free to stop demanding the package and redirect his interest elsewhere. The CPA also avoids the overflow problem by enriching the bidding language in a way that permits accounting for all feasibility constraints when calculating excess demand. Although there is no strict dominance relation between the CPA and the MALS forward auction design in terms of efficiency or revenue, a simulation study suggests that substantial efficiency and revenue gains from using the CPA may be likely.

Re-Offer Unsold Licenses Whatever the design of the forward auction, there is a significant possibility that some cleared licenses will remain unsold. MALS refers to such unsold licenses as "residue." We give several examples in the text below. An important question is

[4] This is distinct from what has sometimes been called the "Package Clock Auction," a version of what we call "Combinatorial Clock Auction" below.

[5] We discuss possible relaxations in the body of our report.

whether unsold licenses should be re-offered by the FCC. This question is less simple than it may initially seem. On one hand, unused licenses are an obvious source of inefficiency, and selling them also generates additional revenues. On the other hand, anticipation of an opportunity to bid for unsold licenses could distort bidding behavior in the primary auction.

Short of using a VCG mechanism for the forward auction (which we do not recommend), we see no way to completely avoid these opposing concerns. However, we believe the scope for distortion of incentives will be limited when only unsold licenses are to be re-offered after the primary clock auction. Incentives for manipulation may be further reduced if the decision whether to re-offer licenses is left to the discretion of the FCC. We therefore recommend that the FCC preserve the option to re-offer unsold licenses. Revenues generated by such sale should be included in the determination of closing conditions.

We recommend use of another clock auction if unsold licenses are re-offered. As long as the Clock Package Auction is used in the primary auction, we believe this will adequately address issues of exposure risk and needs for frequency contiguity. We therefore believe that the supplemental auction could be implemented using the original MALS forward clock auction design.

4.2 Reverse Auction

Single-Pass Reverse Auction An alternative to the sequence of reverse auctions (at declining targets) proposed by MALS is reverse auction conducted in a single pass. The simple idea is that instead of stopping the auction when prices clearing the first provisional clearing target are reached, the auction would continue and trace out the "supply curve" over the range of potential clearing targets.

In our baseline single-pass proposal, the mechanics of the reverse auction would be identical to that of a single reverse auction in the original design. Suppose that the FCC believes that the feasible range of cleared channels per market is between 8 and 16. The auction would begin at prices sufficient to ensure that 16 channels were cleared, and continue until only 8 licenses per market were being offered by current TV licensees. The FCC would record bidders' choices at every round of the auction, including intra-round bids, for later use. The interaction between forward and reverse auction would be exactly as originally proposed by MALS. The forward auction would begin with an attempt to satisfy the closing conditions for a quantity Q_1 (say, $16 \times 6\text{MHz} = 96\text{MHz}$ in every market, less whatever is set aside for guard bands or other use). Revenues from the forward auction would be compared to the total costs of clearing Q_1, obtained from the reverse auction. If the net revenues fell short, the target would be reduced to Q_2. However, rather than reconvening the reverse auction to establish the (lower) prices for clearing target Q_2, these new prices would simply be taken from the supply curve obtained from the single-pass reverse auction.

From a bidder's perspective, the single-pass auction is nearly identical to one of the reverse auctions under the MALS proposal. Just as with the MALS proposal, a bidder would need to come prepared with one number (a reservation value) for each possible relinquishment option. A bidder's options during the auction would be identical to those under the original proposal, and the rules determining prices paid to bidders for each relinquishment option would be no different. Just as in the original proposal, bidders' choices between relinquishment options would be binding offers.

The single pass option offers several advantages. One is elimination of the need for a new reverse auction each time a clearing target fails. This would simplify participation in the reverse auction. It would also avoid delay: rather than pausing the forward auction to reconvene reverse auction bidders each time the clearing target is adjusted, the forward auction could proceed without interruption.

The single-pass design also simplifies participation of reverse auction bidders in a "two-sided closing trial" and would further allow the FCC to minimize the likelihood of "over-shooting" during a closing trial (see below). This could lead to more cleared spectrum.

Another advantage is that bidders would be assured that all broadcaster offers were chosen the same day; unlike the MALS design, here a bidder need not worry that the final offers of different TV stations might be made days or weeks apart.

A slight variation on our proposal is to conduct the single-pass auction using proxy bidding. This idea was suggested in the NPRM and might offer additional benefits. With proxy bidding, each bidder would report his minimum acceptable price for each relinquishment option to a proxy agent (software). The proxy agent would then bid on his behalf in the clock auction, selecting the profit maximizing option for the bidder at each new set of prices. This would be similar to the proxy bidding system on eBay. A potentially important advantage of proxy bidding is that verification of repacking feasibility (and any other underlying optimization subject to repacking constraints) could be performed offline rather than in real time. Depending on the complexity of the feasibility checking problem, offline feasibility checking may allow more complete solutions of this problem, assuring that maximal spectrum is cleared at the lowest possible cost.

Problem-Specific Constrained Optimization The discussion of computational and algorithmic issues in the NPRM lacks critical detail. The most general lesson one should take from research and practical experience with challenging constrained optimization problems is that good solutions are often problem-specific. It is difficult—often impossible—to prescribe efficient algorithms for solutions for very general classes of problems (e.g., integer linear programming problems). However, specific instances of such problems often have special structure that allow known algorithms to perform well, or that permit the design of new algorithms that perform well by tailoring to this special structure. In some cases this can be shown theoretically, while in others this can be demonstrated convincingly through simulation. Because details on the nature of the actual repacking constraints have not been provided, it is impossible to make specific recommendations, or to draw clear lines between which proposals under consideration present computational problems, and which do not. We recommend that such information be provided so that industry experts can assist in the evaluation of algorithmic issues and their interaction with the design of the reverse auction rules.

4.3 Closing Rules

Two-Sided Closing Trial We propose an alternative that departs modestly from the MALS "one-sided closing trial" proposal but offers potential for substantial improvement in efficiency. The main idea is that all competitors participate when a closing trial is triggered. This would allow all potential owners of spectrum rights to compete, expressing their willingness to improve upon the provisional prices in the forward and reverse clock auctions when necessary to ensure that their desired trades are consummated.

In the baseline version of our proposal, no change to the main forward or reverse auction is needed. However, if the forward auction reaches a point where excess demand is zero but the nationwide net revenue condition fails, all parties would be told that the clearing conditions have failed. Prices would then be raised in the forward auction while lowered in the reverse auction. For example, in a sequence of rounds, prices in the forward auction could rise by a percentage ρ while those in the reverse auction fall at the same rate. Bidders on each side would express demand/supply at the clock prices, just as in the original forward and reverse auctions. Prices would continue to adjust until either the net revenue condition is satisfied or prices reach a point at which demand falls short of supply.

The advantage of this approach is that it provides both forward and reverse auction bidders the opportunity to express their willingness to accept less favorable prices to ensure that their trades are executed. This can lead to more efficient allocation of spectrum and to outcomes that are preferred by participants in both the forward and reverse markets.

The use of the single-pass reverse auction would allow significant additional efficiency gains in a two-sided closing trial. This is because a remaining source of potential inefficiency is the miscoordination of the forward and reverse price clocks, which could lead to *overshooting* on one side of the market during a closing trial. The spirit of including reverse auction bidders in the closing trial is that both sides may have some "slack" in the sense that they are willing to accept somewhat less favorable prices if necessary. But it is impossible for the FCC to know which side (or more generally, which bidders) has more slack and, therefore, how quickly to adjust one price relative to another. This leads to the possibility that the clock overshoots valuations on one side before allowing bidders on the opposite side to express their full willingness to adjust the provisional prices. In some cases this could cause a two-sided closing trial to fail when a one-sided trial would have succeeded. With the single-pass reverse auction, much of this inefficiency can be avoided, since the FCC would know the prices at which the clocks would overshoot the valuations of reverse auction bidders. Overshooting on the supply side can be avoided completely, and further adjustments (discussed below) can minimize overshooting on the demand side as well.

5 Conclusion

The FCC and their advisors have developed an ambitious proposal making substantial progress toward design of the incentive auctions. We believe this proposal is an excellent starting point for discussion. Although we see several significant shortcomings, we believe these can be addressed through relatively modest alterations to the MALS design. We have sketched our concerns and proposals above. Below we provide a more extensive discussion of these ideas and others.

Part II

Forward Auction

The purpose of the forward auction is twofold. First, it provides a way for mobile wireless providers to reveal the value they place on spectrum currently allocated for broadcast television use. This is important for assessing how much spectrum has its highest value use in broadcast television vs. mobile wireless service. Second, the forward auction provides a method of determining allocations among mobile wireless providers, and the payments they will make to the FCC. Although the FCC has conducted auctions of spectrum in the past, those auctions served only the second purpose. The interaction between potential buyers (in the forward market) and potential sellers (in the reverse market) is new. In addition, the FCC and its experts seek to take advantage of experience in prior auctions and recent developments in market design.

6 The NPRM and MALS Proposals

6.1 Clock Auction

In the forward clock auction proposed by MALS, the FCC would simultaneously offer licenses in each EA. There will typically be more than one class of license available in each market; e.g., licenses for 10MHz of "paired" spectrum (5 MHz for wireless uplink, 5MHz for downlink), and licenses for 5MHz of unpaired (downlink only) spectrum.[6] Licenses for different combinations of EA and class are treated as distinct *objects* and will have different clocks. Multiple *units* of each object, corresponding to specific frequencies, will typically be

[6] The proposed building block for the forward auction licenses is a 5MHz license. This contrasts with the 6MHz licenses for broadcast television that will be obtained from the reverse auction.

available. However, all units of the same object are treated as generic. At the beginning of the auction, all objects are given low initial (reserve) prices by the FCC. Potential buyers respond to these prices by specifying the number of units of each object that they would wish to purchase at these prices. Prices for objects with excess demand (demand that exceeds the available supply) then rise, and bidders indicate their demands at these new prices. This process continues until the quantity of spectrum demanded in each market is no larger than the associated target quantity. Thus, the forward auction stops at prices just high enough to eliminate excess demand relative to the target quantities.

6.2 SMR Auction

As an alternative, the NPRM mentions the possibility of a Simultaneous Multiround (SMR) ascending auction. The SMR has been used in prior FCC spectrum auctions. In an SMR auction, the FCC would separately identify each license by geography and frequency (there is no generic treatment of licenses). Bidders make price bids on any subset of these specific licenses they desire. The auction proceeds in a sequence of rounds, with prices constrained to move only higher. After each round, the FCC declares a provisional winner for every license. Bidding for all licenses remains open as long as there is bidding activity on any license. The provisional winners become final winners only when bidding activity ceases on all licenses. This feature facilitates aggregation of licenses, but does so most effectively when the bidders regard alternative licenses as substitutes rather than complements.

Although the SMR has a number of virtues, it has a number of significant limitations as well. In an SMR auction bidders must choose which particular license to bid on and how much to raise the current standing bids. This introduces many possible strategies for achieving the same goal, as well as ambiguity in interpreting the behavior of competitors. This inhibits price discovery. Indeed, in early rounds bidders may intentionally "park" their demand on licenses they do not want in order to hide their intentions and inhibit price discovery. The SMR is also inadequate for guaranteeing efficient aggregation when bidders regard alternative licenses as complements, as will be the case whenever a bidder seeks to assemble a group of licenses covering a region larger than a single EA. Because the bidders make offers for specific frequency blocks in the SMR, a hold out problem can arise: a small bidder can easily win a license that makes it impossible for another bidder to obtain multiple licenses with frequency contiguity. Even worse, a speculator may target such a license for the purpose of reselling for a high price in the resale market (e.g., Pagnozzi (2010)). In this sense, SMR does little to guarantee efficient aggregation and vertical contiguity. Bidders may also attempt to signal or coordinate through bids. Further, the fact that provisional winning bids are identified in each round makes this auction format susceptible to collusion. By contrast, the proposed clock auction eliminates the problem of holdout and enables the FCC to guarantee within-region contiguity. The clock design appears to be less susceptible to collusion.

In what follows we focus on the clock auction proposed by MALS. We believe this is a superior starting point for the forward auction design.

7 Primary Concerns

7.1 Exposure Risk

Our most serious concern with the proposed forward auction design is its treatment of complementarities between licenses. Such complementarities will exist for any bidder who will wish to assemble coverage over regions larger than a single EA. This may be the vast majority of forward auction bidders. The clock auction design allows bidders to pursue packages of licenses, but only by offering to buy each individual license that is part of the desired package. This introduces a substantial exposure risk for bidders.

This risk is especially severe if, as the auction proceeds, each bidder is permitted to reduce his quantity demanded only for objects whose prices have risen. We believe this is the intended policy in the MALS proposal.[7] However, this rule creates severe exposure risk for bidders whose valuations exhibit complementarities. This point has been emphasized, for example, by Ausubel, Cramton, and Milgrom (2006). We view the proposed rules as creating unacceptable exposure risk to forward auction bidders. This will inhibit efficient aggregation of licenses and lead to conservative bidding, inefficient spectrum allocations, and low revenues. The revenue implications are especially problematic here, due to the role of the net revenue requirement. Unlike prior FCC auctions, issues of revenue and efficiency cannot be separated: low revenues will lead to inefficiently low quantities of spectrum cleared. Further, even ignoring the role of revenues, both large bidders and small bidders (those not themselves subject to exposure risk) can be harmed by the exposure problem. An example illustrates.

Example 1. *Suppose there are 2 licenses, A and B, and three bidders. Bidders' valuations*

$$\begin{aligned} v_1 &= (6, 0, 6) \\ v_2 &= (0, 10, 10) \\ v_3 &= (5, 5, 10 + \beta) \end{aligned}$$

where, for example, bidder 1*'s valuation vector* v_1 *indicates a value of* 6 *for license A alone,* 0 *for license B alone, and* 6 *for the pair. Bidder 3 places positive value on both license, and obtains a complementarity "bonus" of* $\beta > 0$ *if he obtains both, so that his valuation of the pair is* $5 + 5 + \beta$*. Begin with* $\beta = 5$*. In this case the efficient allocation gives license A*

[7] An alternative would be to permit a bidder to reduce his quantity demanded on any license as long as the price rises for some licenses he previously demanded. In the absence of concerns for frequency contiguity (see section 7.2 below), this alternative would avoid the exposure risk arising from geographic complementarities: a bidder interested in a package of licenses could exit the auction if that package becomes expensive, even if that is due to an increase in the price of only one component EA license. However this alternative rule would weaken incentives for straightforward bidding and could lead to substantial undersell.

to bidder 1 and license B to bidder 2. Suppose that bidders use ***straightforward bidding*** *strategies, i.e., at every set of clock prices, each bidder chooses the demand that would yield him the largest profit if the current clock prices turned out to be the final prices. In this case the auction would proceed as follows*[8]

p_A	p_B	*demand*
1	*1*	*A,B,A+B*
2	*2*	"
⋮	⋮	"
6	*6*	*0,B,A+B*
6	*7*	"
⋮	⋮	"
6	*10*	*0,0,A+B*

Note that once p_B reaches 9, bidder 3 would like to exit the auction altogether. However, because the auction rules forbid this (only the price of B is rising), the combination A+B is the most profitable demand he is permitted to express. The resulting outcome is bad from all bidders' perspectives: bidder 3 makes a loss; bidders 1 and 2, who should have obtained licenses, obtain nothing.

We might expect bidder 3 to avoid the exposure risk by bidding more conservatively. Suppose he follows a ***conservative straightforward bidding*** *strategy: he uses a straightforward bidding strategy except that he refuses to demand any good whose price exceeds his stand-alone valuation for the good. With such a strategy, the value of β becomes irrelevant to the auction*

[8]In our examples we assume arbitrarily small bid increments, denoted by ϵ when necessary. We show only key "rounds" of the clock auction, listing prices in that round and the demands expressed at those prices. The list of demands in each row of the "demand" column show the objects demanded, with commas separating the demands of each bidder. We write "A+B" to indicate demand for a unit of A and a unit of B.

outcome. The auction will proceeds as follows,

p_A	p_B	*demand*
1	*1*	*A,B,A+B*
2	*2*	"
⋮	⋮	"
5	*5*	*A,B,0*

Now bidder 3 avoids the exposure problem. Revenues are substantially lower, which may lead to failure of the net revenue requirement. Otherwise, whether the auction allocation is efficient depends on whether efficiency is helped or harmed by eliminating bidder 3 from competition. For small β *this is efficient; for* $\beta > 6$ *it is not.*

7.2 Frequency Contiguity

A second major concern is that the NPRM and MALS proposals do not make adequate provisions for bidders to obtain contiguous blocks of spectrum. Two types of frequency contiguity are important. First, within a geographic area, wireless service providers seeking to offer LTE or LTE-Advanced services desire significant blocks of spectrum in adjacent frequencies. We understand that such *vertical contiguity* is essential to optimal performance of these technologies. Second, bidders seeking coverage areas encompassing multiple EAs desire licenses in identical frequencies in each EA. We understand that such *horizontal contiguity* minimizes interference at boundaries and allows use of more efficient network deployment.

The current proposals for assigning individual licenses to winners of generic licenses in each EA may address the need for vertical contiguity within a single EA. But they would not allow bidders to express their broader needs for frequency contiguity.

The MALS proposal suggests the use of a random priority rule for assigning particular frequencies to winners of the clock auction. The NPRM mentions the possible use of a bidding mechanism for assignment. The currently envisioned version of the random priority rule is problematic if bidders care about horizontal contiguity. Assuming that the generic auction is held for each EA, a random priority rule would work as follows: 1) Each winner rank-orders all possible sets of frequency blocks that equal to the number of licenses he won in the clock auction. 2) The FCC randomly orders all the winners. 3) The winner first on the list is assigned his most preferred blocks; the second on the list is assigned his most preferred block of those remaining, and so on. This procedure will generate inefficient allocations when bidders value horizontal contiguity. This can be illustrated by a simple example.

Example 2. *Suppose there are two EAs, EA_1 and EA_2, and suppose each EA_i has three frequency blocks: A_i, B_i, C_i for $i = 1, 2$. Assume that in the reverse auction all three blocks are cleared in EA_1. In EA_2, only two of them are cleared. Suppose there are three bidders, 1, 2, 3 in the forward auction. Bidder 1 and bidder 2 each won 1 license in each EA, whereas bidder 3 won only one license in EA_1 in the forward generic auction. Suppose 1 and 2 rank: $A_i - B_i - C_i$ in that order in each EA_i, for $i = 1, 2$. And suppose 3 ranks: $C_1 - B_1 - A_1$. Then, bidder 3 gets C_1 in EA_1, but 1 and 2 get (A_1A_2, B_1B_2), (A_1B_2, B_1A_2), (B_1A_2, A_1B_2), (B_1B_2, A_1A_2) with equal probability, 1/4. If it is more valuable for a bidder to win the same frequency block across the EAs, this outcome is inefficient.*

Further, there is no dominant strategy for bidders in the random assignment phase. To illustrate, return to the example and suppose winning A_1A_2 is not much more valuable than winning B_1B_2. It is an equilibrium for 1 to rank $A_i - B_i - C_i$ in each EA $i = 1, 2$ and for 2 to rank $B_i - A_i - C_i$ in each EA_i, $i = 1, 2$. This leads to an efficient assignment of (A_1A_2, B_1B_2, C_1) for bidders 1, 2, and 3, with probability one. While this may suggest that bidders may be able to coordinate on an efficient outcome in equilibrium, in practice such coordination may be difficult to achieve. The lack of strategy-proofness is a drawback. Of course, if both 1 and 2 value A_1A_2 sufficiently more than B_1B_2, then the above inefficient allocation will be the unique equilibrium.

7.3 The Overflow Problem

The MALS forward auction provides no language for bidders to express whether their demand for a particular license is driven by interest in a larger group of licenses. Without such language, bidders seeking small groups of licenses can face rising prices even when their own demands are not a source of scarcity, leading to bias against small bidders and inefficient allocations.

An example will illustrate the problem. Consider the Boston MEA, which includes the Bangor, Portland, and Boston EAs. Suppose that six 10MHz paired licenses are available in Bangor and Portland, but only five in the Boston EA. This implies that at most five bidders could obtain MEA-wide coverage. Suppose that at some point in the auction "regional bidders" seeking MEA-wide coverage (or coverage in a larger area that includes the MEA) are demanding seven licenses, while there is only one Portland EA license demanded by a firm seeking coverage in Portland alone. Suppose the regional bidders have an all-or-nothing strategy in this MEA: they are not interested in any strict subset of the 3 EAs. In such a situation, we would like the auction to "hold on" to the offer of the Portland-only bidder: we know that only five units of demand from the regional bidders can be satisfied. So selling all licenses requires allocating to the Portland-only bidder. However, under the MALS design, the auction will count eight units of demand for the Portland license, conclude that this license is in excess demand, and raise its price. This will force the Portland-only bidder to either pay a higher price or exit, even though his demand is not a source of scarcity. This will tend to force such a bidder out of the market unnecessarily.

We term this phenomenon the *overflow problem*. With a richer bidding language, the auction design could account for feasibility constraints and limit the extent to which demand for large packages "flows down" to be counted as demand for the smaller packages and individual licenses nested in the larger package. Without such a language, too much demand flows down. This creates artificial competition for smaller bidders. This bias against smaller bidders will lead to undersell and/or inefficient allocations of licenses.

Within the simple clock auction design proposed by MALS, we see no easy fix. Indeed there is a tension between avoiding the inefficiency introduced by the exposure problem and avoiding those introduced by the overflow problem. The following example illustrates.

Example 3. *Suppose there are two markets, with two licenses available in market A but only one license in market B. Suppose there are six bidders with valuations for A, B, and the pair A+B given by*

$$
\begin{aligned}
v_1 &= (1,0,1) \\
v_2 &= (3,0,3) \\
v_3 &= (0,6,6) \\
v_4 &= (0,8,8) \\
v_5 &= (2,3,7) \\
v_6 &= (3,5,12).
\end{aligned}
$$

This type of configuration is highly plausible: markets in which less spectrum can be cleared will tend to be those where licenses are more valuable. The unequal clearing across markets limits the feasible supply of the "package" AB desired by bidders 5 and 6 to one unit. The efficient allocation involves assigning 1 unit of each object to bidder 6, and the remaining unit of object A to bidder 2.

Under the MALS proposal we may expect bidders 5 and 6 to bid cautiously to avoid the *exposure problem. If bidders use conservative straightforward bidding strategies, the auction will proceed as follows:*

p_A	p_B	*demand*
1	1	0,A,B,B,A+B,A+B
2	2	0,A,B,B,A+B,A+B
$2+\epsilon$	$2+\epsilon$	0,A,B,B,B,A+B
$2+\epsilon$	3	0,A,B,B,0,A+B
⋮	⋮	⋮
$2+\epsilon$	$5+\epsilon$	0,A,B,B,0,A
$2+\epsilon$	6	0,A,0,B,0,A

The allocation is inefficient. Further, the magnitude of this inefficiency can be made arbitrarily large by changing bidder 6's package valuation.

We might hope that package bidders would bid less cautiously, so that the implications of exposure risk are not so dire. If they do, this will introduce the overflow problem. Supposing that all bidders follow straightforward bidding strategies, the auction will proceed as follows:

p_A	p_B	*demand*
1	1	0,A,B,B,A+B,A+B
2	2	0,A,B,B,A+B,A+B
3	3	0,0,B,B,A+B,A+B
3	4	0,0,B,B,A,A+B
3	5	"
3	6	0,0,0,B,A,A+B
3	7	"
3	8	0,0,0,0,A,A+B

Revenue is higher but the allocation is still inefficient. This time the inefficiency is due to the overflow problem. Although only one A+B pair is feasible, bids by the package bidders drive bidder 2 out of the market, leading to misallocation of one of the market A licenses. This is undesirable, but the MALS auction design does not allow for any distinction between demand for A alone and demand for A as part of the AB package. In addition to the misallocation, bidder 5 falls prey to the exposure risk he ignored and is forced to pay more than his valuation for the license he wins.

7.4 Other Sources of Inefficiency

Nonexistence of Market Clearing Prices

As pointed out by Ausubel and Milgrom (2002), market clearing prices need not exist when some licenses are complements. Suppose for example that there are two licenses (A and B) available and two bidders with valuations

$$\begin{aligned} v_1 &= (4, 5, 15) \\ v_2 &= (7, 9, 9)\,. \end{aligned}$$

Here bidder 1 obtains a significant complementarity when obtaining both licenses, whereas bidder 2 does not. The efficient allocation gives both licenses to bidder 1. For a pair of prices (p_A, p_B) to yield no demand from bidder 2 requires $p_A \geq 7$, $p_B \geq 9$. But at such prices bidder 1 demands no licenses. In the MALS auction, with conservative straightforward bidding, the auction will proceed as follows:[9]

p_A	p_B	demand
1	1	A+B,B
1	1+ϵ	A+B,B
⋮	⋮	⋮
1	3	A+B,A
1+ϵ	3	A+B,B
1+ϵ	3+ϵ	A+B,A
⋮	⋮	"
3	5	A+B,A
3+ϵ	5	A+B,B
3+ϵ	5+ϵ	A,A
3+2ϵ	5+ϵ	A,B

The allocation is inefficient, and revenues are low relative to the surplus available. With straightforward bidding, the allocation would be worse, with one of the two licenses failing to sell.

We emphasize that the potential lack of market clearing prices is a limitation not just of a clock auction, but of any mechanism in which the same prices are offered to all potential buyers. Anonymity of prices is an attractive feature and may be essential if bidders are to view the auction as fair. However, this feature may lead to some inefficiency, sometimes in the form of undersell.

[9]We assume bidders resolve indifferences between licenses by choosing A. This has no effect on the final allocation and an arbitrarily small effect on revenue.

Miscoordination of Price Clocks

Synchronization of price clocks can be important in any clock auction. A simple example illustrates.

Example 4. *Suppose there are two licenses available, A and B. There three bidders with valuations*

$$\begin{aligned} v_1 &= (5,0,5) \\ v_2 &= (0,9,9) \\ v_3 &= (3,6,12) \end{aligned}$$

where, for example, bidder 3 places value 3 on license A alone, 6 on license B alone, and 12 on the "package" AB. Here the efficient allocation excludes bidder 3. Depending on the synchronization of clocks, the MALS auction could proceed as follows under straightforward bidding:

p_A	p_B	*demand*
1	*1*	*A,B,A+B*
⋮	⋮	"
5	*5*	*0,B,A+B*
5	⋮	*0,B,A+B*
5	*7*	*0,B,A+B*
5	*8*	*0,B,A+B*
5	*9*	*0,B,A*

The allocation is inefficient (and bidder 3 faces a loss).[10]

[10] Note that bidder 3 would like to exit the auction entirely at the prices (5,7) but is not permitted to, since only p_B is rising at this point. The most profitable option available to bidder 3 at this point is to continue demanding A+B.

However, if the clocks moved at different speeds (that for B moving twice as fast), efficiency could be achieved with straightforward bidding:

p_A	p_B	*demand*
1	*2*	*A,B,A+B*
2	*4*	"
3	*6*	"
4	*8*	*A,B,0*

This gives the efficient allocation.[11]

Of course, optimal coordination of clocks depends on the realized valuations and the strategies used by bidders. Perfect synchronization is therefore infeasible. Inefficiency may arise no matter how the price clocks are controlled. However, there may be general principles for synchronization of clocks that can enhance efficiency. For example, clock speeds might be calibrated based on expected final prices, perhaps based on prices in prior FCC spectrum auctions. This issue should be investigated further.

8 Clock Package Auction

MALS mentions the possibility of adding packages to the forward auction. Two ideas are suggested: holding a full Combinatorial Clock Auction ("CCA") as explored in Ausubel, Cramton, and Milgrom (2006), or adding a limited number of packages to the clock auction. We prefer the latter.

The key elements of the CCA are a clock phase used for price discovery, followed by a sealed bid phase that determines winners. We refer readers to Ausubel, Cramton, and

[11] With conservative straightforward bidding, both clock speeds yield the efficient allocation.

Milgrom (2006) and Cramton (2012) for details. Although the CCA has a number of attractive features, it has several others we view as problematic for the FCC forward auction. One concern is the fact that all licenses remain "up for grabs" at the end of the clock auction, with winners not determined until after a sealed bid phase. A second is that the "VCG-nearest core" pricing rule used for the critical sealed-bid phase is opaque, making it difficult for bidders to understand the strategic environment they face. Finally, we are uncertain what effect the sealed bid phase might have on bidding behavior in the clock phase of the auction, particularly given the substantial complementarities between licenses. In the worst case this might lead to poor price discovery, substantial uncertainty for bidders, low revenues, and inefficiently low quantities of spectrum cleared.

We believe that substantial gains can be obtained following the simpler option of adding packages to the clock auction. Here we provide a detailed proposal for doing so, using a simple hierarchical structure to define the set of packages offered. We propose offering packages that cover each Major Economic Area (MEA), each Regional Economic Area Group (REAG), as well as the entire nation. This particular choice is not essential, and our formal description below incorporates the possibility of other "multi-tree" structures. But the EA-MEA-REAG-nationwide structure is natural, and we believe it would capture most of the significant geographical complementarities between licenses.

The introduction of packages to the clock auction is motivated by the issues of exposure risk, need for horizontal contiguity, and the overflow problem, all discussed above. Each is a problem that, if left unaddressed, could lead to inefficient allocations of cleared spectrum, low revenue, and inefficient reduction in the quantity of spectrum traded in the combined forward and reverse auctions. The inclusion of packages in the forward clock auction can address all three issues.

Before proceeding with the details, we emphasize that our proposal

- involves a very modest change to the original proposal;
- does not introduce significant complexity to the auction from the perspective of bidders or the FCC;
- does not set aside any spectrum for packages;
- does not require equalized clearing of spectrum across markets;
- results in no thinning of competition for any license;
- creates no advantage (or disadvantage) for package bidders;
- will not lead to violation of the property that identical licenses sell for identical prices, except when feasibility constraints and excess demand for packages requires superadditive package prices;
- does not appear to introduce new opportunities for undesirable strategic bidding behavior.

8.1 Objects for Sale

As with the original MALS clock auction, the auction is a multi-*unit* multi-*object* auction. An *object* is class of generic license covering a particular geographic area. Typically there will be more than one license of each such class, i.e., more than one *unit* of each object.

For simplicity we assume here that all licenses covering a particular geography are generic. Our proposal easily extends to allow multiple mutually exclusive classes of licenses (e.g., paired vs. unpaired) that are generic within each class. Objects are then defined by their geographic coverage. The elementary geographic building block is the EA. A package object is a collection of single-EA objects; for example, an MEA license for Philadelphia consists of one EA license in each of the three EAs comprising this MEA .

Let $\mathcal{E}$ denote the set of EAs and $\mathcal{S}$ the set of packages offered. Then

$$\mathcal{O} = \mathcal{E} \cup \mathcal{S}$$

represents the set of objects for sale. Complexity considerations dictate that $\mathcal{O}$ not include all possible subsets of $\mathcal{E}$. Rather, $\mathcal{S}$ should include the packages of greatest importance to bidders. We will assume that $\mathcal{S}$ is chosen so that $\mathcal{O}$ can be placed in a hierarchical structure defined below.

A notion of (sufficient) horizontal contiguity is defined in advance. All objects are specified as consisting of horizontally contiguous allocations of spectrum.[12] No spectrum is set aside for package licenses. Competition in the auction (and feasibility constraints) will determine how much spectrum, if any, is to be allocated to package licenses.

8.2 Hierarchical Structure

For objects k and j in $\mathcal{O}$, we write $k \subset j$ (equivalently, $j \supset k$) if the set of EAs included in object k is a strict subset of those included in j. When $k \subset j$ we will say that j *contains* k. We will say that object j is a *parent* of object k (equivalently, k is a *child* of j) if both (i) $k \subset j$ and (ii) $\nexists \ell \in \mathcal{O}$ such that $k \subset \ell \subset j$. An object may have no parent, one parent, or multiple parents. Let π_j denote the set of j's parents and let χ_j denote the set of j's children.

We represent the parent-child relationships with a directed graph Γ containing one vertex for each object $j \in \mathcal{O}$. For $j, k \in \mathcal{O}$, Γ has a directed edge from j to k if j is a parent of k.

[12] Alternatively, smaller packages (say, MEAs and REAGs) could be specified with a guarantee of horizontal contiguity, with larger packages (say, a national package) coming only with the promise of contiguity when possible.

Figure 1 illustrates with a simple example in which seven objects are for sale: one for each of four EAs, one for each of two MEAs, and one for the "nation" as a whole.

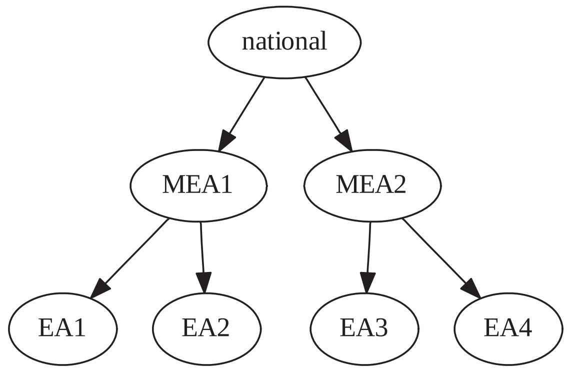

Figure 1: A simple multitree structure.

Because the parent-child relationship is defined based on strict subsets, there can be no directed cycles in Γ. We will further require that Γ be a *multitree,* i.e., that there be at most one undirected path between any two vertices in Γ.[13] The structure in Figure 1 satisfies this restriction. Another example is given in Figure 2, where Regional Economic Area Groups (REAGs) are included as well. Both examples generate a *tree* structure for Γ, which is a special case of a multitree. Another possible multitree structure is one identical

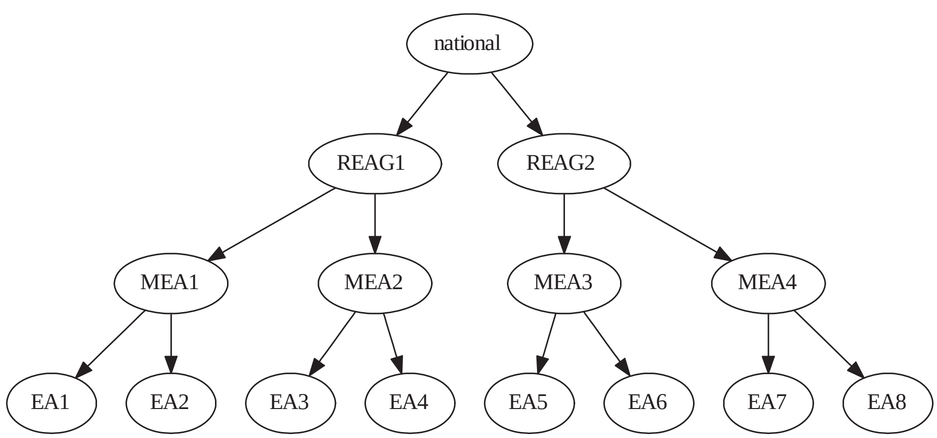

Figure 2: A larger multitree structure.

[13]Equivalently, the set of vertices reachable from any given vertex forms a tree.

to that in Figure 2, but without the national license. For the forward auction, the multitree structure is likely to cover the types of packages needed. As shown below, a key feature of the multitree structure is a lack of ambiguity about which objects are in excess demand.[14]

Each object can be assigned to a unique *tier* in Γ. This assignment can be constructed iteratively as follows: objects which are single-EA licenses are assigned to tier 1; parents of tier-1 objects are assigned to tier 2; parents of tier-2 objects to tier 3; etc. Let $\tau^{\max}$ denote the maximum non-empty tier. In the prior examples the assignments to tiers are simple: for example, tier 1 corresponds to the EA objects, with tier $\tau^{\max}$ corresponding to the national object.

8.3 Overview of the Clock Package Auction

The auction includes price clocks for each object. Initial (reserve) prices $p_k^{\min}$ are set for each EA license. Reserve prices for packages are additive:

$$p_j^{\min} = \sum_{\substack{k \in \mathcal{E}: \\ k \subset j}} p_k^{\min}. \tag{1}$$

The auction begins at these prices and proceeds through multiple rounds with progressively higher prices. In each round, bidders express nonnegative integer quantities demanded for each object at the current clock prices. Bidders' expressed demands are binding offers. A bidder expressing positive demand for multiple objects is agreeing to purchase all of those objects if the auction ends at the current prices. Prices continue to rise until there is no excess demand (defined below) for any object, at which point the auction terminates. As

[14] It may be useful to consider an example of a structure that violates the multitree restriction. Let there be three EAs: A,B, and C, and suppose the packages offered are AB,BC, and ABC. The graph of this structure is not a multitree because there are two directed paths from ABC to B. This could be accommodated (imperfectly) in our design by defining only one of the packages AB or BC to be a child of ABC, yielding a modified graph that is a multitree. The choice of the child would be arbitrary and could affect the outcome of the auction. Further, the CPA avoids the overflow problem entirely when Γ is a multitree, but only partially when Γ must be modified to obtain a multitree.

suggested in the MALS proposal, intra-round bidding can be used to minimize overshooting due to discrete price increments.

8.4 Feasible Supply and Feasible Allocations

For each $j \in \mathcal{O}$, let q_j^f denote the *feasible supply* of object j. This is the maximum number of units of the object that could be allocated. For objects consisting of a single EA license, this is just the number of available licenses in that EA. Because every package object is a collection of smaller objects, the feasible supply of a package object will be limited by the minimum feasible supply among its components (i.e., its children). In particular, for any object j such that $\chi_j \neq \emptyset$,

$$q_j^f \leq \min_{k \in \chi_j} q_k^f.$$

With package licenses specified as horizontally contiguous, the feasibility of horizontal contiguity might further constrain the feasible supply, yielding a strict inequality.

The set of quantities $\{q_j\}_{j \in \mathcal{O}}$ is a *feasible allocation* if for all $k \in \mathcal{O}$,

$$q_k + \sum_{\ell \supset k} q_\ell \leq q_k^f. \tag{2}$$

Typically the set $\left\{q_j^f\right\}_{j \in \mathcal{O}}$ will not be a feasible allocation.

Example 5. *Suppose there are 4 EAs, labeled A,B, C, and D. Let $\mathcal{O}$ be comprised of the singleton EA objects, two regional objects AB, CD, and a national object ABCD. Suppose sufficient spectrum has been cleared to allow allocation of up to 3 licenses in EAs A,B, and*

C but only 2 licenses in EA D. If there are no other feasibility constraints, we have

$$\begin{aligned} q^f_A &= 3 \\ q^f_B &= 3 \\ q^f_C &= 3 \\ q^f_D &= 2 \\ q^f_{AB} &= 3 \\ q^f_{CD} &= 2 \\ q^f_{ABCD} &= 2 \end{aligned}$$

An example of a feasible allocation is

$$\begin{aligned} q_A &= 1 \\ q_B &= 1 \\ q_C &= 2 \\ q_D &= 1 \\ q_{AB} &= 1 \\ q_{CD} &= 0 \\ q_{ABCD} &= 1. \end{aligned}$$

This feasible allocation also utilizes all the available licenses.

To avoid incentives for manipulation, we require that the expressed demand of a bidder at any price be a feasible allocation.[15] This constraint is easily imposed: if quantities$\{q^d_{ij}\}_{j\in\mathcal{O}}$

[15]Demands for infeasible allocations cannot be filled. Thus, attempts to distort the outcome of the auction with such bids have no cost. The same concern would arise in the MALS auction, where ruling out demands for infeasible allocations is also simple.

are demanded by a bidder i, the auction interface can verify that

$$q_{ij}^d + \sum_{k \supset j} q_{ik}^d \leq q_j^f \quad \forall j \tag{3}$$

before entering the bid, asking the bidder for a corrected demand vector if (3) is violated.

8.5 Total Demand, Excess Demand

Critical to the forward clock auction is the determination of which objects are in excess demand at a given set of prices. Without packages this is simple: the total number of units of an object demanded is compared to the total number of units available. With packages, we must account for the fact that demand for a package implies demand for the objects contained in the package. With the multitree structure this is straightforward. Further, because bidders express demand for package objects separately from demand for smaller objects, we can now avoid the overflow problem (see section 7.3).

Begin by defining the *first-order demand* for object j to be the quantity demanded of the object alone at the current prices. For example, first-order demand for an EA license is the quantity of this license demanded alone (not as part of a package object). Let $d_j^1(p)$ denote the first-order demand for object j.

Object j's *second-order demand,* denoted $d_j^2(p)$, and *total demand*, denoted $d_j^T(p)$ are then defined iteratively. Second-order demand for an object j represents the demand for j embedded in the feasible demand for objects that include j. Total demand for j is simply the sum of the first-order and second-order demand. The iterative definition starts from the top (tier $\tau^{\max}$) of the graph Γ and proceeds down. For each object j in tier $\tau^{\max}$ let

$d_j^T(p) = d_j^1(p)$. We then proceed to tier $\tau^{\max} - 1$. For each object j in this tier, let

$$d_j^2(p) = \sum_{k \in \pi_j} \min \left\{ q_k^f, d_k^T(p) \right\} \quad (4)$$

$$d_j^T(p) = d_j^1(p) + d_j^2(p). \quad (5)$$

We take the minimum of q_k^f and $d_k^T(p)$ in (4) to account for the fact that in some cases not all demand for a parent can flow to its children, due to feasibility constraints. For example, in Example 5 above, at most two units of demand for the package CD would ever be counted as demand for C.[16] This process continues through tier 1, applying equations (4) and (5) at each tier.

Finally, object j has *excess demand* at prices p if total demand for j exceeds its feasible supply, i.e., if

$$d_j^T(p) > q_j^f.$$

8.6 Price Clocks

If no object has excess demand at the current clock prices, the clock auction terminates. Otherwise prices rise for all objects that (i) have excess demand or (ii) contain objects with excess demand. For specificity only, we assume bid increments are specified as a constant percentage increase.

Whereas the determination of total demand works from the top down in the graph Γ, determination of new clock prices works from the bottom up. Let p_j denote the current price of an object j; the new price p'_j is to be determined. For each tier-1 object j such that $d_j^T(p) > q_j^f$, the price will be incremented to $p'_j = (1+\delta)\, p_j$. Moving then to tier 2, the price of an object j in this tier will be incremented if either

[16] One way to represent this is to assign capacities to the edges in the graph Γ. The edge from j to any child $k \in \chi_j$ would have capacity q_j^f. The maximum demand that can flow down from parent to child is determined by the capacity of the relevant edge.

(a) the price of some object $k \in \chi_j$ has already been incremented; or

(b) $d_j^T(p) > q_j^f$.

If condition (a) holds,

$$p'_j = \max\left\{p_j, \sum_{k\in\chi_j} p'_k\right\}$$

regardless of whether condition (b) also holds.[17] If only condition (b) applies—i.e., if $p'_k = p_k$ for all $k \in \chi_j$, but there is excess demand for j—the new price is $p'_j = (1+\delta)\,p_j$. Only in this latter case is the price of a package *not* additive in the prices of its components (children).

This process continues through each of the higher tiers until all price adjustments have been determined. The new prices are then offered to bidders in the next round. As in the MALS proposal, bidders would be allowed to reduce the quantity they demand only for objects whose prices have risen.

8.7 Examples

Example 6 (Elimination of Exposure Risk). *Our first example illustrates the elimination of the exposure problem when the clock package auction is used and $\mathcal{O}$ includes all packages relevant to bidders. Let there be two markets, A and B, with two identical licenses*

[17] Alternatively, the price increment could be made larger if both conditions hold. Whether this aids efficiency is ambiguous in general under straightforward bidding (recall our discussion of clock synchronization). However, we believe that minimizing deviations from additive package price is desirable for encouraging straightforward bidding, as a bidder will know that he cannot slow down the rise in a package price by withholding demand for the package until later rounds and bidding instead on its components in early rounds.

available in each. There are five bidders with valuations

$$
\begin{aligned}
v_1 &= (5, 0, 5) \\
v_2 &= (4, 0, 4) \\
v_3 &= (0, 5, 5) \\
v_4 &= (0, 4, 4) \\
v_5 &= (3, 3, 11).
\end{aligned}
$$

Here the CPA proceeds under straightforward bidding as follows:

p_A	p_B	p_{AB}	*demand*
1	*1*	*2*	*A,A,B,B,AB*
2	*2*	*4*	"
3	*3*	*6*	"
4	*4*	*8*	*A,0,B,0,AB*

This yields the efficient allocation. Compare this to what may arise without including packages in the auction. Suppose the clock auction proposed by MALS were held. If bidder 4 avoids the exposure problem by adopting a conservative straightforward bidding strategy, that auction would proceed as follows:

p_A	p_B	*demand*
1	*1*	*A,A,B,B,A+B*
2	*2*	"
$\vdots$	$\vdots$	"
3+ϵ	*3+*ϵ	*A,A,B,B,0*

All units sell, but the exposure problem leads to inefficiency and low revenue: total revenue is 25 percent lower and total surplus (ignoring the potential adverse effect of revenue losses on closing conditions) is 14 percent lower.

Example 7 (Elimination of the Overflow Problem). *Return now to Example 3 (page 29). Previously we saw that the MALS auction design leads to inefficiency and/or undersell and there was a tension between addressing the needs of large bidders (facing the exposure problem) and those of small bidders (facing the overflow problem). Adding package clocks to the auction eliminates both problems. With the same setup considered in Example 3, the CPA would proceed as follows:*

p_A	p_B	p_{AB}	*demand*
1	1	2	0,A,B,B,AB,AB
1	2	3	"
⋮	⋮	⋮	"
1	5+ϵ	6+ϵ	0,A,B,B,A,AB
1+ϵ	5+2ϵ	6+3ϵ	"
2-ϵ	6	8-ϵ	0,A,0,B,A,AB
2	6+ϵ	8+ϵ	0,A,0,B,0,AB
2	7	9	"
2	8	10	0,A,0,0,0,AB

This yields the efficient allocation. Notice that the price of object A pauses once the sum of its first-order demand and the feasible second-order demand falls to 2 units (in the first *round displayed). At this point*

$$\begin{aligned} d_A^T &= d_A^1 + \min\left\{q_{AB}^f, d_{AB}^T\right\} \\ &= 1 + \min\{1, 2\} \\ &= 2 \\ &= q_A^f. \end{aligned}$$

The total demand equals the feasible supply; thus, there is no excess demand for A. This illustrates how the CPA allows the excess demand calculation to distinguish between demand for A alone and demand for A that is embedded in demand for a package. This distinction is impossible in the simple (MALS) clock auction, but is essential if we are to obtain efficiency. Stopping the price at 1 allows the auction to "hold on" to the demand of bidder 2 while still raising the price to the package bidders. Of course, if one of the package bidders later drops its demand for the package and instead demands a unit of A, p_A *will rise again. This occurs in the example when* p_B *reaches 5+*ϵ *and bidder 5 finds license A more profitable than the package.*

Example 8 (Superadditive Package Prices). *Let there be two markets, A and B, with two identical licenses available in each. However, interference constraints limit the number* *of feasible horizontally contiguous AB packages to 1. There are six bidders with valuations*

$$\begin{aligned} v_1 &= (5,0,5) \\ v_2 &= (6,0,6) \\ v_3 &= (0,5,5) \\ v_4 &= (0,6,6) \\ v_5 &= (3,3,11) \\ v_6 &= (4,4,14)\,. \end{aligned}$$

Complementarities in the two package bidders' valuations are contingent on horizontal contiguity. With straightforward bidding, the CPA would proceed as follows:

p_A	p_B	p_{AB}	*demand*
1	*1*	*2*	*A,A,B,B,AB,AB*
⋮	⋮	⋮	"
5	*5*	*10*	*0,A,0,B,AB,AB*
5	*5*	*11*	*0,A,0,B,0,AB*

The price clocks for the single-market licenses stop at 5, even though three units of each are demanded. This is because only 1 of the 2 package units demanded can be part of a final allocation. For example

$$\begin{aligned} d_A^T &= d_A^1 + \min\left\{q_{AB}^f, d_{AB}^T\right\} \\ &= 1 + \min\{1,2\} \\ &= 2 = q_A^f. \end{aligned}$$

However, there is excess demand for the package at this point, so the package price p_{AB} continues to rise. This yields a package price that exceeds the sum of the component prices p_A and p_B. Note that the degree of superadditivity, the total revenues, and the total surplus could be made arbitrarily high in this example by driving up the package valuations of bidders 5 and 6. With the original MALS proposal and conservative straightforward bidding, the exposure problem (this time the exposure of obtaining packages that lack horizontal contiguity) leads to a worse outcome

p_A	p_B	*demand*
1	1	A,A,B,B,A+B,A+B
⋮	⋮	"
3+ϵ	3+ϵ	A,A,B,B,0,A+B
4+ϵ	4+ϵ	A,A,B,B,0,0

Revenue is nearly 25 percent lower, while total surplus is 15 percent lower. Both losses can be made arbitrarily large by driving up the complementarities in the two package bidders' valuations.

8.8 Properties of the CPA

Success

The CPA is guaranteed to "succeed" in the sense that it will terminate at a feasible allocation. We show this with the following two results.

Theorem 1. *If the clock package auction terminates at price vector p^*, the implied allocation $\left\{d_j^1(p^*)\right\}_{j\in\mathcal{O}}$ is feasible.*

Proof. Since the auction terminates when there is no excess demand

$$d_j^T(p^*) \leq q_j^f \quad \forall j \in \mathcal{O}. \tag{6}$$

Recall that for all objects j in tier $\tau^{\max}$, $d_j^T(p^*) = d_j^1(p^*)$. Now consider an object j in tier-($\tau^{\max} - 1$). Any object $k \supset j$ must be in a higher tier. So by (4), (5), and (6), $d_j^T(p^*) = d_j^1(p^*) + \sum_{k \supset j} d_k^1(p^*)$. Continuing this iteration through tier 1, we obtain

$$d_j^T(p^*) = d_j^1(p^*) + \sum_{k \supset j} d_k^1(p^*) \quad \forall j \in \mathcal{O}.$$

By (6) this implies

$$d_j^1(p^*) + \sum_{k \supset j} d_k^1(p^*) \leq q_j^f \quad \forall j \in \mathcal{O}$$

i.e., $\left\{d_j^1(p^*)\right\}_{j \in \mathcal{O}}$ is a feasible allocation. □

Theorem 2. *Suppose that (i) the bid increment is bounded from below by* $\delta > 0$*, (ii) bidder valuations are finite, and (iii) there exists* $L < \infty$ *such that no bidder expresses demand which, if accepted, would yield the bidder a loss greater than* L*. Then the clock package auction will terminate in a finite number of rounds.*

Proof. Suppose to the contrary that for every round $r = 1, 2, \ldots, \infty$ there is excess demand. This requires that there be at least one object j that is in is excess demand in an infinite number of rounds. Because the bid increment is bounded from below by δ, the price of j (and any object containing j) must diverge to infinity. Because valuations are finite, this requires that there be some bidder demanding units of j (or an object containing j) who would face an arbitrarily large loss if his demand were accepted. This contradicts part (iii) of the hypothesis. □

No Exposure Problem

As long as the set $\mathcal{S}$ includes all of the types of packages desired by bidders (i.e., all those involving complementarities or the need for horizontal contiguity), the exposure problem is absent.

No Overflow Problem

As illustrated in the examples, the overflow problem and its bias against small bidders is eliminated. A bidder's price rises only when his own demand is a source of scarcity.

Simplicity for the FCC

The form and complexity of the proposed clock auction is essentially the same as that in the MALS proposal. We have added a relatively small number of package licenses to the auction: if there are K EAs and L packages, the number of objects in the auction is $K + L$ vs. K in the original proposal. Operation of the auction involves only the calculation of excess demand and adjustment of price clocks. Our description of these calculations translate directly to an algorithm for controlling the price clocks. The complexity of this algorithm is linear in the total number of objects for sale.

Simplicity for Bidders

The addition of packages to the ascending clock auction leaves the fundamental decisions for bidders unchanged. As in the original MALS proposal, a bidder merely indicates its most preferred purchases at each round of prices. The price of a package license is never less than the sum of prices for each component EA license. This keeps bidders' decisions simple (there are no tempting arbitrage opportunities that involve exposure risk) and aids price discovery. With the restriction to bids that are themselves feasible allocations, we see no new opportunities for strategic bidding behavior created by the addition of packages to the clock auction.

Fairness

The clock package auction as we have described it creates no advantages or disadvantages for large bidders (e.g., those bidding for packages) or small bidders (e.g., those bidding for individual licenses). The allocation of spectrum to package vs. single licenses is not preset but is determined by prices. A package bidder has no advantage because it must always pay a price at least as large as the sum of the component prices for the elements of its package. It is not disadvantaged either: although package prices may be superadditive, this happens only when this is necessitated by excess demand for the package itself. Similarly, the elimination of the overflow problem removes a bias against small bidders that is present in the MALS design.

Efficiency and Revenue

The discussion and examples above reveal that adding packages to the clock auction can eliminate some sources of inefficiency in the clock auction proposed by MALS, and can enhance revenues as well. However, there is no strict dominance relation: one can construct examples in which adding package clocks may reduce efficiency, depending on how clocks are synchronized and how package bidders handle the exposure problem in the auction without packages.

Example 9. *Suppose there are two licenses (A and B) and three bidders with valuations*

$$\begin{aligned} v_1 &= (3,0,3) \\ v_2 &= (0,6,6) \\ v_3 &= (2,2,7). \end{aligned}$$

The efficient allocation gives license A to bidder 1 and license B to bidder 2. In the MALS auction, conservative straightforward bidding will yield the efficient allocation, proceeding as follows:

p_A	p_B	*demand*
1	*1*	*A,B,A+B*
2+ϵ	*2+ϵ*	*A,B,0*

In the CPA, under straightforward bidding the auction may proceed as follows:

p_A	p_B	p_{AB}	*demand*
1	*1*	*2*	*A,B,AB*
2	*2*	*4*	*"*
3	*3*	*6*	*0,B,AB*
3	*4*	*7*	*0,B,0*

Although revenue is the same, good A goes unsold.[18] *This undersell can be seen as arising from miscoordinated price clocks (recall section 7.4). For example, if the clock for license B moved at twice the speed of that for license A, the CPA would instead proceed as follows:*

p_A	p_B	p_{AB}	*demand*
1	*2*	*3*	*A,B,AB*
2	*4*	*5*	*"*
$\frac{7}{3}$	$\frac{14}{3}$	*7*	*A,B,0*

The allocation is now efficient. Further, revenues are now 75 percent higher than in the MALS auction.

To provide some comparisons of outcomes under alternative auction rules we have run

[18] To obtain strictly higher revenue from the MALS auction, modify the example so that $v_3 = (2+\epsilon, 2+\epsilon, 7)$.

a set of simulations using a stylized representation of the forward auction. We discuss the details of the simulation design in Appendix A. At a broad level, however, we generate many randomly drawn bidding environments capturing key qualitative features likely to be present in the forward auction. We use these to simulate actual bidding behavior in both the original MALS auction design and in the CPA. This enables us to compare the potential performance of the two auction designs in terms of efficiency and revenue.

Our results are presented in detail in Appendix A. They suggest that substantial efficiency and revenue gains from the CPA may be typical. For example, in our baseline simulation design, total surplus and total revenue are higher under the CPA in the vast majority of our 1,000 replications. Average revenue (across all replications) is 70% higher under the CPA. The average surplus is 17% higher. The CPA achieves an average of 97 percent of the (infeasible) first-best efficiency. Compared to the infeasible first best (fully efficient) allocation, the CPA achieves at least 90 percent efficiency in 91 percent of the replications, whereas the MALS auction achieves this level in only 37 percent of the replications. Results in alternative specifications are qualitatively similar, with larger gains when complementarities for package bidders are stronger.

9 Supplementary Bidding

As discussed before, both the Clock auction proposed by the NPRM and MALS and the CPA proposed here could leave some items unallocated. It should be an option, if the FCC finds it necessary, to resell the unsold items immediately following the main clock auction. We imagine this option to be triggered only if the FCC finds significant undersell to have resulted from the main round. Should the FCC decide to resell the unsold items, an important consideration in the auction format design is to minimize possible adverse effects it may have on the main clock auction. For instance, if the unsold items were sold in a "pay-as-you-bid auction," bidders may have incentives to signal low values in the clock phase in an attempt to lower the prices they face in the supplementary round.

For this reason, a clock auction, which is relatively simple and has a transparent pricing rule, is preferred. We do not envision package bids to be allowed for this round, in light of the fact that package bidding in the main round will have treated much of the exposure problem. We do not envision package bids to be allowed for this round, in light of the fact that package bidding in the main round will have addressed much of the exposure problem. A proxy version of the clock auction could be used if a speedier resolution of the forward auction is an important concern.

An alternative more comprehensive approach would put all licenses, including those sold in the initial clock phase, back into sale in the supplementary round. This is the approach prescribed by the Combinatorial Clock Auction (CCA), mentioned as an option in the NPRM (for package bidding). CCA begins with a clock phase (with a different bid withdrawal rule than the MALS clock auction), but adds a supplementary round of proxy bidding for packages following the clock phase. In the (supplementary) proxy phase, all clock bids (i.e., prices for winning quantities) are added as default package bids, but bidders are allowed to submit additional sealed bids for these packages and to bid, with some limitation, for other packages. Winners are then determined to maximize the total bids, and are charged with the VCG-nearest core prices.[19]

There are several concerns with such a comprehensive supplementary bidding approach. First, the pricing rule (i.e., the VCG-nearest core pricing) means that the final clock bids and the prices winners eventually pay would differ substantially (even when the winning

[19] The VCG rule charges each winning bidder the social opportunity cost associated with his winning licenses—the net social value that was foregone due to the winning by the bidder of these licenses. A core pricing rule is a profile of prices that would leave no profitable deviation either by an individual or by a group of individuals through reallocation of licenses within them or through their non-participation. A VCG-nearest core pricing selects the VCG prices if they are in the core, and selects the core point that is the closest to, and in equal distance across all winning bidders away from, the VCG in case the VCG is not in the core. See Ausubel, Cramton, and Milgrom (2006) and Cramton (2012) for detail.

allocation does not change across the two phases),[20] and the relationship between the two phases and the activity rule governing it are complex and may be perceived as opaque by the bidders.

Second, the outcome of the proxy phase is unpredictable in the sense that the winners of the clock phase may not secure the licenses they have won from the clock phase. The prevailing activity rule provides some protection in this regard, for each winner of the clock phase may reclaim the winning package from the the clock phase by raising his bid for that package by the final clock prices of the unsold items (see Cramton (2012) and Ausubel and Cramton (2011)). But that may prove too expensive for this option to be of practical value to the bidders.

Third, when all items are sold by the end of the clock phase, the allocation remains unchanged, but the prices the bidders pay depend on their opponents' bids only. Since their winning licenses remain unchanged and their payments do not depend on their bids, the bidders become indifferent among all feasible package bids (allowed by the standard revealed preference activity rule). As explained in detail in Appendix B, this ambiguous bidding incentive in the proxy phase could lead to excessively low revenue for the seller, susceptibility to collusion, and distorted incentives in the clock phase and inefficiencies.

Overall, the feature that "everything is up for grabs again in the supplementary round" can be perceived as introducing too much complexity/uncertainty into the process for the participants. Such uncertainty may in turn make it difficult for the bidders to form a valuation for the clock phase. And this could hinder price discovery (of even individual items) and may entail underbidding in the clock phase.

Limiting the scope of supplementary bidding to only those items that have not been sold by the clock phase makes the procedure much simpler and more predictable and transparent

[20] In the UK 10-40 GHz auction which used a version of CCA, the final prices of licenses were 20-30% of the winning clock bids in the initial phase, even though the relative prices were stable across the two phases (see Cramton (2008)).

in terms of the consequence of the bidding behavior in the clock phase. In particular, those items that are sold in the initial clock phase will not be subject to any further uncertainty in terms of allocation and prices. The fact that the items sold in the clock phase will be unavailable and the uncertainty on whether the FCC will opt to resell the unsold items immediately will also encourage more sincere bidding in the clock phase, contributing to the efficient allocation of the licenses through the clock phase.[21]

10 Improved Assignment Phase

As noted earlier, the current procedure for assignment does not allow bidders to express preferences for horizontal contiguity across licenses won in different EAs. Here, we propose simple modifications of the MALS assignment procedure that can enhance efficient assignment and horizontal contiguity. Obviously, the problem of allocational efficiency and horizontal contiguity can be treated most effectively by introducing package bids, as envisioned by our Clock Package Auction proposal. The methods discussed here will be most useful if such a comprehensive bidding approach is not adopted, but may be useful even with its use, to the extent that the number of packages allowed may be limited, or when bidders desire vertical contiguity between a package and single EA license in an area covered by the package.

[21]Some demand that would be present if all units are available for sale may not be present if only some items are available for sale, and this may make the supplementary bidding not as competitive as it could be, and in principle this may create incentives for bidders to strategically reduce demand in the clock phase. We do not believe that this is a significant issue. The risk of the desired items being sold in the clock phase (and thus not being available for the supplementary phase), and the uncertainty regarding the FCC's exercise of the option for immediate resale will make such strategic behavior very costly.

10.1 Random Priority with Enriched Preferences

The NPRM suggests a possibility of a Random Priority ("RP") assignment of specific frequency blocks. Efficiency could be enhanced by enriching the preference language for alternative frequency blocks across multiple EAs, and applying the Random Priority rule with respect to these preferences. More precisely, instead of rank-ordering alternative frequency blocks in each EA separately, the FCC would allow bidders to rank-order alternative *combinations* of frequency blocks across different EAs. To illustrate, recall Example 2 introduced earlier: there are two EAs, EA_1 and EA_2; each EA_i has three frequency blocks: A_i, B_i, C_i for $i = 1, 2$; all three blocks are cleared in EA_1 but only two of them are cleared in EA_2; and the remaining station in EA_2 can be moved to any block. There are three bidders, 1, 2, 3 in the forward auction. Bidder 1 and bidder 2 each won 1 license in each EA, whereas bidder 3 won only one license in EA_1 in the forward auction.

Suppose bidder 1 ranks: $A_1A_2 - B_1B_2 - A_1B_2 - B_1A_2 - \cdots$. Bidder 2 may rank similarly: $A_1A_2 - B_1B_2 - B_1A_2 - A_1B_2 \cdots$. One can then define the random priority rule analogously but over both EAs. To illustrate, suppose that bidder 3 ranks $C_1 - B_1 - A_1$. In this case, again bidder 3 will get C_1 in EA_1. And bidders 1 and 2 will get (A_1A_2, B_1B_2) and (B_1B_2, A_1A_2) with equal probability 1/2. Hence, in this way, we achieve full horizontal contiguity.

The example was (deliberately) simple. Matters become more complicated if bidder 3 ranks: $A_1 - B_1 - C_1$, and the remaining station in EA_2 must keep C_2, or C_2 is too impaired to be of any value to the bidders. In that case, with probability 1/3, bidder 3 goes first and will pick A_1 in EA_1, in which case the next bidder will get B_1B_2, and the last bidder will get C_1A_2. More precisely, each of the following six assignments,

$$(A_1A_2, B_1B_2, C_1), (B_1B_2, A_1A_2, C_1), (A_1A_2, C_1B_2, B_1)$$

$$(C_1B_1, A_1A_2, B_1), (B_1B_2, C_1A_2, A_1), (C_1A_2, B_1B_2, A_1)$$

will be selected with probability 1/6 (where again the first component denotes bidder 1's assignment, the second denotes bidder 2's assignment, and the third denotes bidder 3's).

In this case, the new version of the RP assignment does not work as well in preserving horizontal contiguity.[22] A further improvement can be made if the FCC restricts the set of admissible assignments to be those that guarantee (or optimize on) horizontal contiguity. For instance, if it is known that bidder 3's gain from having A instead of B is not very large whereas the loss from not having inter-EA contiguity is very large, then the FCC may declare only two assignments, (A_1A_2, B_1B_2, C_1) and (B_1B_2, A_1A_2, C_1), to be admissible. In this case, horizontal contiguity can be guaranteed, although bidder 3 may be worse off. Of course, the FCC may take a middle ground by declaring C_1A_2 or A_1C_2 to be too inefficient to be a part of an admissible set of assignments. In that case, the RP will be run so that each winner at his turn can only choose from the set of assignments that does not include these two outcomes.

More generally, the improved version of the RP assignment procedure can be stated as follows.

- The FCC picks an extended area $\mathcal{X}$ (which could be the entire nation or one or multiple MEAs or REAGs) which contains multiple EAs. The extended area $\mathcal{X}$ should be chosen to be the minimal region such that a failure to attain horizontal contiguity across different EAs within $\mathcal{X}$ would entail a nontrivial loss to a bidder.

- The FCC next picks the set of "admissible" ways of allocating the cleared blocks in the EAs contained in $\mathcal{X}$ to the winners of the corresponding objects. For instance, the FCC could first impose vertical contiguity (to the extent this is guaranteed, as with the NPRM and MALS) and horizontal contiguity for the packages (for which hori-

[22] Nevertheless, this rule does a better job of accounting for bidders' preferences for horizontal continuity than does the NPRM proposal. In fact, the new version produces an assignment that is Pareto efficient, whereas the outcome from simple the RP assignment fails to be Pareto efficient. Furthermore, the procedure is strategy-proof, making it a weakly dominant strategy for each bidder to reveal his preferences truthfully.

zontal contiguity is guaranteed) and identify the set of feasible assignments satisfying these guarantees. Next, the FCC may impose a suitably-chosen degree of horizontal contiguity it deems as desirable based on an appropriate metric. For instance, the FCC may require that each admissible assignment should satisfy horizontal contiguity over a certain minimal fraction of areas over which each bidder has won licenses (e.g., measured by the number of pairs of adjacent EAs in which a bidder has won licenses).

- Each winner of any license within $\mathcal{X}$ ranks his possible allocations from the set of admissible allocations.
- The FCC next runs the RP assignment procedure based on the preference rankings submitted by the bidders.

The next result is standard:

Theorem 3. *The generalized RP is strategyproof and produces an assignment that is Pareto efficient within the set of admissible allocations.*

Vertical contiguity and horizontal contiguity may sometimes be in conflict with each other, and it may be impossible to satisfy both. The current proposal will resolve the tradeoff in three ways: (1) through the contiguity guarantee that comes with a package, (2) through the set of admissible assignments determined by the FCC, and (3) through the rich preferences submitted by the winners.

To illustrate, suppose there are three EAs: EA_1, EA_2 and EA_3. EA_1 and EA_2 have each 3 blocks, A, B and C, cleared from the reverse auction. EA_3 has 2 blocks: A and B. Suppose all three EAs are pairwise adjacent. Two bidders have won in these EAs. Bidder x won two licenses in EA_1 and one license in each of EA_2 and EA_3. Bidder y won two licenses in EA_2 and won one license in each of EA_1 and EA_3.

Let us first see that it is impossible to satisfy both vertical and horizontal contiguity of assignments. Vertical contiguity for bidder x in EA_1 means that he should win either AB

or BC in EA_1. For the same reason, bidder y should win either AB or BC in EA_2. In case bidder x wins AB in EA_1, horizontal contiguity between EA_1 and EA_2 demands that bidder y should win BC in EA_2. Finally, horizontal contiguity for bidder x between EA_1 and EA_3 means that he should get A in EA_1. But then bidder y must get B in EA_3, violating the contiguity between EA_1 and EA_3 for that bidder. The case in which bidder x gets BC has a similar violation of horizontal contiguity.

The proposed assignment scheme would resolve the conflict in the following way. First, horizontal contiguity guaranteed for a package must be respected. Second, the FCC may ensure vertical contiguity within each EA for the winners (to the extent it can). Third, the FCC may maximize the number of pairs of adjacent EAs for which horizontal contiguity of licenses (won as single licenses and between single licenses and package of licenses) is satisfied.

To illustrate, assume that the three EAs form a single MEA. Suppose first that each of the two bidders won one MEA package and one single EA license (one in EA_1 for x and one in EA_2 for y). Then, the above procedure results in two admissible assignments.

Table 1: Admissible Assignments with Two Package Winners

	A	B	C	A	B	C
EA_1	x	y	x	y	x	x
EA_2	x	y	y	y	x	y
EA_3	x	y		y	x	

Notice that vertical contiguity must be violated for one bidder, since the horizontal contiguity guarantee that comes with each MEA package must be respected first. Of these two admissible assignments, one will be chosen by the RP rule. Unless the specific frequency block matters much, the first assignment will be chosen if bidder y is selected to go first (since it ensures vertical contiguity for y but not for x) and for the same reason, the second assignment will be chosen if bidder x is selected to go first.

Suppose next that only bidder x has won MEA package (and two single EA licenses), and bidder y won all licenses as single EA licenses. In this case, the assumed procedure yields a unique admissible assignment.

Table 2: Admissible Assignment with One Package Winner

	A	B	C
EA_1	x	x	y
EA_2	x	y	y
EA_3	x	y	

Bidder x enjoys vertical contiguity across licenses within each EA and horizontal contiguity across EAs. Bidder y attains vertical contiguity of licenses within each EA, but the horizontal contiguity between EA_1 and EA_3 is not satisfied.

10.2 Assignment Auction

The random priority assignment is sensible only when the bidders do not find the value differences across particular frequency blocks to be large, and the FCC can limit the set of admissible assignments to guarantee contiguity of frequency assignments across regions for the winners of geographically adjacent licenses. An alternative would be to hold an auction to determine the assignment. One possibility is to use the assignment auction prescribed by Cramton (2012), also used in the UK 10-40 GHz auction. According to this method, winners of the principal auction stage bid for preferred assignments. The winners can submit multiple (or possibly no) bids for their preferred assignments that are consistent with their winning awards of the principal auction, and the auction manager selects a collection of bids, at most one bid from each bidder, that are maximal among all possible collections that produce feasible assignments. The winning bidders are then charged the VCG-nearest core prices for their assignments.

In the example from Section 10.1, even without the FCC limiting the admissible assignments, such an auction is likely to produce contiguous assignments across the regions for bidders 1 and 2. If bidder 3 has weak preferences over alternative frequencies in EA_1, then he will bid very little, if any, for his preferred frequency block, and bidders 1 and 2 will bid seriously for A_1A_2 and B_1B_2 (to the extent that they value contiguous assignments), so they will outbid bidder 3 and secure a contiguous assignment. At the same time, the need to respect the horizontal contiguity for the package items will still limit the set of admissible assignments.

While there may be potentially many assignments a bidder can bid for, the auction will be relatively straightforward from the perspective of bidders since important matters such as how many licenses they win in each region will have been already settled. In particular, a bidder can, and will typically, bid for a small number of select assignments (as was the case with the UK auction). The pricing rule also limits the incentives for non-truthful bidding and is envy free.

11 Other Concerns and Recommendations

11.1 Generic Licenses

The treatment of licenses as generic within market and class of license is essential to the MALS design and our CPA proposal. This feature has a number of desirable properties as long as licenses designated as generic actually are equivalent. It is essential that the band plan be designed to minimize interference or other impairment of licenses, so that genericity of licenses is genuine. If in limited cases licenses do unavoidably differ substantially, they must be treated as non-fungible in the forward auction. Failure of the auction design to distinguish adequately between heterogeneous licenses would lead to a significant exposure problem, conservative bidding, and inefficient allocations.

11.2 Bidding Credits

The NPRM proposes offering bidder credits of 15 percent to "small businesses" and 25 percent to "very small businesses." Bidding credits have been used toward this end in past FCC spectrum auctions, and the FCC proposes to do so again in the forward auction.[23]

However, an important difference from past auctions is the interaction between the forward and reverse auctions. Consequently, the result of favoring designated entities through discounts will not merely be to favor designated entities in the allocation of spectrum for wireless services. It will also favor the status quo, i.e., allocation of spectrum to current broadcast television licensees. Discounts offered to designated entities may result in a failure of closing conditions for a target quantity of spectrum, even when bidders in the forward auction are offering prices sufficient to meet the net revenue requirement.[24] The result will not be the intended one: instead of tilting the allocation toward designated entities in the mobile wireless industry, the quantity of spectrum made available to mobile wireless—including to designated entities—will be reduced.

What is needed is an alternative that preserves the desired preferential treatment of designated entities when it comes to the question of *how* mobile wireless spectrum is allocated, but does so without adversely influencing the question of *whether* spectrum is allocated to mobile wireless. This is possible with a simple modification of the NPRM proposal. Bidding credits could be made contingent on satisfaction of the net revenue requirement, and phased in gradually to allow the maximum feasible bidding credit (up to the 15 and 25 percent

[23] Ayres and Cramton (1996) point out that bidding credits could sometimes enhance revenues by creating competition for licenses attracting few large bidders. This outcome may be less likely with the proposed auction design due to the use of generic licenses. Nonetheless, our proposal below would not affect any revenue enhancing feature of bidding credits.

[24] Consider an example with one license and two bidders: a *very small business* with valuation 65 and a non-designated entity with valuation 80. Suppose that the net revenue requirement is 62. The designated entity's 25% bidding credit allows it to become the provisional winner in the clock auction at a price of 80. However, its payment would be only 60. Thus the net revenue requirement fails. Here this occurs even though both bidders would be willing to pay more than the required net revenue of 62 for the license.

levels) that can be accommodated without leading to a revenue shortfall.[25] This need not introduce uncertainty for designated entities. They can be told the bidding credit they are eligible for at each price vector. Nor would the auction be disrupted when the bidding credits "kick in," since the only effect would be an increase in demand. This modification of the bidding credit program would require only relaxation of the activity rule for designated entities, allowing them to take advantage of the credits. This should be straightforward.

This proposal is a sharply targeted remedy to the problem that bidding credits could lead to net revenue failure. Under straightforward bidding, the proposed modification would have no effect when the net revenue requirement would have been satisfied under the original proposal. Likewise, if the net revenue requirement fails under our proposed bidding credit system, it would also fail under the original system. Thus the only cases in which our proposed change would have an effect is when the net revenue condition would be satisfied under our proposal but would have failed under the original proposal.

11.3 Activity Rule

The purpose of the Activity rule is to prevent bid sniping[26] and encourage straightforward bidding, both of which are important for effective price discovery. The basic idea behind such an activity rule is to constrain bidding behavior to be consistent with straightforward bidding. Traditionally, the activity rule used by the FCC in past auctions has been a "monotonicity rule," requiring that the aggregate quantity demanded by a bidder—typically measured by

[25] In the simple example above, bidding credits would kick in as soon as the price reached 62. At a price of 65, for example, the very small business could receive a bidding credit of 3; at a price of 70, a bidding credit of 8.

[26] Bid sniping refers to the strategy of waiting (i.e., bidding low or insincerely) until the last minute of the auction to bid seriously. When permitted, bidders have incentives to engage in such behavior in order to conceal their preferences and thereby increase their chances to win desired items at low prices.

MHz × Population—not increase over the course of the auction.[27] This rule works well when the items are homogeneous, but not well when they are heterogeneous. For instance, the prices of different EA licenses may vary widely on a per MHz × Pop basis. In that case, straightforward bidding behavior may require switching to a larger package when its price becomes relatively cheaper, but this behavior will not be allowed by the monotonicity rule. This activity rule is also susceptible to "parking," a strategy of bidding initially on underpriced or low-value objects with high quantity so as to keep the the prices on desired objects from rising quickly. Such behavior inhibits price discovery, and may distort prices and hinder efficient allocation.

An alternative "revealed preference" activity rule proposed by Cramton (2012) and Ausubel, Cramton, and Milgrom (2006) is more flexible and is immune to parking behavior. In words, the revealed preference requirement would allow a bidder to switch to a larger package as long as that package has become relatively cheaper than the package she demanded at the previous round. The hybrid version suggested by Ausubel and Cramton (2011), which uses the traditional eligibility monotonicity rule but allows bidders to switch to large packages as allowed by the revealed preference requirement,[28] appears to offer further flexibility to the bidders. These rules appear to be sensible for both the clock auction proposed by the NPRM and the CPA proposed here.

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[27]Specifically, according to this rule, each bidder i must bid within his eligibility point E_t^i, which is set initially (say at E_0^i) based on the deposit posted by the bidder, and evolves over time to equal the smaller of his bidding activity $e \cdot q_t^i$ at round t and the eligibility point E_{t-1} at the previous round (where e refers to the vector of eligibility points for the products the bidder is bidding on, q_t^i refers to the quantity bidder i demands or bids at round t.)

[28]More precisely, under the Simplified Revealed Preference rule, at any round in the Clock Phase a bidder can bid on a larger package than would be permitted by the bidder's current eligibility, provided that the package satisfies "revealed preference" with respect to each prior round's bid in which eligibility was reduced (Ausubel and Cramton (2011)). The rule states that in Clock round t, bidder i can bid an package q_t if

$$e \cdot q_t \leq E_t^i,$$

or

$$(p_t - p_s) \cdot (q_t - q_s) \leq 0,$$

for each previous round s at which her eligibility was reduced.

Part III

Reverse Auction

12 The NPRM and MALS Proposals

Broadcast television bands currently occupy 294 MHz of spectrum in five different bands allocated for broadcasting use. The density of broadcast television licensees varies significantly across markets, and licensees themselves differ substantially in their geographic coverages ("contours"), depending on such factors as location, transmitter power, antenna height, local terrain, etc. In many markets, substantial spectrum in the broadcast television allocation is unused. Because television and mobile wireless uses are incompatible, reallocating portions of the spectrum to mobile wireless uses requires relocating television licensees. By shifting broadcasters to a single portion of the spectrum, sufficient bandwidth may be cleared to permit viable mobile wireless services to be offered in the remaining portion.

Although legislation gives the FCC authority to relocate ("repack") broadcast television licensees within or across the broadcast television blocks, this must be done in a way that does not unduly affect licensees' contours. The ease of meeting this requirement will vary considerably across markets and across incumbent licensees within markets. Further, in some large metropolitan markets there are many television licensees. In such markets, even after repacking there might be relatively little spectrum available for wireless services. Such markets may, however, also be some of the most valuable markets for mobile wireless services.

The Spectrum Act authorizes the FCC to resolve the competing demand for the scarce spectrum through voluntary participation in "incentive auctions" in which television licensees can offer to relinquish their licenses. The market design problem for the FCC is to provide an auction mechanism that selects voluntary offers to relinquish while also accounting for the constraints on repacking broadcasters in a way that adequately respects their contours. This need to repack without introducing interference introduces significant complications to the design of the reverse auction. Repacking constraints are "global," in the sense that placing a broadcaster in one market in a particular frequency can affect the feasibility of placing broadcasters in the same or nearby frequencies in other markets. For example, Washington, D.C. and Philadelphia, PA are sufficiently proximate that use of a UHF television channel in one may preclude use of the same channel in the other market. For the same reason, decisions regarding channel usage in Philadelphia may affect New York City, which then may affect Bridgeport, CT, and then Boston, MA. Similarly, a Washington D.C. assignment may affect feasible assignments in Baltimore, MD and possibly Richmond, VA. Thus, channel usage can have a "daisy-chain" effect that can link major metropolitan areas—as well as smaller communities—that are hundreds of miles apart.

12.1 Descending Clock Auction

MALS proposes a simultaneous descending clock design for the reverse auction. The reverse auction would begin with provisional target quantities of spectrum to be cleared in each market. Although there would be a single nationwide reverse auction, it will utilize different clocks for each market and each *relinquishment option*—e.g., go off the air, share with another station, or move to a lower band. The option to "remain in the current band" (i.e., to be repacked) is always available as well. Bidders who do not participate in the auction are automatically assigned to this option. Participation in the auction is strictly voluntary. Clock prices may differ across bidders within a market and may move at different speeds, due to differences in coverage areas, in the feasibility of each option, in the costs of repacking, or in the effect of repacking one station on the ease of repacking others.

The auction clocks begin with high initial ("reserve") prices chosen by the FCC for each feasible relinquishment option in each market. Each bidder selects the option he finds most attractive at these prices. Roughly speaking, prices are then lowered, bidders again express their preferences, and the process iterates until prices reach a level just low enough that the quantity of spectrum offered in each market equals the target quantity.

Complicating this simple description is the fact that feasibility considerations will limit repacking options. Each clearing target determines the amount of spectrum available for repacking bidders who choose to stay on the air. Before prices are adjusted during the auction, the feasibility of repacking each remaining bidder (i.e., the feasibility of adding that bidder to the set to be repacked) must first be checked. If it becomes infeasible for a given bidder to be repacked, his current relinquishment offer will be provisionally accepted at the current clock price for that option. The feasibility checking iterates until it is the case that each remaining bidder could feasibly be repacked. Only at this point do clock prices decline for these remaining bidders.

A significant virtue of the clock design is its simplicity from the perspective of bidders. Although complicated repacking constraints operate in the background, the rules of the auction are transparent, making it easy for bidders to see how their actions determine outcomes and payments. Further, given appropriate rules for controlling clock prices,[29] bidders have clear and strong incentives to follow a straightforward bidding strategy—i.e., always to select the option that would maximize one's own profit if the current clock prices were to be the final prices.

Theorem 4. *Consider the MALS reverse auction and assume that clock prices decline continuously and with speeds that do not depend on the actions of bidders. Then straightforward bidding is a weakly dominant strategy for all bidders.*

Proof. See Appendix C. □

[29] The NPRM and MALS are not explicit about the rules that will govern adjustment of clock prices for feasible relinquishment options. As Theorem 4 suggests, we see some potential for distortion of incentives if the rates at which prices adjust depend on bidder offers.

12.2 Other Reverse Auction Designs

The NPRM also suggests an alternative to the descending clock auction design. Details are not provided, but broadly the proposal involves collecting price offers from reverse auction bidders and choosing winners (or, perhaps, provisional winners in each of a sequence of rounds) to minimize payments to bidders subject to repacking constraints.

13 Primary Concerns

13.1 Computational Complexity

Constraints on repacking broadcasters cause issues of auction design and computational complexity to be intertwined here in a way that has not previously been well explored. As already noted, two types of auction design have been proposed by MALS and the NPRM. One is a descending clock auction, using real-time feasibility checking. The other is an unspecified sealed bid auction design in which optimization over relinquishment offers (subject to feasible repacking) might play a prominent role.

In general the types of feasibility checking and constrained optimization problems involved in such auction designs can present severe computational challenges. The NPRM is vague about the nature of the repacking constraints and imprecise about the amenability of the constrained optimization problem to solution using standard or specialized algorithms. In some cases the NPRM discusses use of approximate solutions. In the worst case, approximation approaches can perform extremely poorly. Further, here the quality of an approximation depends not only on proximity to the true solution, but also on how the optimization affects the performance of the auction as a whole.

Without details on the repacking constraints, it is impossible to evaluate the likely performance of potential full solution or approximate solution algorithms. Although certain auction designs may ease or worsen the computational complexity or performance of approximate solutions, similar issues are likely to arise in any of the contemplated reverse auction designs.

We elaborate on these issues in section 15 below.

13.2 Delay and Uncertainty Due to Target Failures

If the FCC chooses initial clearing targets appropriately,[30] these will be quite high, reflecting an attempt to maximize opportunities for efficient trade. This will almost certainly lead to multiple clearing targets being tried before one is found that allows satisfaction of closing conditions for the grand auction.

Under the current rules, each time the target is reduced, the forward auction must pause while reverse auction bidders are reconvened in order to obtain new "supply" prices for the reduced target quantities. In addition to the burden this creates for bidders, this feature of the auction design will delay completion of the grand auction. It may also introduce uncertainty for reverse auction bidders, since truthful bidding would require some broadcasters to take actions that commit them to assignments days or weeks ahead of other bidders.

14 Single-Pass Reverse Auction

An alternative to the proposed sequence of reverse auctions contemplated by MALS is to conduct the reverse auction in a single pass. The simple idea is that instead of stopping the auction when prices for the most optimistic target Q_1 are reached, the auction would continue and trace out the "supply curve" over the entire range of potential clearing targets. We describe two variations of the basic idea. These differ in whether bidder optimization over the alternative relinquishment options is done by bidders themselves or by proxy.

[30]MALS propose to set initial clearing targets based on the supply offered at high initial reserve prices.

We see at least four potential gains from the single-pass option:

1. greater simplicity for reverse auction bidders;
2. avoidance of delay and inconvenience for bidders when a closing target fails;
3. enabling improved efficiency in a two-sided closing trial (see section 18);
4. avoiding real-time feasibility checking.

The first three gains can be achieved with our baseline proposal. The last takes advantage of proxy bidding.

14.1 Single-Pass: Baseline

Our baseline proposal would alter the reverse auction design only slightly. Instead of conducting a sequence of reverse auctions, each with a progressively smaller clearing target, just one reverse auction would be held. Let $Q_1, Q_2, \ldots, Q_T$ represent all possible clearing targets entertained by the FCC, listed in descending order. In the single pass reverse auction, the FCC would start the auction at prices more than sufficient to clear Q_1, just as it would in the first of its sequence of reverse auctions under the MALS proposal. But instead of stopping when prices for this clearing target are reached, the auction would continue with prices falling until the quantity of spectrum offered in each market reaches Q_T. Bidders' exit prices would be recorded, allowing the FCC to "replay" the reverse auction from the beginning to any desired clearing target. These "replayed" auctions would be used to reveal the payments required for any possible clearing target.

The interaction with the forward auction would then be exactly as originally proposed. The clearing target Q_1 would be tried first in the forward auction. If the revenues obtained there exceeded the total cost implied by the reverse auction bids for the target Q_1, the grand auction would end. Otherwise the forward auction would continue with the lower clearing target Q_2. However, rather than reconvening the reverse auction to establish the (lower) prices for the new target, these new prices would simply be reconstructed from the single-pass reverse auction.

The single-pass option changes bidding in the reverse auction very little. A bidder must still come to the auction with a reservation value in mind for each of the possible relinquishment options. A bidder's options during the auction would be identical to those under the original proposal, and the rules determining prices paid to bidders for each relinquishment option would be no different. Just as in the original proposal, bidders' choices between relinquishment options would be binding offers. We also see no change in a bidder's incentive for straightforward bidding.[31]

The single pass option offers several advantages. One is elimination of the need for a new reverse auction each time a clearing target fails. This would simplify participation in the reverse auction. It would also avoid delay: rather than pausing the forward auction to reconvene reverse auction bidders each time the clearing target is adjusted, the forward auction could proceed without interruption.

The single-pass design also simplifies participation of reverse auction bidders in a "two-sided closing trial" and would further allow the FCC to minimize the likelihood of "overshooting" during a closing trial (see section 18 below). This could lead to more cleared spectrum. Another advantage is that bidders would be assured that all broadcaster offers were chosen the same day; unlike the MALS design, here a bidder need not worry that the

[31] Consider a simple example with 3 bidders in a market where the clearing target may be 1 or 2 licenses. Suppose that the only options for bidders are to go off the air or remain in their original band (exit). We can divide the auction into two phases: phase 1 covers the period before any bidder exits, and phase 2 is the period between the first and second exits. Suppose that exits are revealed during the reverse auction, so that bidders know the current phase. Consider phase 1. Exiting implies obtaining zero profit (zero gain over the value of remaining on the air). So by standard arguments, exiting at a price above one's valuation is weakly dominated. Further, since the price will only decline further in phase 2, standard arguments imply that remaining in the auction at prices below one's valuation is weakly dominated. Now consider phase 2. The price for clearing 2 licenses has already been determined. So what remains is a single-unit descending auction, where standard arguments imply that bidding one's reservation value is weakly dominant.

final offers of different TV stations might be made days or weeks apart. Such concerns could adversely affect participation and bidding in the auction if bidders believe that new information affecting valuations might arrive in the period between the first MALS reverse auction and the last.

A possible concern is that reverse auction bidders might be reluctant to reveal information unnecessarily. For example, consider a bidder with a reservation value for going off the air that is well below what turns out to be the final market clearing price. Under the MALS design, such a bidder would have no need to reveal just how low a price he would have been willing to accept, whereas a single-pass reverse auction may collect this information.[32] However, we understand that the FCC has well established procedures for protecting confidential information.

14.2 Single-Pass with Proxy Bidding

Conducting the single-pass auction by *proxy* may offer additional benefits. With proxy bidding, bidders would be asked once to report reservation values for each relinquishment option through proxy-agent software. The rules of the auction for a given target Q_t would be identical to that originally proposed. However all bidding would be done by the proxy agent. The proxy agent would implement a straightforward bidding strategy for each bidder. Specifically, at each vector of prices offered by the auction's descending price algorithm, the proxy agent for bidder i would choose the relinquishment option (or "exit") that is most profitable to i according to i's reported reservation values. The proxy auction would stop when prices for the target Q_T are reached.

The primary gain from proxy bidding is that there would be no need for real-time feasibility checking or other optimization. This will ensure that the auction will not stall between

[32] Any such bidder is one who ends up accepting an offer to clear, and bidders giving up their licenses may be less concerned about revealing their reservation values.

rounds due to computational difficulties. It may also allow more complete solutions of the underlying repacking problems, leading to more efficient clearing, i.e., assurance that the most favorable supply offers are accepted.

15 Addressing Algorithmic Issues

Stepping back from any particular reverse auction design, optimal clearing of a given target quantity of spectrum would minimize the total costs of clearing (i.e., the value of relinquished spectrum allocations) and of repacking television broadcasters who remain on the air. Reaching this fully optimal solution is certainly infeasible. One problem is that of *incentive compatibility*: current licensees cannot be compelled to reveal the values they place on their current spectrum allocations and must voluntarily agree to the terms of any relinquishment. This alone makes it impossible to pay each cleared television licensee the lowest price it would accept for its license. A second problem is the computational complexity involved in finding a cost-minimizing allocation. This complexity arises from the effect of repacking one station on the feasibility (or cost) of repacking others. Even if bidders' reservation values were known to the FCC, the cost minimization problem is an integer programming problem of a type known to be computationally hard in general. The unique challenge in the reverse auction design is to handle adequately both incentive compatibility and computational feasibility while also keeping the rules sufficiently simple and transparent. Here we discuss this challenge in further detail.

The MALS clock auction offers an approach in which each bidder has a strong incentive to offer to relinquish whenever the price exceeds his true reservation value (recall Theorem 4). Taking these strategies as given, the MALS clock auction design replaces minimization of total cost with minimization of payments to cleared licensees, subject to sequential verification of repacking feasibility. As long as more bidders than necessary are volunteering to be cleared, prices fall, leading to repacking of bidders with the highest reservation values. In the simplest form of the proposed design, repacking constraints would enter only through verification that it is feasible to repack a bidder before he is offered a new set of prices. This approach puts priority on the cost of clearing relative to the cost of repacking. Only when it is infeasible for a bidder to be repacked would the auction allow repacking costs to affect the choice of which broadcasters are to be cleared. Possible variations include using repacking constraints to adjust relative clock speeds or to score bidders' offers. This would allow repacking costs to play a larger role, but repacking would still be done sequentially, possibly leading to suboptimal choices.

Of course, the clock auction design is also what leads to the simple and transparent incentives for bidders. Other auction designs might improve on the repacking optimization *given* a set of price offers from bidders, but these offers may be systematically different from those in a clock auction. How such tradeoffs could affect the overall efficiency of the MALS design relative to other options may depend on the nature of the repacking constraints, about which the FCC has thus far released little information.

Further, the MALS clock auction does not avoid significant computational challenge. Formally, feasibility checking takes the form of a "graph coloring problem," which is known to be computationally hard in the worst case. Without knowing the fine details of repacking constraints, it is impossible to determine whether the feasibility checking required by the MALS proposal is itself computationally feasible.

The NPRM also entertains other types of reverse auction designs, where full optimization over bids and repacking costs might play a more prominent role. Few details are provided, making a careful analysis of such options difficult.

Below we elaborate on these issues. However, our overriding message is that by releasing more detailed specifications of the repacking constraints involved, the FCC would enable outside experts to better assess the likely performance of the proposed reverse auction design and alternatives. Such analysis could clarify which of the mechanisms under consideration may be vulnerable to the computational concerns we discuss, and allow development of new specialized algorithms for the repacking problem that might yield substantial improvements in performance.

15.1 Background and Context

Both the NPRM and the MALS proposal discuss several variations for the structure of the reverse auction. The overarching goal of the reverse auction is to clear a targeted amount of broadcast spectrum in an efficient manner, ideally while minimizing overall costs (in the form of direct payments to the broadcasters and costs associated with repacking broadcasters remaining on-air). Both documents correctly emphasize that the primary computational challenge in meeting this goal arises from the *repacking constraints*. In the absence of such constraints, the reverse auction would require nothing more than selecting those broadcasters demanding the lowest prices (regardless of the auction mechanism details), and paying them to clear their spectrum (up to the targeted amount), while repacking the highest bidding broadcasters. In the presence of repacking constraints, minimizing costs for clearing the desired spectrum may become computationally intractable, and force consideration of various heuristic and suboptimal algorithms.

The repacking constraints are physical, engineering and policy constraints that are exogenous to broadcaster prices, bids and valuations. Broadly speaking, they arise from interference considerations due to the physical and frequency proximity of broadcasters — i.e. if two broadcasters are close enough geographically, and are assigned the same or adjacent channels, interference may occur. Geographic terrain also influences the strength of broadcaster signals. Such sources of interference already exist within the current assignments of broadcasters to frequencies, and define the current "coverage area" or "contour" of a given broadcaster. The repacking constraints arise from the regulatory requirement that any broadcaster choosing to remain within their current band must be repacked in a way that adequately respects the broadcaster's current coverage area or service contour.[33]

To a first approximation, then, repacking constraints can be thought of as constraints between certain *pairs* of broadcasters: if broadcasters A and B are geographically proximate, and currently on sufficiently different channels that neither interferes with the other's service area, but relocating them on the same or adjacent channels would create too much new interference (as defined by the regulatory requirements), then they cannot be repacked together in this way.

We can view these pairwise constraints as forming a *network* or *graph* in which there is a node or vertex representing each broadcaster in the nation, and there is an edge or link between any pair of vertices whose corresponding broadcasters are constrained in the above manner (i.e. cannot be relocated on the same or adjacent channels without creating too much new interference). This is a large, complex and dense network, and its meaning for the repacking process is clear: once we have decided to repack broadcaster A on channel c, we are then unable to repack any network neighbor of A on any of channels $c-1, c, c+1$. Repacking decisions thus have consequences that *propagate* in both the spatial and frequency dimensions: repacking A on channel c not only constrains repacking geographically proximate broadcasters on c (spatial propagation), but also on channels $c-1$ and $c+1$ (frequency propagation). In other words, "local" decisions can have "long-distance" effects.

It is such long-distance influences through the repacking constraint network that give rise to a computationally challenging repacking problem for *any* proposal for the reverse auction mechanism. The simplest version of the repacking problem is referred to as *feasibility checking* in the NPRM and MALS documents, and can be formulated as follows: *Given the repacking constraint graph G over n broadcasters (vertices) described above, and a subset R*

[33] The NPRM discusses several different possible interpretations of this requirement, none of whose details materially alter the subsequent discussion.

of the broadcasters to be repacked, determine if there is an assignment of the broadcasters in R onto k contiguous channels in a way that obeys all the repacking constraints in G. In an *optimization* version of the problem, there is a price or valuation associated with each broadcaster, and the goal is to find the set R of size k that can be feasibly repacked whose sum of prices or valuations is maximized (e.g. thus minimizing the cost for clearing the set of winning broadcasters).

As we shall discuss, depending on the details of the particular reverse auction mechanism under consideration, one of these variants of the repacking problem arises.[34] But the computational complexity of all of these problems is that they are all provably intractable on worst-case inputs.[35] This means that if the graph G is arbitrary, then the best algorithm for solving these problems will require computation time that scales *exponentially* with the number of broadcasters; in practice, this would mean that these problems could be feasibly solved only for very small instances, much smaller than the number of broadcasters who will participate in the reverse auction.

Given this worst-case computational intractability of repacking problems, there are three broad approaches that can be considered:[36]

1. Employ general-purpose algorithms known to find optimal or exact solutions, even though their worst-case running time may be computationally infeasible, and hope that the actual running times on the "real data" (most importantly, the actual repacking constraints, but also potentially including clearing targets, prices, etc.) will in reality be manageably modest.

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[34]The first variant arises in the descending clock auction, while the second arises in sealed-bid mechanisms.

[35]This can be proven via a reduction from the well-known graph coloring problem.

[36]It is worth remarking that some variations of the mechanisms under consideration could greatly ease the real-time computational burdens of the reverse auction. For example, the single-pass clock auction with proxy bidding discussed in Section 14.2 would permit feasibility checking computations to be performed "offline" (that is, not during the auction itself, but after the fact), which might allow slower algorithms that find better solutions.

2. Employ general-purpose algorithms known to be computationally efficient, but which may find suboptimal or approximate solutions.

3. Design special-purpose algorithms that exploit the structure of the actual repacking constraints, hopefully achieving fast computation time and near-optimal solutions.

Approaches 1 and 2 require no advance knowledge of the repacking constraints (though knowledge of them may permit simulations that shed light on the efficacy of these approaches, which we discuss later), while Approach 3 inherently assumes such knowledge. Note further that hybrids of the three approaches are also possible, and may yield the most appealing tradeoffs.

We now proceed to discuss the computational challenges presented by repacking in the context of the specific reverse auction mechanisms under consideration, concentrating primarily on the descending clock auction of the MALS proposal. Our primary intent is again to show that the release by the FCC of considerably more detailed data on and models for the repacking constraints could allow (a) the assessment of how likely drawbacks are to arise in the actual auction, via computational simulations; and (b) the development of new, specialized approaches to repacking that may yield considerably better outcomes in terms of the amount of spectrum cleared, and the expenditures to do so.

15.2 Computational Complexity in a Descending Clock Auction

The NPRM and MALS documents both describe a descending clock auction, which in its simplest form gradually reduces the price offered to each broadcaster to clear their spectrum.[37] Once a broadcaster declines the current clock offer and exits the auction, it must be repacked. Since it is a weakly dominant strategy for each broadcaster to remain in the auc-

[37] Throughout we shall simplify the discussion by assuming the only options available to broadcasters are to clear their spectrum entirely, or to be repacked in their current band. The NPRM discusses a number of other possible options as well, none of which would ease the computational challenges we are discussing.

tion until its clock offer falls below its private valuation for remaining on-air, this auction can be thought of as processing broadcasters in descending order of valuations (since this is the order in which they will choose to exit the auction under their weakly dominant strategies), and is thus mathematically equivalent to a setting in which the broadcaster valuations are known and we use a "greedy" algorithm that attempts to repack higher-valuation bidders first.

In this mechanism or algorithm, we must ensure the invariant that at each point, each broadcaster currently remaining in the auction could be feasibly repacked, given the set of broadcasters we have already committed to repack. Thus, upon adding a new broadcaster b to the current feasible set R of repacked broadcasters, we must check that each remaining broadcaster b' could be added to R and still obey all repacking constraints. If not, then in the language of the proposals, b' must be *frozen* at its current clock price and paid this price to clear its spectrum, since it can no longer be repacked. The version of the repacking problem that is thus relevant to this mechanism is *feasibility checking* — we must determine whether the given set $R \cup \{b'\}$ can be feasibly repacked — and under weakly dominant strategies, the broadcaster valuations determine the sequential construction of a set of $k = n - t$ broadcasters to repack, where t is the clearing target.

As noted, the major source of computational complexity in the clock auction is the repeated feasibility checks to determine which bidders must be frozen as the repacking set R is gradually expanded; this problem is computationally intractable in general. As per the algorithmic approaches mentioned above, one approach is to solve each feasibility check exactly — that is, employ an algorithm guaranteed to correctly answer each feasibility check, and to produce an assignment of the repacked set R to the targeted number of contiguous channels.

There are indeed a number of general-purpose algorithms for solving such problems, including what are broadly known as *satisfiability* or *constraint satisfaction problem* algorithms.[38] While many such algorithms yield fast computation times on various large-scale, real-world problems and benchmarks, the question of how well they would perform in the context of the actual reverse auction is an empirical one that can only be approached via simulation and detailed knowledge of the repacking constraints.

The NPRM and MALS proposals seem to suggest the intention to use such a general-purpose, exact algorithm for feasibility checking in the clock auction. An alternative approach would be to employ an algorithm for feasibility checking that is ensured to be computationally efficient, but may find suboptimal or inexact solutions. We describe one simple such algorithm here. Suppose that for the current repacking set R we have already found an assignment of each broadcaster in R to one of the k contiguous channels $1, \ldots, k$, in a way that violates no repacking constraint. Then to decide if broadcaster b is a feasible addition to R, and to extend the assignment to channels, we can simply consider assigning b to the first free (not already assigned) channel for which no neighbor of b in the repacking constraint network has already been assigned to an adjacent channel. If no such channel exists, we declare that b cannot be repacked given the current R and assignment. The difference between this algorithm and an exact feasibility check is that once a repacked broadcaster is assigned to a channel, the algorithm never considers changing that assignment in order to accommodate later repackings. For this reason we shall refer to this as the *myopic* feasibility checking algorithm, since it makes short-term repacking decisions without regard for later or global consequences.

While it is not difficult to find xamples where this myopic procedure produces suboptimal

[38] `http://en.wikipedia.org/wiki/Constraint_satisfaction_problem`

results, in the sense of being forced to clear more than the desired number of broadcasters,[39] it is an empirical question whether this heuristic, or some other one, would badly underperform in the actual auction. With enough knowledge about the repacking constraints, it is possible that a feasibility checking procedure balancing performance with computational efficiency could be designed.

Repacking Inefficiency in the Descending Clock Auction

In addition to the computational challenges and trade-offs discussed so far, the descending clock auction format may limit the ability to reach near-optimal solutions as measured by repacking efficiency (that is, the sum of valuations of the repacked bidders), because repacking decisions are made sequentially. In its simplest form the FCC would process the broadcasters "greedily" by descending valuation in a completely myopic fashion, without regard to the constraints on later repacking such greedy decisions might impose. A simple example can illustrate the issue.

Example 10. *Suppose there are 5 broadcasters A, B, C, D, E, currently broadcasting on channels 2, 4, 6, 8 and 10 (thus there are no current interferences at all, since no pair is on adjacent channels), and that the target is to clear three of them and to repack the other two onto channels 1 and 2, thus freeing up channels 3 through 10 for wireless spectrum. Suppose that the geographical proximities of the broadcasters transmissions are such that the following pairs of broadcasters* cannot *be repacked together: AB, AC, and AD. We can think of A as a high-power, wide-area broadcaster who would have interference with the nearby broadcasters*

[39] For example, let there be five broadcasters A, B, C, D, E to be considered in turn for repacking on 5 channels, and assume that the geographical proximities are such that the following pairs of broadcasters *cannot* be repacked together: AB, AC, BC, BD. We first add A to R, assigning it to channel 1. When we next consider B, we must repack it on channel 3, since the constraint AB prevents assignment to channel 2. Broadcaster C is assigned to channel 5 due to the constraints AC and BC. At this point, D cannot be added to this assignment: only channels 2 and 4 are free, and B is adjacent to both, and BD is a constraint. E can be placed on channel 2, giving a final ordered assignment of $AEB * C$, where $*$ denotes an unused channel, and we have failed to repack all broadcasters. However, it is easily verified that there *is* a way to repack all five: the assignment $ADCEB$ violates no constraints.

B, C and D; whereas E is sufficiently distant (or low power) that it can be repacked with any of the others.

Table 3: Bidder Values from Repacking

Broadcasters	A	B	C	D	E
Values of being on air	100	99	99	1	1

Suppose the bidders' values for remaining on air are as shown in Table 3.[40] *Given the repacking constraints, the clock auction proceeds as follows. Suppose the price clock is initially set at 100 and descends gradually. Again, it is a weakly dominant strategy for each bidder to drop out when the clock price reaches their valuation. At the initial price of 100, A is first to drop out, so is added to the repacking set R. But once A is added to R, broadcasters B, C and D cannot be added to R due to repacking constraints. This leaves only E, which can be added to R, so the price clock runs down until E drops out. Hence, we get* $R = \{A, E\}$, $W = \{B, C, D\}$. *If we evaluate the clock auction by the sum of the valuations of the repacked broadcasters, in this example that sum is* $100 + 1 = 101$. *However, there is a much better solution: we could instead choose* $R = \{B, C\}$, $W = \{A, D, E\}$, *which yields a total repacked valuation of* $99+99 = 198$, *almost twice as much as for the clock auction. The problem in this example is that while A has the highest valuation, it also represents the greatest constraints on subsequent repacking.*

Other simple examples (see for instance Example 11) also show that the actual *expenditures* of the clock auction for clearing the desired amount of spectrum can be much more than under other mechanisms such as VCG.

This is an extreme example intended to illustrate a point. We emphasize that the excess

[40] The great variation in valuations in this and subsequent examples is deliberate, and simply intended to show the extent to which different mechanisms can yield very different outcomes, expenditures, etc. Of course, it is worth noting that such variation in broadcaster valuations must be the primary source of any such differences.

expenditure of the clock auction presented in this example is not meant as a critique of the mechanism, but simply to illustrate the potential for suboptimal results, and the way in which it may depend on repacking constraint details.

Hybrid Algorithms, Discounting, and Variable Clock Rates

Of course, there are other algorithms for feasibility checking that could give different tradeoffs than an exact or optimal algorithm which may take exponential time, or a suboptimal or approximate algorithm guaranteed to run efficiently. In particular, it is quite common in such situations to consider various *hybrid* algorithms. For example, we could implement some form of *lookahead* in creating the channel assignment which tried to determine whether repacking a broadcaster will cause severe constraints on further repackings within the next few steps.[41]

The NPRM suggests the possibility of modifying broadcaster bids to exit their spectrum through the use of *weights* or *scores* that would combine the actual bids with a variety of other factors, including the potential difficulties posed by repacking a broadcaster. In particular, one proposal is to score or reweight the bids of broadcasters whose repacking could greatly constrain further repacking, thus making the optimization problem more difficult. Scores would be used for determining the greedy order in which broadcasters are considered for repacking, while the original (unscored) bids would still be used to determine prices for clearing.

Similarly, in the context of a descending clock auction, the analogous idea would be to have the clock prices of "difficult to repack" broadcasters descend more slowly, thus making

[41] Various types of lookahead are quite common, as one example, in chess-playing computer programs, where rather than simulate all possible future trajectories of play after a contemplated move (which would be computationally infeasible), one simulates just the next few possible moves, and chooses moves based on the "quality" of these intermediate states. The idea is to balance near-term reward with longer-term consequences — in the same way we may want to defer repacking a high-valuation broadcaster now because of the future constraints that would impose, we may want to sacrifice capturing a pawn on this move if it could put us in danger of checkmate several moves from now.

them less likely to exit the auction early and thus constrain the repacking process. While the specific goals of introducing such schemes are not discussed at length in the NPRM, presumably one hope is that such scored or modified bids might elicit more computationally and economically efficient behavior from the repacking algorithms.

Another simple example demonstrates the difficulty of designing such scoring or variable clock rate schemes in a way that would have the intended effects, and that actually circumvent the algorithmic problems we have already highlighted. The example also shows a case where the expenditures for clearing are much higher for the clock auction than for VCG.

Example 11. *Let there be the seven broadcasters* A, B, C, D, E, F, G *whose valuations are as shown in Table 4.*

Table 4: Bidder Valuations

Broadcasters	A	B	C	D	E	F	G
Values of being on air	100	99	3	1	1	1	1

Suppose we wish to clear 5 broadcasters and repack 2. Let the pairs of broadcasters who cannot be repacked on the same or adjacent channels be AB, AC, BD, CE*. Then similar to an earlier example, this is a case where the greedy (or clock auction) approach will find a suboptimal solution: First* A *drops out at price* 100*. Then* B *and* C *can no longer be repacked, so their clock prices are frozen at* 100*. We then repack* D *and clear* E, F *and* G *at their valuations. The total expenditure is thus* $100 + 100 + 1 + 1 + 1 = 203$*. But a better feasible solution is to repack* B, C*, and clear* A, D, E, F, G*, and the associated total expenditure (under VCG) is:* $101 + 2 + 2 + 2 + 2 = 109$*.*

In order for some scoring scheme to avoid this excess expenditure when a greedy algorithm or clock auction is applied, it would need to make the scored price (or clock rate) of A less (or slower) than that of B and C, since it is the first step of choosing A that dooms greedy in this example. But on what principle would such a scoring or varying rate be based? Notice that all of A, B and C occur in exactly two constraints each, so a simple "constraint counting" scheme will not suffice. Furthermore, this example can be easily modified to actually have B and C have many *more* constraints than A, and yet still elicit the same suboptimal behavior from the greedy or clock approach. It thus seems quite difficult to imagine a simple scheme of this type that eradicates the drawbacks of this approach. The basic problem is that the mere number of constraints a broadcaster appears in, or even information about the more detailed structure and interaction of these constraints, may not help in designing scoring schemes, since one cannot know in advance which constraints will actually matter in (that is, affect the choices of) the greedy repacking process. In this example, the constraints BD and CE were irrelevant once B and C had to be cleared.

15.3 Computational Complexity in Sealed-Bid Auctions

In addition to the descending clock mechanism, the NPRM raises the possibility of some form of sealed-bid reverse auction. In a sealed-bid mechanism, broadcasters are asked to each submit once a price or bid offered for going off-air and clearing their currently used spectrum. Let $B = \{1, \ldots, n\}$ denote the set of n broadcasters or bidders, and for simplicity let us suppose that there is only a single option on which to bid — vacating broadcast spectrum entirely, or not at all. Let $V = \{v_1, \ldots, v_n\}$ denote the *private valuations* the broadcasters assign to remaining on air — that is, the lowest payments they would accept in order to go off-air. Depending on the mechanism, the broadcasters may bid differently than their valuations — for instance, in a pay-as-bid mechanism, there will be an incentive to bid higher than one's valuation, whereas in VCG it is a weakly dominant strategy to bid one's valuation. Thus we also introduce *prices* or *bids* $P = \{p_1, \ldots, p_n\}$ submitted by the broadcasters.

In the same way that the descending clock auction encounters the feasibility checking variant of the repacking problem, sealed-bid approaches entail a distinct but related computational problem. More precisely, for a targeted number $t \leq n$ of channels to be cleared, we now have an easily stated optimization problem with a well-defined objective function: find that subset W of t "winning" broadcasters to be cleared such that the total expenditure $E(W) = \sum_{i \in W} p_i$ is minimized, *subject to the constraint* that the remaining $n-t$ broadcasters in the set $R = B - W$ can be repacked together. Again, it is this complex combinatorial constraint that makes the computation of the optimal W intractable in general. Note that in the special case of a VCG auction, broadcasters bid their private values as a weakly dominant strategy, so the expenditure $E(W)$ coincides with the *value function* $V(W) = \sum_{i \in W} v_i$, whose minimization corresponds to maximizing the sum of valuations in R.

The NPRM outlines two classes of algorithmic approaches to this problem: *global* algorithms that are generally guaranteed to find an optimal (cost-minimizing) or approximately optimal solution, but that may require inordinate computation time to do so; and what the NPRM refers to as *sequential* (but which could instead be termed *local* or *greedy*) approaches whose solution may be far from optimal, but whose computation time is more modest.

The NPRM further suggests that the repacking constraints are such that the optimization problem falls into the well-known class of Integer Linear Programming (ILP) programs — that is, a linear objective function optimization (in our case, the expenditure function $E(W)$) subject to linear constraints over both continuous and discrete variables. The discreteness arises from presumably multiple sources, including but not limited to the fact that broadcasters must either vacate or not, as opposed to vacating "fractionally," and certain pairs or sets of broadcasters being either mutually exclusive for repacking purposes, or entirely compatible, but not "partially repackable."[42]

The NPRM discusses[43] taking standard algorithmic approaches to the ILP problem. There exist a wide variety of both open-source and commercial software packages for encoding and solving (integer) linear programming problems.[44] In the case of large scale ILP, none of these packages can guarantee rapid convergence (computation time scales polynomially with the number of variables and constraints) to an optimal or even approximately optimal solution. Since presumably the number of constraints defining the feasibility function F_C is almost certainly in the thousands (possibly even tens or hundreds of thousands), we cannot claim that we have a "small" instance of ILP and argue that brute-force search for the optimal solution would succeed in a reasonable amount of time. The NPRM suggests, without providing any supporting evidence or detail, that existing ILP packages might suffice for finding good approximately optimal solutions. This conjecture is an empirical one that cannot be evaluated without simulation studies using specific ILP packages and specific ILPs based on the actual repacking constraints (or at least constraints broadly similar in their number, size and structure).

15.4 Empirical Evaluations of Reverse Auction Mechanisms

The discussion and examples above emphasize the problems of computational and expenditure inefficiency that *could* arise under the reverse auction designs in the NPRM and MALS proposals. But these examples were obviously based on hypothetical repacking constraints

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[42]For example, suppose for each broadcaster i and channel c, we introduce a binary variable $x_{i,c}$ that assumes value 1 if and only if i is relocated to c under repacking, and is 0 otherwise. Then if broadcasters i and j are connected to each other in the repacking constraint graph, in an ILP approach we could introduce, for all channels c, the linear constraints $x_{i,c} + x_{j,c} \leq 1$, $x_{i,c} + x_{j,c+1} \leq 1$, and $x_{i,c} + x_{j,c-1} \leq 1$. These constraints, along with the restriction that the variables $x_{i,c}$ are integer-valued (more precisely, binary), encode the fact that i and j cannot be assigned the same or adjacent channels. (To satisfy each inequality, only one of its two variables may assume value 1.) In this way we can simply represent the repacking constraint network as an ILP.

[43]NPRM, para. 45

[44]See e.g. `http://en.wikipedia.org/wiki/Linear_programming`

of the type suggested in the NPRM, and were designed to exhibit worst-case behaviors of various kinds (though they were simple and did not require complex or unrealistic constructions). The extent to which such difficulties would be encountered in the actual auction depends on a variety of unknown factors, including the detailed structure of the repacking constraints and the likely valuations and bidding behavior of broadcasters.

While eliciting or accurately estimating broadcaster valuations and behavior may be infeasible, doing so for the repacking constraints is not, and would alone permit a variety of activities that could shed significant light on the likely outcomes and potential problems with the various reverse auction designs. Even in the absence of full specification of all repacking constraints, crude approximations — such as identification of which pairs of broadcasters are likely to create interference if repacked on the same or adjacent channels (the aforementioned repacking constraint network) — would allow much better assessment of the proposed auction designs than is currently possible.

The two main activities that would be permitted by having model repacking constraints are simulation studies and the design of specialized algorithms and mechanisms. We discuss each in turn.

Simulation Studies

With model repacking constraints, one could extensively simulate the main proposals for the reverse auction — the descending clock auction and its variants employing algorithms for repacking feasibility checking, and sealed-price mechanisms combined with global optimization using Integer Linear Programming for winner determination. Of course, in both cases one would have to also have a model for broadcaster valuations and bidding behavior, but one could run simulations over a wide variety of assumptions on broadcasters. For example, the simplest model for bidder behavior in the descending clock auction (and the one that in fact motivates that design) is staying in the auction until the clock drops below the broadcaster's valuation. Armed with models for repacking constraints and valuations, we could thus run large-scale simulations that answered some of the key questions we have raised here:

- For a greedy or descending clock approach, how difficult is the exact feasibility checking problem? How effective are heuristic repacking solutions, such as the myopic feasibility algorithm discussed above? And for these and other variants, how suboptimal are the solutions found in terms of expenditures and amount of spectrum cleared?

- For the sealed-bid/ILP approach, is the computation of winning bids subject to repacking likely to be impractically difficult and slow? If so, is the ILP approach able to at least find acceptable approximately optimal solutions quickly? How much worse are these approximations than the global optimal solution?

To illustrate the kinds of simulation studies that could be performed, the questions they could help investigate, and the benefits of more detailed repacking constraint information, we have implemented a very preliminary simulation framework that allows us to compare various quantities (such as clearing expenditures, computation time, and many others) for the main reverse auction mechanism proposals (as well as potential future variants thereof), and for mechanisms that provide useful economic benchmarks. For example, our framework (which is currently implemented in a mixture of Matlab code and open-source optimization algorithm software) permits the simulation of the following mechanisms:

- The descending clock auction, using either an ILP or constraint satisfaction solver for feasibility checking, under the assumption that broadcasters remain in the auction until their offer falls to their valuation for remaining on-air.

- The descending clock auction, but with the exact solution for feasibility checking replaced by the aforementioned and much faster myopic construction of the repacked set and assignment to channels.

- The sealed-bid VCG mechanism, again under truthful bidding, using an ILP solver for the selection of the expenditure-minimizing set of broadcasters to clear.

- A sealed-bid pay-as-bid (first price) reverse mechanism, evaluated under full-information Nash equilibrium bidding, and again employing ILP for the selection of broadcasters to clear.

Essentially any other variant or proposed mechanism could be incorporated into our simulation framework. Our framework also permits the specification of essentially any set of pairwise repacking constraints and broadcaster valuations. For instance, this allows us to simulate varying numbers of broadcasters, with varying density or number of repacking constraints; varying distributions (parametric forms, variances and means, etc) of broadcaster valuations, etc.

In Appendix D, we provide some very preliminary sample simulation results, which highlight the ways in which different reverse auction mechanisms are influenced by repacking constraints and broadcaster valuations, and the fact that which mechanism is "best" (in several senses) depends strongly on these constraints and valuations, thus emphasizing the need for real data. We expect the scale and sophistication of our simulations to improve with such data and the development of our framework.

Design of Specialized Algorithms

The second activity enabled by model repacking constraints would be the design of specialized mechanisms and algorithms that exploit the special structure of these constraints. The majority of algorithmic work in computer science is devoted to finding good algorithms for special cases of problems that are intractable in the general case. A classic example of this is the Traveling Salesman Problem (TSP), which is NP-hard without any assumptions on the network of "cities" and "distances" between them, but for which there is a very efficient algorithm yielding very good approximations to the optimal solution if the network corresponds to a physical, 2-dimensional map. TSP is more generally an excellent example of a hard problem on which tremendous practical progress has been made through a combination of simulation studies and designs for special cases.[45]

Armed with model repacking constraints, one could similarly investigate whether a specialized algorithm for cost-minimizing winner selection in a sealed-bid auction could be considerably faster than with a generic ILP approach. The same could be done for the previously discussed problem of feasibility checking in the descending clock auction. Fast and (near-)optimal algorithms for these problems could materially change the assessment of the corresponding mechanism proposals.

15.5 Specific Requests for Repacking Constraint Data

In light of the discussion, examples, and simulations above, it is clear that repacking complexity is a central concern in any reverse auction mechanism, and that the details of the actual repacking constraints and broadcaster valuations will significantly impact the potential computational complexity, costs for clearing, and other properties of the reverse auction. We therefore now discuss various types of information that could greatly aid the study of reverse auction proposals, and lead to the development of better repacking algorithms specialized to the actual constraints.

An ideal along these lines would be a complete mathematical description of all the known repacking constraints, for all broadcasters across the nation, specified in some formal, machine-readable language or notation (e.g. a network of pairwise incompatibilities for adjacent channels, an integer linear program or constraint satisfaction program, etc.) Related, but slightly more general (and thus less informative about the actual specific constraints) would be a program or parametric model that permits the generation of such complete

[45] See e.g. `http://en.wikipedia.org/wiki/Travelling_salesman_problem`

descriptions under varying assumptions (e.g. power and interference assumptions, service contours, acceptable degradations of service, etc.)

It would also be extremely valuable to have a simple network or networks giving pairwise repacking constraints over broadcasters (i.e. the repacking constraint network), or again, parametric mathematical models for the repacking constraint network under varying assumptions. Related data of value would be geographical (latitude and longitude coordinates) for the transmission location(s) of every broadcaster eligible for participation in the reverse auction.

Part IV

Closing Rules

16 The NPRM and MALS Proposals

An important component of the grand auction design is the integration of the forward and reverse auction. This is where "supply" and "demand" meet to determine how much spectrum will be allocated to TV stations, and how much to wireless carriers. Of course, there are important differences from the classic supply and demand setting. One is that prices paid by forward auction buyers must cover not only payments to reverse auction sellers, but costs of repacking and of the auction itself. Another is that the forward and reverse auction prices do not reflect the valuations of bidders on the margin.

Example 12. *Suppose there are two bidders on each side of the market. One of the forward auction bidders ("buyer H") has valuation 100 and the other ("buyer L") has valuation 50. Likewise, one of the reverse auction bidders ("seller H") has valuation 100, and the other ("seller L") has valuation 50. Suppose the target is to clear one license and that there are no repacking constraints. The reverse auction will end at a price of* 100*, the second lowest seller valuation. Seller L will be provisionally assigned to sell his license. The forward auction will end at a price of* 50*, with bidder H the provisional winner. Clearly the net revenue requirement fails: the provisional forward auction revenue is only half the reverse auction cost. This is not because the transfer of spectrum from seller L to buyer H is inefficient; indeed, this trade would double the surplus created by the spectrum. The problem is that instead of comparing buyer H's willingness to pay with seller L's valuation, the auction rules compare buyer L's willingness to pay to seller H's valuation. This comparison has nothing to do with whether trade should occur.*

The fundamental problem is the separation of the forward and reverse auction markets. Although the purpose of the incentive auctions is to allocate scarce spectrum competitively, a clock auction in the forward market induces competition only between mobile wireless carriers; these bidders compete to be among the provisional winners, but do not compete with television stations for final ownership of spectrum. Likewise, a reverse clock auction creates competition only between current broadcast television licensees. Reverse auction bidders compete to be provisional sellers, but the two sides never compete directly with each other to resolve the question of how much spectrum should be transferred from potential sellers to potential buyers. The result is that both the forward and reverse auction will tend to stop too soon, i.e., at prices that are unnecessarily low in the forward auction and unnecessarily high in the reverse auction. Uncorrected, this could undermine efficiency, just as in the example above.

This limitation is recognized by MALS and underlies their proposal that when net revenue falls short the forward auction clock prices would continue rising until either the revenue condition is satisfied or demand falls short of the current target. This can be interpreted as a one-sided *closing trial* for the grand auction at the current target. It allows forward auction bidders to express their willingness to pay prices above the provisional forward auction prices if necessary to achieve satisfaction of closing conditions at the clearing target.

17 Primary Concerns with the One-Sided Closing Trial

The one-sided closing trial contingency still leaves more room for inefficiency than necessary. This is easily seen in Example 12. A price anywhere strictly between 50 and 100 would result in trade. However, no trade will occur even with the closing trial because buyer H will not be willing to beat the provisional reverse auction price of 100. The problem is that while a one-sided closing trial allows forward auction bidders to express their willingness to offer better prices when necessary to ensure that they are allocated the licenses they seek, there is no such provision for the supply side. Reverse auction bidders lack the opportunity to improve their price offers in the event that closing conditions fail. Thus, even if there are reverse auction bidders who would like to offer prices substantially below those obtained in the reverse auction (recall that these are the maximum prices that clear exactly the target), those bidders will have no opportunity to express this. Put differently, with a one-sided closing trial there is still a barrier to full buyer-seller competition for ownership of the marginal units of spectrum. The result will be unnecessary inefficiency. Some trades that buyers and sellers would all prefer will not take place, not due to the inherent constraints of incentive compatibility, but due to the auction design.

18 Two-Sided Closing Trial

One possible solution to the limitations of a one-sided closing trial seems clear: allow both sides of the market to participate in the closing trial when this phase is required.

18.1 Baseline Proposal

In the baseline proposal, no change to the main forward or reverse auction is needed. However, if the forward auction reaches a point where excess demand is zero but the net revenue condition fails, all parties would be told that the clearing conditions have failed. Prices would then be raised in the forward auction while lowered in the reverse auction. For example, in a sequence of rounds, prices in the forward market could rise by a percentage ρ while those in the reverse auction fall at the same rate. Intra-round bids could be used to avoid overshooting. Bidders on each side would express demand/supply at the clock prices, just as in the original forward and reverse auctions.

Let p_D and p_S denote vectors of clock prices in the forward and reverse auctions, respectively. Let $D(p_D)$ denote the vector of total quantities (e.g., MHz) demanded of each object at the forward market prices, with $S(p_S)$ denoting the vector of total quantities offered for supply at the reverse auction prices. Let Q_t denote the current target clearing quantities.[46] At the beginning of the closing trial we have

$$D(p_D) = S(p_S) = Q_t.$$

The two-sided closing trial would end as soon as either

(a) the revenue requirement is satisfied, or

(b) $\min\{D(p_D), S(p_S)\} < \alpha Q_t$ for some $\alpha \in (0, 1]$.

Condition (b) corresponds to the case in which the closing trial fails. Here $<$ represents the component-wise strict partial order. Thus, if $D^m(p_D)$, $S^m(p_S)$ and Q_t^m are the components of $D(p_D)$, $Q_S(p_S)$ and Q_t corresponding to market m, the condition

$$\min\{D(p_D), Q_S(p_S)\} < \alpha Q_t$$

holds iff *both*

$$\min\{D^m(p_D), S^m(p_S)\} \leq \alpha Q_t^m \ \forall m$$

and

$$\min\{D^m(p_D), S^m(p_S)\} < \alpha Q_t^m \ \exists m.$$

Thus, if $\alpha = 1$, part (b) stops the auction as soon as one bidder (from either side of the market) exits the closing trial. Smaller values of α (for example, $\alpha = \frac{Q_{t+1}}{Qt}$, if clearing targets are uniform across markets) would allow a degree of incomplete clearing of the current target Q_t before triggering a nationwide adjustment to the target to Q_{t+1}. Variations on this rule

[46] Recall that we denote the sequence of clearing targets, beginning with the first, as $Q_1, Q_2, \ldots$.

are possible. For example, one might set the trigger threshold in condition (b) based on nationwide or regional clearance levels rather than by the "worst" market.

If the closing trial ends with condition (a) above, the closing trial succeeds. The grand auction would then terminate. If condition (b) terminates the closing trial, the attempt to clear the target Q_t would fail, the target would be reduced to Q_{t+1}, and the forward and reverse auctions would continue from this point as envisioned in the original MALS proposal.

As long as the clocks in auction $t+1$ are started from the final prices in the auction t closing trial (i.e., strict continuation from the closing trial), incentives for reverse auction bidders are identical to those in the original reverse auction design. The proposed modification does not appear to change incentives in the forward auction either.

The following example illustrates the two-sided closing trial.

Example 13. *Consider a one-market example in which the FCC is attempting to clear 90MHz of spectrum (fifteen channels) in the reverse auction and sell 80MHz of this spectrum in the forward auction in the form of eight 10MHz (5+5) paired licenses (the remaining 10 MHz is reserved for guard bands or other uses). Suppose that the reverse auction stops at a price of 75, whereas each of the 15 stations electing to go off the air at this price values its license at only 50. In the forward auction, prices stop at 100 per license, although the 9 provisional winners each place a value of 130 on a license. Comparing the provisional forward auction revenue of 800 to the provisional clearing cost of 1125, we have a shortfall. Furthermore, a one-sided closing trial is certain to fail: the maximum willingness to pay among the 8 provisional winners in the forward auction is 1040 (i.e., 8×130). Thus, with the MALS auction design, this target will fail and the target quantity will shrink. A two-sided closing trial could avoid this inefficient outcome. Suppose that in the closing trial, prices rise in the forward auction and fall in the reverse auction at equal speeds, adjusting by 1 unit per round. After 15 such rounds, the forward auction revenue (*$8 \times 115 = 920$*) would (more than) cover the reverse auction cost (*$15 \times 60 = 900$*). Thus the closing conditions* *can be satisfied. Note that all bidders should be happy to accept these prices in the closing trial: exiting the closing trial implies moving to a lower target, where the reverse auction price will fall to 50 and the forward auction price will rise to 130. Thus, there will be many information structures in which bidders will voluntarily participate in the closing trial.*

18.2 Closing Trial with Proxy Bidding

Including reverse auction bidders in the closing trial would require them to be called back to the auction when the original MALS proposal would not have required this. This would be required only when net revenue falls short and would be required under the MALS proposal as well unless the one-sided closing trial succeeded. Nonetheless, under the proposal for a single-pass reverse auction (see section 14), this could be avoided altogether, potentially simplifying the closing trial contingency. With the single-pass reverse auction, each bidder will have indicated his willingness to relinquish his license across the full range of prices consistent with the possible clearing targets. This means that the FCC could serve as a proxy bidder for TV stations in a closing trial, keeping a bidder in the auction as long as necessary if the price remains above the minimum price acceptable to the bidder.

We emphasize that taking advantage of this does *not* require that the single-pass auction itself be conducted with proxy bidding.

As long as reverse auction bidders trust the proxy system, this would not alter reverse auction bidders' incentives in the single-pass reverse auction. And up to the inherent ambiguities created by the repacking optimizer, it would be possible for bidders to verify the actions of the proxy agent ex post.

The next section will demonstrate an additional advantage of the single-pass reverse auction in promoting efficiency in the closing trial.

18.3 Avoiding Overshooting

A remaining source of inefficiency in the closing trial arises from potential miscoordination of price clocks. For example, consider a market in which one license is to be cleared. Normalize the net revenue requirement to zero. Suppose the provisional prices and valuations of provisional winners are as follows:

	Forward	Reverse
provisional price	6	9
valuation of provisional winner	10	8

In the closing trial, if the forward and reverse auction clocks move at equal absolute speeds, the closing trial will fail when the forward auction clock reaches a price of 7 and the reverse auction reaches a price of 8. This causes the target to fail even though there are substantial gains to trade remaining. Here, in fact, two-sidedness of the closing trial harms the outcome: a one-sided closing trial would have cleared the target at a price of 9.

Ideally, the clocks would by synchronized in a way that reflected the relative gaps between prices and valuations. In the example above, if the forward price moved at a speed proportional to $(10 - 6)$ while the reverse clock moved at a speed proportional to $(9 - 8)$, the closing trial would end successfully with forward and reverse prices equated at 8.4. Even imperfect coordination can improve outcomes. For example, if the forward clock moves at twice the reverse clock speed, the auction again ends successfully, this time at prices equated at 8. Unfortunately, whether it is better to speed up or slow down one clock relative to another depends on bidders' (unknown) valuations.

Note, however, that the key to efficiency is not actually that the relative clocks speeds be coordinated perfectly, but that the clock does not *overshoot* valuations on one side before allowing bidders on the opposite side to express their full willingness to adjust the provisional prices. With the single-pass reverse auction, substantial progress toward this goal can be achieved using variable clock speeds, exploiting the fact the supply at any possible price vector is already known. As a simple example, forward and reverse auction clocks could be adjusted at identical speeds in the closing trial as long as this does not cause the closing conditions to fail due to a drop in supply, i.e., as long as at the new prices p_S

$$Q_S(p_S) \geq \alpha Q_t.$$

When the proposed change in the clock prices would violate this condition, the reverse auction clocks would stop, and only the forward auction clocks would continue. Variations on this example could achieve the same goal. The key is that, with a single-pass reverse auction, overshooting on the supply side can be avoided.

This modification does not prevent overshooting in the forward auction during the closing trial. Of course, the risk of such overshooting is only worse in a one-sided closing trial. Further, by beginning the two-sided closing trial with the forward auction clock moving more slowly than the reverse auction clock (the latter controlled to avoid overshooting as described above), one could minimize the chance of any overshooting.

One possible concern with our proposed strategy for avoiding overshooting is that it creates incentives for reverse auction bidders to manipulate their bids, exiting at prices a bit above their reservation values. However, because there will be many reverse auction bidders, the expected gain to a single bidder from such behavior may be small relative to the cost of forgoing profitable selling opportunities.

Appendices

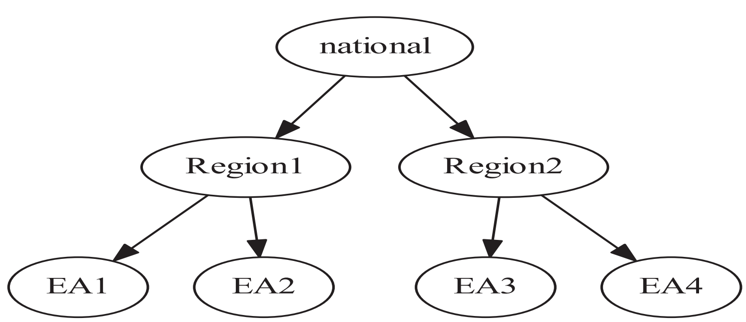
A Clock Package Auction Simulations

The goal of these simulations is to explore the potential performance of the CPA relative to the simple clock auction proposed by MALS. The simulations are not intended to be "realistic" in all aspects. For example, we consider a small number of EAs and do not attempt to create valuations that are realistic in terms of dollar values. Rather, our goal is to illustrate relative performance of the CPA design in a variety of settings aimed at capturing the essential qualitative features likely to be present in the forward auction. The code is written in Matlab and is sufficiently flexible to allow a wide range of further modifications.

A.1 Simulation Design

Licenses and Objects

We specify a three-tier hierarchical structure in which the objects for sale are EA objects, regional objects, and national objects. The scale of the hierarchical structure is controlled by a parameter m. There are $4m^2$ EAs and $2m$ disjoint regions. For our baseline specification we set $m = 1$, yielding the structure illustrated below



The feasible supply of each object is controlled by a parameter k. In our baseline specification we model the case of partially unequal spectrum clearing across EAs. For three quarters of the EAs, we set the feasible supply equal to k; for the remaining quarter, we set the feasible supply to $k-1$. For package objects j, $q_j^f = \min_{k \in \chi_j} q_k^f$ (recall that χ_j represents the children of object j). We set $k = 4$ in the baseline specification.

Bidders

We model three types of bidders: *national bidders*, *regional bidders*, and *local bidders*. National bidders value all objects and obtain additional value (complementarities) when they can obtain a regional license or nationwide license with a guarantee of horizontal contiguity. Each regional bidder is associated with a single region. A regional bidder values the EA licenses in "his" region, and obtains complementarity when he can obtain the regional license with its guarantee of horizontal contiguity. Each local bidder is associated with a single EA and desires one only a license for that EA.

For our baseline specification we have 4 nationwide bidders. We start with 3 regional bidders for each regional license and 2 local bidders for each EA license. However, we use randomization to reduce the numbers of regional and local bidders in each replication, creating variation in the level of competition. This is explained below.

Valuations

Valuations are generated randomly for each replication. The valuations of bidder i are characterized by a vector $V_i = (V_{i1}, \ldots, V_{iJ})$ with one component for each object. We construct the components of each bidder's valuation as

$$V_{ij} = V_{ij}^{+} \times \tilde{V}_{ij}$$

where V_{ij}^{+} is either zero or one (indicating which objects bidder i may place positive value on) and $\tilde{V}_{ij}$ is a random variable reflecting the valuation itself.

For a nationwide bidder i, we set $V_{ij}^{+} = 1$ for all objects j. For a regional bidder i, $V_{ij}^{+} = 0$ for all j with probability α_1. With probability $(1 - \alpha_1)$, $V_{ij}^{+} = 1$ for j corresponding to his regional package and the EAs contained in that package. The randomization has the effect of creating variation across simulations in the number of regional bidders, since assigning a bidder to have no positive valuations is equivalent to eliminating this bidder. Similarly, for a local bidder i, $V_{ij}^{+} = 0$ for all j with probability α_2. With probability $(1 - \alpha_2)$, $V_{ij}^{+} = 1$ for the EA j with which i is associated. All other V_{ij}^{+} are zero. In our baseline simulations we set $\alpha_1 = 1/3$ and $\alpha_2 = 1/2$. This leads to an average of 1 local bidder for each EA license, and an average of 2 regional bidders for each regional package.

For EA objects j, we let

$$\tilde{V}_{ij} = \begin{cases} 0 & \text{w.p. } \alpha_3 \\ \text{u}[0,1] & \text{w.p. } 1 - \alpha_3 \end{cases}$$

for each bidder i (equivalently, for each i such that $V_{ij}^{+} = 1$), where u$[0, 1]$ indicates a random draw from a standard uniform distribution. The parameter α_3 is used to create random variation in the number of local bidders, but also introduced additional heterogeneity in package bidders' valuations. These valuations for EA licenses are the building blocks of all valuations. We set $\alpha_3 = 0.1$ in the baseline simulations.

For regional objects j, we let

$$\tilde{V}_{ij} = (1 + \beta_1) \sum_{k \in \chi_j} \tilde{V}_{ik}$$

where β_1 is a random parameter determining the degree of complementarity. For our baseline specification we let β_1 =u$[0, 1/2]$. Similarly, for the nationwide object, we specify

$$\tilde{V}_{ij} = (1 + \beta_2) \sum_{k \in \chi_j} \tilde{V}_{ik}$$

where β_2 is a random parameter. For our baseline specification we let β_2 =u$[0, 1]$.

Observe that the vectors V_i have length equal to the number of objects. From V_i we construct valuations for all possible subsets of objects i might purchase. For simplicity, we allow bidders to value at most one unit of each object and at most one unit of each EA, whether purchased alone or in a package. To obtain valuations for bundles, when these bundles are not themselves objects, we assume additive valuations. Thus, for example, a national bidder i who purchases a regional license j and an EA license k, where $k \not\subset j$, has valuation for this bundle equal to $V_{ij} + V_{ik}$. Similarly, a regional bidder who obtains two EA licenses j and k without the guarantee of horizontal contiguity provided by the package is assumed to value the component EA licenses with no complementarity, so that his valuation is $V_{ij} + V_{ik}$ for the pair. We let $\bar{V}_i$ denote the resulting extended vector of valuations, which now includes valuations for all objects and for all bundles of objects consistent with the restrictions above.

Note that the costs of the bundles which are not themselves objects in the package auction can be constructed from clock prices in the same way as done for the valuations. For example, if $P = (P_1, \ldots, P_J)$ denotes the current clock prices associated with each object, the cost of purchasing two objects j and k (for example, the regional license and excluded EA license described above) is $P_j + P_k$. We let $\bar{P}$ denote the resulting extended price vector.

Clock Package Auction

The auction begins at a reserve price $r = 10^{-6}$ for each EA object. Reserve prices for package objects are additive in the reserve prices of the EAs they contain We use a negligible bid increment of 10^{-4}.

We run the CPA with straightforward bidding. At each new vector of prices, for each bidder i we construct the vector

$$\bar{\Pi}_i = \bar{V}_i - \bar{P}$$

which is a list of profits the bidder would obtain from each possible choice of demand if the current prices turned out to be the final prices. Bidder i expresses 1 unit of demand for the profit-maximizing choice from this list. Once the straightforward demands of all bidders are constructed, we calculate the excess demand for each object as described in the text. Clock prices then adjust on objects with excess demand until we reach prices at which there is no excess demand for any object.

MALS Clock Auction

Using the same valuations, we also run the clock auction without packages. The setup is the same, but there are no package objects for sale. Because straightforward bidding in the MALS auction involves significant exposure risk to bidders who have complementary valuations,[47] we assume conservative straightforward bidding. For purposes of generating bids, this is equivalent to setting the complementarity parameters (β_1 and β_2) to zero and having bidders follow straightforward bidding strategies. Thus, given this modification of valuations (and the elimination of package objects in the auction), we generate bidders' profit maximizing demands at each price vector exactly as described above. However, we do not eliminate the complementarities when assessing the value bidders obtain from the final allocation. Thus, if a bidder ends up with all components of a package, we assume he receives a payoff reflecting his full complementarities. This gives a potentially significant "unfair" advantage to the MALS clock auction, since it ignores the fact that complementarities may

[47] Further, in some cases—for example, with symmetric information in Example 1—it requires such bidders to follow strategy that is certain to lead to a loss.

depend on frequency contiguity. Thus the performance gains we find for the CPA will tend to be understated.

A.2 Results: Baseline Specification

We begin with the results under the baseline specification described above. Table 5 summarizes the parameters of our baseline specification.

Table 5: Baseline Parameters

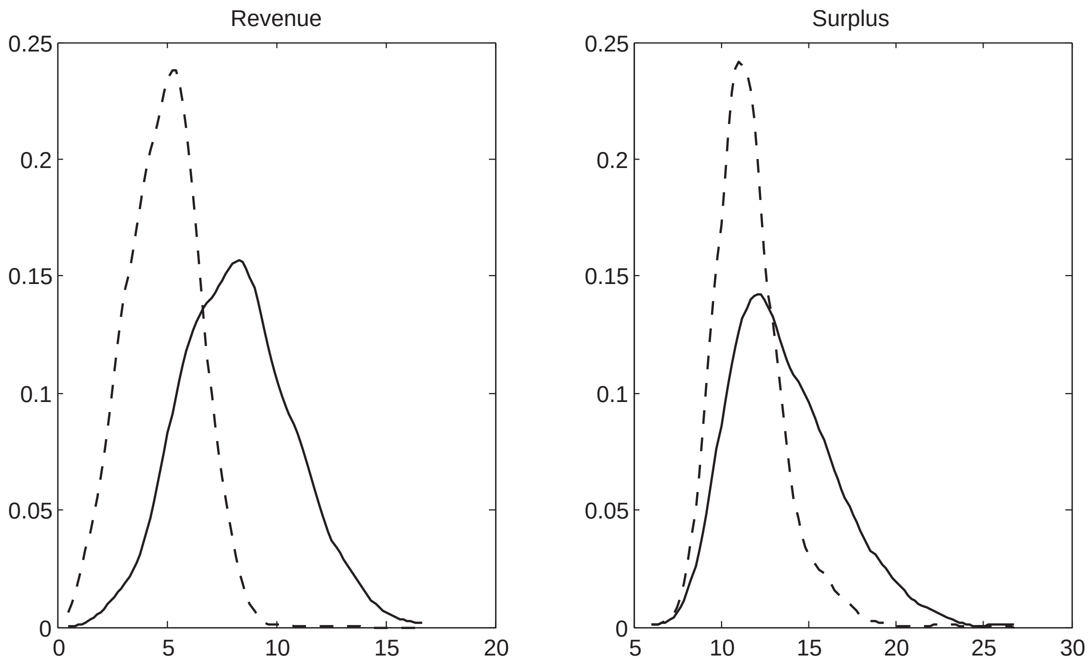
m	=	1
k	=	4
α_1	=	0.33
α_2	=	0.5
α_3	=	0.1
β_1	=	u[0,0.5]
β_2	=	u[0,1]
increment	=	10^{-4}
reservation price	=	10^{-6}
n simulations	=	1000
national bidders	=	4
regional bidders	=	6
local bidders	=	8

Note that we use the same simulation draws for both the CPA and MALS auction. This avoids introducing simulation error into comparisons and will allow us to make pointwise (replication by replication) comparisons between the two types of auction.

We first examine the performance of the CPA and the MALS clock auction in terms of revenue and efficiency. We measure efficiency by total surplus, i.e., the sum of winning bidders' valuations for the objects they win. This ignores the potential influence of revenues on efficiency via the quantity of spectrum ultimately cleared. Because the CPA consistently generates higher revenue in our simulations, this implies that our comparisons of efficiency are (again) likely to understate the gains from the CPA. We also do not consider re-offering of unsold licenses when undersell occurs. As seen below, there is slightly more undersell in the CPA, so allowing the re-offer of unsold licenses would only further strengthen the relative performance of the CPA.

Figure 3 illustrates the revenue and surplus outcomes using kernel density estimates applied to the realized revenue and surplus across all 1000 replications.[48] The left panel shows a significant rightward shift in the distribution of revenue when moving to the CPA, indicating significant gains in revenues. There is a similar but smaller rightward shift in the distribution of surplus.

Figure 3: Distribution of Revenue and Surplus



Note: This figure shows kernel density fits to the realized revenue (left panel) and surplus (right panel) across 1000 replications. Solid curves = CPA; dashed curves = MALS auction.

Table 6 shows key summary statistics for these outcomes. Here we report the mean, standard deviation, median, 25th percentile, and 75th percentile of each outcome measure.

[48]Kernel density estimates can be thought of as smoothed histograms. Thus, they illustrate the relative "frequencies" (more precisely, estimates of the relative "likelihoods") of outcomes across the sample.

On average the CPA produces 70 percent higher revenues than the MALS auction, and surplus that is 17 percent larger. In both cases the standard deviation of outcomes is somewhat higher under the CPA. However, much of this is can be accounted for by a change in scale corresponding to better outcomes under the CPA. In particular, the coefficient of variation for revenue is only slightly higher under the CPA than under the MALS design, and the coefficient of variation for surplus is smaller under the CPA.

Table 6: Summary Statistics

	Revenue		Surplus		% Units Sold		Surplus/Optimal	
	CPA	MALS	CPA	MALS	CPA	MALS	CPA	MALS
Mean	8.2	4.8	13.6	11.6	96.8%	99.7%	97.2%	84.3%
Std	2.5	1.6	3.0	1.9	5.1%	1.5%	4.4%	11.1%
Median	8.1	4.9	13.0	11.4	100%	100%	100%	85.5%
q25	6.4	3.7	11.4	10.4	93.3%	100%	95.0%	76.5%
q75	9.9	5.9	15.4	12.6	100%	100%	100%	94.2%

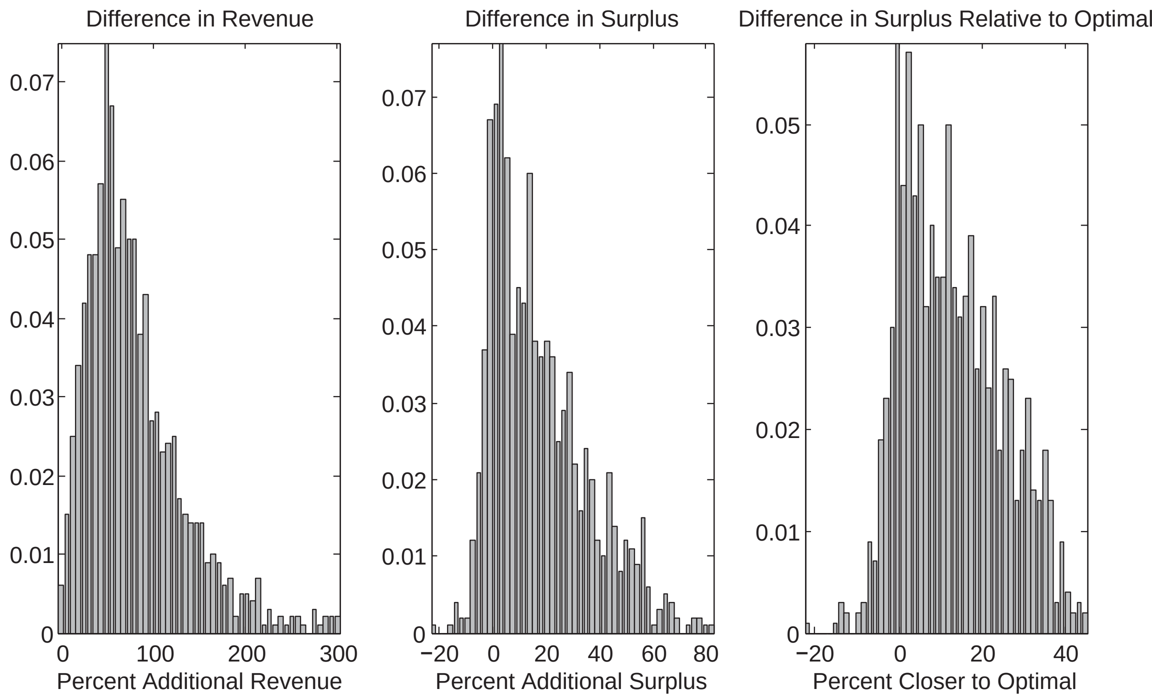
Note: This table reports summary statistics for the baseline simulations. The labels "q25" and "q75" indicate the 25th and 75th (respectively) percentile values of each outcome measure.

Table 6 also shows the share of licenses sold. The CPA sometimes results in undersell, typically a failure to sell one unit of one EA license. This undersell is occasionally due to lack of demand for some EA object (recall that the number of regional and local bidders with positive valuations is stochastic), but most often due to imperfect coordination of price clocks (recall Example 9). Nonetheless, the degree of undersell is small, with 97 percent of licenses selling on average, and 100 percent of license selling in more than half the replications. Undersell occurs rarely in the MALS auction under conservative straightforward bidding.

Table 6 also reports the surplus achieved as a share of that in the efficient (i.e., first-best) allocation. This comparison shows that the gains in efficiency simply could not be much greater than what is achieved by the CPA. The CPA significantly outperforms the MALS auction, achieving 100 percent of the efficient surplus more than half the time, and 97 percent on average. The MALS auction achieves full efficiency much less often and yields only 84.3 percent of the efficient surplus on average.

Figure 4 provides another way of looking at the same outcomes. Here we show histograms of the differences in revenue, surplus, and share of the efficient surplus *replication by replication*. We knew that there was no strict dominance relation between the two auction designs. Further, even if the CPA outperforms the MALS auction with high probability, there could in principle be replications in which the CPA does much worse. Figure 4 reveals that this is not the case. Each panel summarizes the pointwise (replication by replication) differences in performance. These difference can be interpreted as the gains from the CPA, although the "gains" can be negative. The first panel shows that the CPA generates higher revenues in virtually all (99.6 percent) of the replications. The magnitude of the gap is often quite large. As shown in Table 7, the median percentage gain in revenue from the CPA is 67.1 percent, and the 75th percentile gain is more than 100 percent.

Figure 4: Distribution of the Percent Difference



Note: This figure shows the histogram of the pointwise (replication by replication) differences between the CPA and MALS auction designs in terms of revenue, surplus, and share of first-best surplus achieved.

The second panel of Figure 4 shows that CPA also generates strictly greater efficiency than the MALS auction in a large majority (85.9 percent) of the replications. Table 7 shows that the median efficiency gain is 13.5 percent. In 11.7 percent of the replications the CPA yields lower surplus, but on average the CPA yields a 17.5 percent gain.

Table 7: Summary Statistics : Pointwise CPA Gains

	Revenue	Surplus
Mean	82.8%	17.5%
Std	70.0%	18.0%
Median	67.1%	13.5%
q25	42.7%	3.2%
q75	105.1%	28.0%

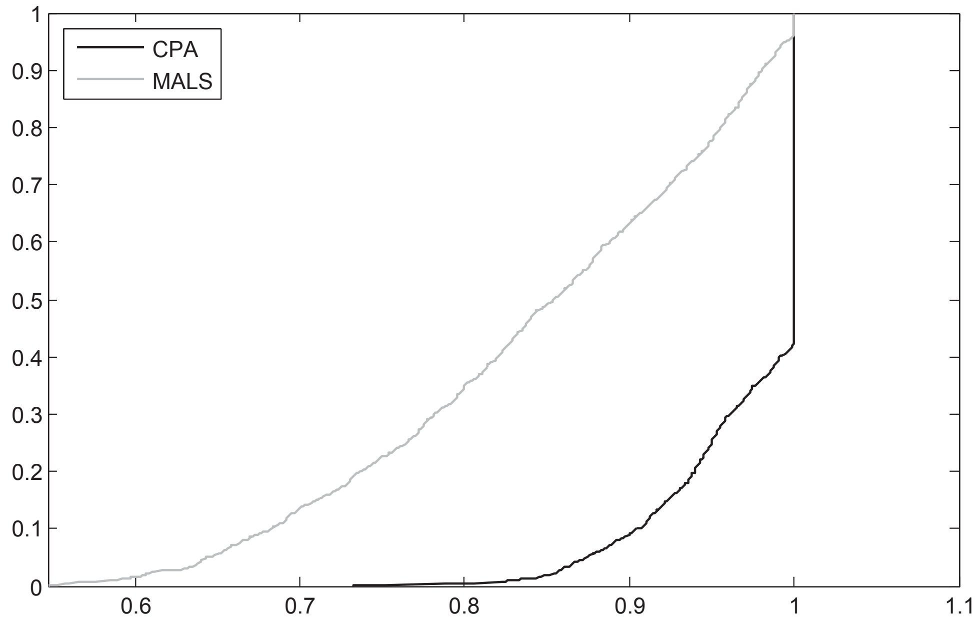
Note: This table summarizes the distribution of outcomes obtained across 1000 simulations. The numbers represent the percent differences between the outcome in the CPA and that in the MALS auction.

In the third panel of Figure 4 we normalize (divide) the surplus outcomes by the first-best surplus, showing the distribution of differences in the share of the efficient surplus achieved (i.e., share under CPA minus share under MALS). As already shown, there are some replications in which the MALS auction is closer to full efficiency. However the gap in these cases is typically small, and a preponderance of replications demonstrate significant efficiency gains from the CPA design. As already shown in Table 6, these gains reach full efficiency in more than half the replications.

Figure 5 illustrates the efficiency outcomes in yet another way, showing the empirical distribution of total surplus achieved normalized by the first-best surplus.[49] Here we see the substantially stronger performance of the CPA. For example, the CPA achieves at least 90 percent efficiency in over 91 percent of the replications. The MALS auction achieves this level of efficiency in only 37 percent of the replications.

[49] The *empirical distribution* plots the share of replications (on the vertical axis) in which the efficiency achieved is *lower* than the level indicated by the horizontal axis. For example, at 100 percent efficiency (horizontal axis equal to 1) the figure shows that fewer than 42 percent of the replications yielded less than full efficiency. Put differently, the vertical segment of the CPA empirical distribution at 100 percent efficiency indicates that more than 58 percent of replications achieved the first-best allocation.

Figure 5: Empirical Distribution of Surplus Relative to the Optimal



A.3 Sensitivity Analysis

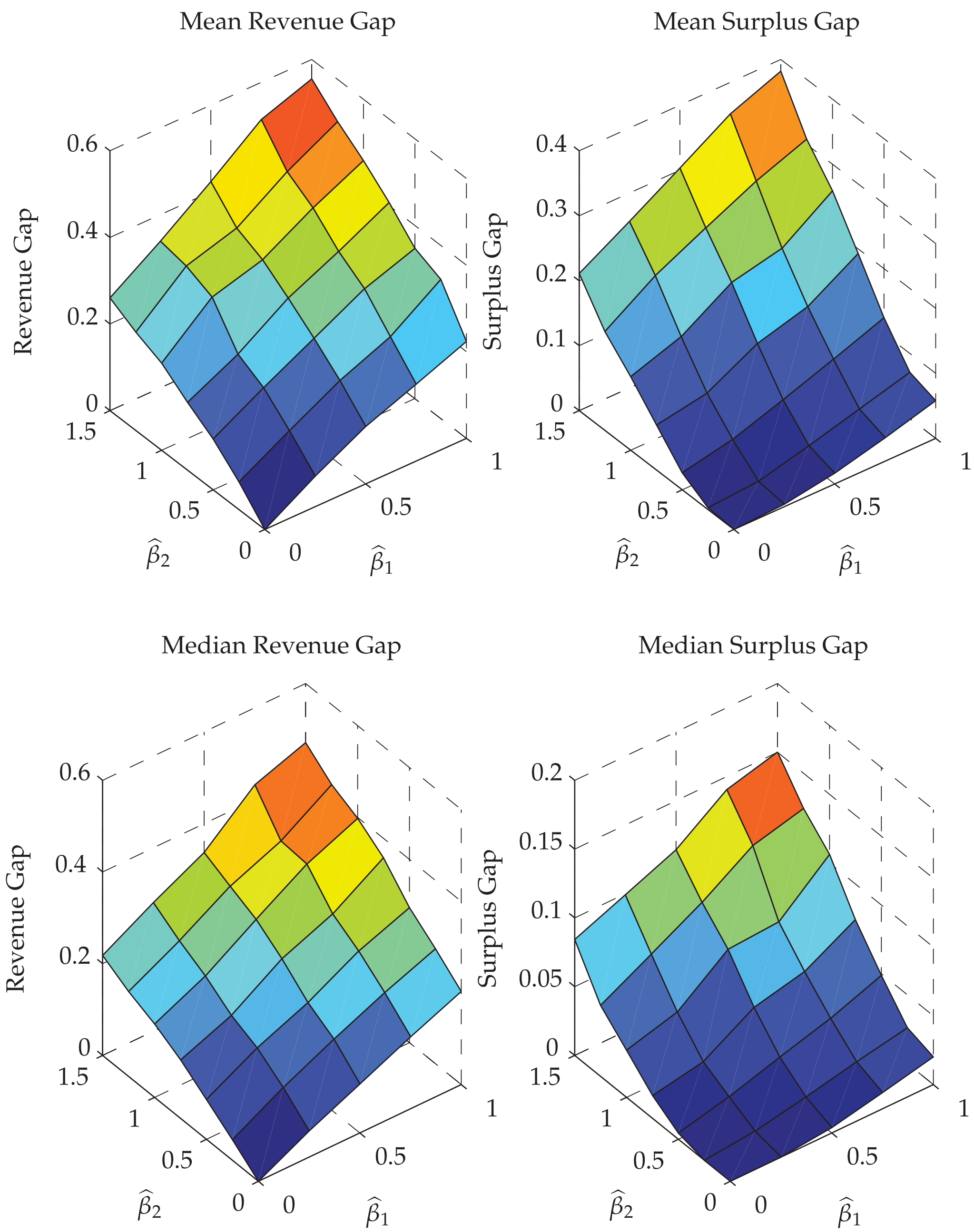
Strength of Complementarities

We first consider the sensitivity of the simulation results to the strength of the complementarities, as determined by the distributions of the parameters β_1 and β_2. We do this by altering the supports of the uniform distributions. Specifically, we let $\beta_1 \sim u[0, \hat{\beta}_1]$ and $\beta_2 \sim u[0, \hat{\beta}_2]$ and re-run the simulations using different combinations of values for the parameters $\hat{\beta}_1$ and $\hat{\beta}_2$. We let $\hat{\beta}_1$ vary between 0 and 1, while $\hat{\beta}_2$ varies between 0 and 1.5. At each pair of values $(\hat{\beta}_1, \hat{\beta}_2)$ we run 500 simulations.[50]

We summarize our findings in Figure 6, where we show the mean and median gains in revenue and surplus from the CPA at each combination of parameters. As we would expect, larger values of the complementarities lead to larger gains from the CPA. However, the qualitative findings remain consistent with those in our baseline specification.

[50]Results change very little with a larger number of draws.

Figure 6: Sensitivity Analysis



Note: These figures show the percent difference (CPA minus MALS) in revenue and surplus of the mean and the median comparison over 500 simulations. For each set of simulations, β_1 and β_2 are drawn randomly for each auction from uniform distributions with supports $[0, \hat{\beta}_1]$ and $[0, \hat{\beta}_2]$, respectively.

Within-Auction Complementarity Heterogeneity

In our baseline specification the complementarity parameters β_1 and β_2 were drawn once for each auction replication. Thus the same complementarity parameter applied to every regional and national bidder. Because these parameters interact multiplicatively with valuations, this implies different levels of complementarities for each bidder. Nonetheless, it may be more reasonable to treat the complementarity parameters as themselves varying across bidders. We have therefore re-run the baseline simulations drawing β_1 and β_2 randomly for each bidder in each simulated auction. We summarize the results in Tables 8 and 9. Comparing the results to those in Tables 6 and 7, we see that the outcomes are nearly identical to those in the baseline specification.

Table 8: Summary Statistics

	Revenue		Surplus		% Units Sold		Surplus/Optimal	
	CPA	MALS	CPA	MALS	CPA	MALS	CPA	MALS
Mean	8.1	4.9	13.7	11.6	96.2%	99.7%	97.0%	82.6%
Std	2.0	1.6	2.4	1.8	5.9%	1.6%	4.8%	8.9%
Median	8.1	4.8	13.5	11.5	100%	100%	100%	82.7%
q25	6.8	3.8	12.1	10.5	93.3%	100%	95.0%	76.3%
q75	9.5	6.0	15.3	12.5	100%	100%	100%	89.2%

Note: This table summarizes outcomes obtained across 1000 simulations using random draws of β_1 and β_2 for each bidder.

Table 9: Summary Statistics : Pointwise Comparison

	Revenue	Surplus
Mean	77.9%	18.9%
Std	55.0%	15.4%
Median	66.2%	17.8%
q25	44.3%	7.4%
q75	96.9%	29.1%

Note: This table summarizes the distribution of outcomes obtained across 1000 simulations using random draws of β_1 and β_2 for each bidder. The numbers represent the percent differences between the outcome in the CPA and that in the MALS auction.

B Other Concerns with CCA Supplementary Bidding

We provide further details on the concerns associated with a combinatorial clock auction (CCA) approach to supplementary bidding in Section 9.

First, bidders in the proxy phase may not have appropriate incentives for bidding for all relevant packages, and clock bids that are used to impute for the missing package bids may not accurately price the "social opportunity costs." To illustrate, suppose there are four units of a (homogeneous) license, and there are two bidders 1 and 2, each of whom values his i-th unit at $5 - i$, for $i = 1, 2, 3, 4$. Suppose the clock price rises in small increments each round from zero. If bidders bid straightforwardly and truthfully, then the clock auction ends with each bidder winning two units at the price of 2. All four units are sold, and this means that under the suggested activity rule (e.g., Ausubel and Cramton (2011)) the allocation is not changed, and more importantly, any profile of feasible strategies by the bidders forms an equilibrium, meaning that each bidder is indifferent across all feasible package bids (see Levin (2011) for instance). It is then conceivable that the bidders may not bid at all or only bid for their winning packages. Suppose the bidders make no supplementary bids.[51] In that case, the "proxy agent" uses the clock bids to impute the bidder's package bids, so the bids of 4, 6 and 4 will be entered as bids for packages of 4, 3 and 2 units, respectively.[52] Notice that these package bids understate the bidder's true values, which are $10(= 4+3+2+1), 9(= 4+3+2)$ and $7(= 4+3)$, for 4, 3 and 2 units, respectively. Since the preferences are substitutable, the prices charged for the winning bidders are VCG, but importantly they are computed based on the "imputed" package bids. Consequently, each bidder pays $2 = 6 - 4$,[53] below the true VCG payment, $3 = 10 - 7$.

[51] A similar result holds if they bid only for the winning package of the clock phase.

[52] Interestingly, the package bids can be non-monotonic. Of course, the non-monotonicity may arise even if the bidders do make supplementary bids, given their indifferences. Incidentally, such a non-monotonicity has been observed in the UK 10-40GHz auction.

[53] That is, the price is the value of the most profitable package for the opponent, which is no more than 6, minus the value of the opponent's winning package, which is 4.

Second, the ambiguous bidding incentives in the proxy phase may make CCA susceptible to collusion. Bidders have a relatively simple way to support a collusion. They can (tacitly) agree to divide the licenses among themselves, and they can drop their demands quickly (but dropping one unit at a time for each increment) to their agreed-upon levels. In the proxy phase, they can simply go "quiet" and not make any supplementary bids, unless some bidder has deviated in the clock phase, in which case they all bid truthfully for the packages in the proxy phase.[54] This behavior can be supported as an equilibrium due to the feature that bidders become indifferent over all of their feasible bids in case all license are sold; the indifference means that a bidder has no obvious incentive to deviate from a collusive agreement. More importantly, even bidding straightforwardly during the clock phase can lead to a collusive outcome, as illustrated above.

Third, the ambiguous bidding incentives in the proxy phase could compromise bidders' incentive for bidding truthfully in the clock phase and could lead to an inefficiency, as pointed out by Levin (2011). To see this, consider the above example again. There exists an equilibrium in which bidder 1 bids slightly more than 3 for the third unit to win 3 units and bidder 1 bids true marginal values to win only 1 unit, and thereafter in the proxy phase bidder 1 bids truthfully and bidder 2 goes "quiet" and makes no bid. As noted, since all items are sold in the clock phase, any strategy profile, including the described behavior, is an equilibrium in the proxy phase. Given this behavior in the proxy phase, the described behavior of the bidders in the clock phase is supported as an equilibrium.[55] Clearly, the resulting allocation is inefficient.

[54] This is possible if they had demanded such packages in the clock phase.

[55] Against the truthful bidding by bidder in the proxy phase, bidder 2 can do no better than bidding true marginal values in the clock phase. Against the "quiet" behavior by bidder 2, bidder 1 pays only $2 = 6 - 4$, whereas if he wins only 2 units, he will still pay $2 = 6 - 4$ (as observed earlier).

C Proof of Theorem 4

Theorem 4. *Consider the MALS reverse auction and assume that clock prices decline continuously and with speeds that do not depend on the actions of bidders. Then straightforward bidding is a weakly dominant strategy for all bidders.*

Proof. Consider an arbitrary bidder i. Let $v_{i0}, v_{i1,} \ldots, v_{iK}$ denote i's valuations of each option, where option 0 is "exit" (remain in one's currently assigned band) and options $1, \ldots, K$ are relinquishment options. Let $p_i = (p_{i0}, \ldots, p_{iK})$ denote the clock prices offered to bidder i at a given instant in the auction. The price p_{i0} is always zero. The price of an option k that is infeasible for bidder i is specified as $p_{ik} = -\infty$. For each $k = 0, \ldots, K$ let

$$\pi_{ik}(p_i) = v_{ik} + p_{ik} - v_{i0}.$$

Thus, $\pi_{ik}(p_i)$ represents the gain (relative to the "exit" option) i would realize at the current clock prices by choosing option k. Of course, $\pi_{i0}(p_i) = 0$. Let

$$\bar{\pi}_i(p_i) = \max_k \pi_{ik}(p_i).$$

We will need to consider three possible types of deviations from the straightforward bidding strategy (the strategy of choosing option $k^*(p_i) \in \arg\max_k \pi_{ik}(p_i)$ at every price vector p_i). (i) Suppose i exits when $\bar{\pi}_i(p_i) > 0$. Since exit is irreversible, this deviation has no effect on i if he would ultimately have been assigned option 0 by following straightforward bidding. So this deviation can affect i only when it changes his final assignment from some $k > 0$ to 0. Since straightforward bidding guarantees i a payoff of at least v_{i0} and could have led to strictly higher payoff (e.g., if all others exited immediately at their current price offers), exiting when $\bar{\pi}_i(p_i) > 0$ is weakly dominated at p_i. (ii) Now suppose i chooses an option $k > 0$ even though $\bar{\pi}_i(p_i) < 0$. Relative to straightforward bidding when the price vector is p_i, the only way this choice could change i's payoff is if i ends up assigned to some option $k > 0$. But in such cases i's payoff is strictly less than v_{i0}. Since v_{i0} is the lowest payoff i can receive under straightforward bidding, this deviation is weakly dominated. (iii) The remaining possible deviations from straightforward bidding involve choosing an option $j \notin \arg\max_{k>0} \pi_{ik}(p_i)$ when $\bar{\pi}_i(p_i) > 0$. Because this choice does not change the subsequent price offers to i or other bidders, the only way this deviation from straightforward bidding can change i's payoff is when p_i turns out to be the final price vector i is offered. In this case, he would have been strictly better off following the straightforward bidding strategy. Thus, at any p_i, straightforward bidding is weakly dominant.

D Reverse Auction Simulations

In this section we describe simulation results for the following four reverse auction mechanisms:

- The descending clock auction, using either an ILP or constraint satisfaction solver for feasibility checking, under the assumption that broadcasters remain in the auction until their offer falls to their valuation for remaining on-air. In the figure legends below, this mechanism is referred to as *descending clock*.
- The descending clock auction, but with the exact solution for feasibility checking replaced by the much faster myopic construction of the repacked set and assignment to channels. In the figure legends below, this mechanism is referred to as *myopic clock*.
- The sealed-bid VCG mechanism, again under truthful bidding, using an ILP solver for the selection of the expenditure-minimizing set of broadcasters to clear. In the figure legends below, this mechanism is referred to as *VCG*.
- A sealed-bid pay-as-bid (first price) reverse mechanism, evaluated under full-information Nash equilibrium bidding, and again employing ILP for the selection of broadcasters to clear. In the figure legends below, this mechanism is referred to as *pay-as-bid full-info Nash*.

For these four mechanisms and equilibrium concepts, we consider all six pairwise comparisons of expenditures for clearing 7 broadcasters out of 10 and repacking 3. In Figure 7, the repacking constraint network is generated randomly at various edge densities (0.25, 0.5, and 0.75) over the 10 broadcasters, and the valuations of broadcasters are independent random values between 0 and 100. Each point in Figure 7 corresponds to a single such random trial, and shows the expenditure for clearing three broadcasters for each method on that trial. Points above the diagonal are a win for the mechanism+equilibrium concept on the x-axis, while points below are a win for the one on the y-axis. In general, the problems become harder with higher edge density (more repacking constraints). Figure 8 is similar in spirit, but instead shows comparisons at the fixed edge density 0.5, but with the number of broadcasters k to be repacked varying from 2 to 4.

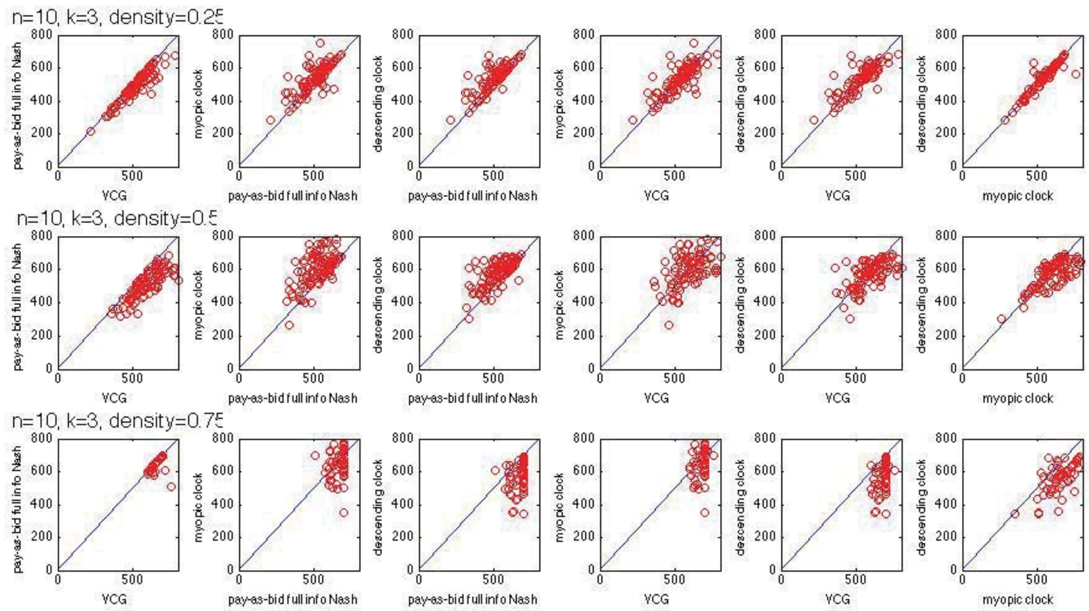


Figure 7: Comparison of mechanism expenditures with varying repacking constraint network edge density.

The main takeaways from these preliminary plots are:

- Even within this artificial family of generated networks, valuations, etc., there are significant differences between the expenditures and other properties of the solutions found by the various mechanisms, and these differences depend on the parameters.

- For example, the clearing expenditures of the clock auction are generally higher than those of pay-as-bid Nash at low edge density, but are generally lower at high edge density. The clock auction expenditures are generally comparable to those of VCG, except at high edge density, where the clock auction tends to spend less.

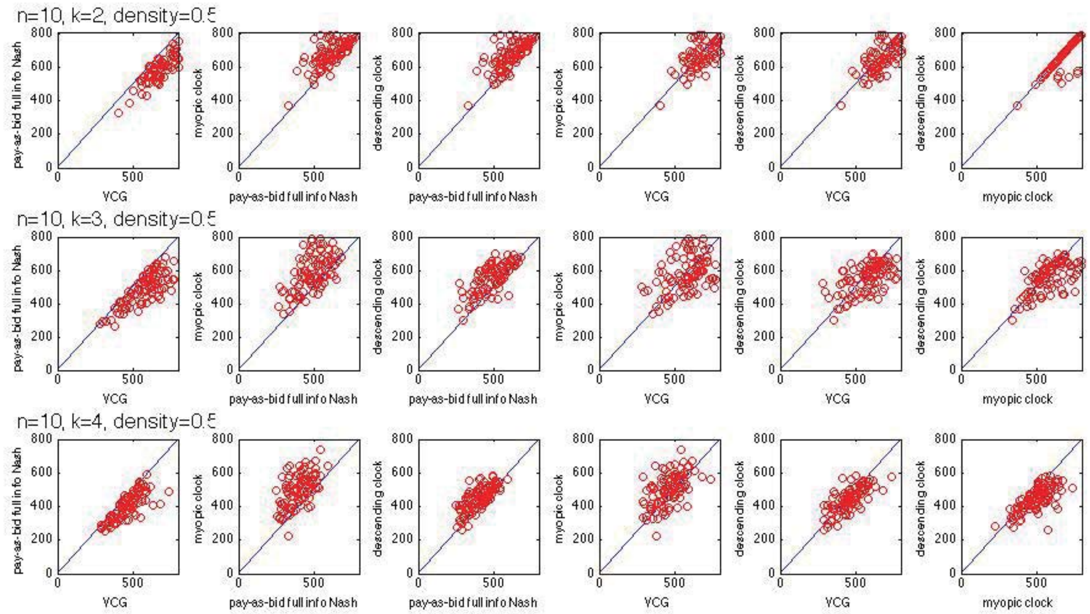


Figure 8: Comparison of mechanism expenditures with varying clearing targets.

- The myopic clock auction — the version of the clock auction where exact feasibility checking is replaced by the computationally efficient myopic assignment algorithm — competes well with the standard clock auction at low to moderate edge density. At high edge density, it starts to underperform due to its clearing more stations than necessary.

- Related observations can be made for varying numbers of broadcasters to repack.

Figure 7 and 8 compare the mechanism expenditures in absolute (“dollar”) terms, but perhaps more relevant are the relative expenditures, as measured by the ratio of expenditures for each pair of mechanisms as we vary edge density or clearing target. The maximum and minimum ratios for each pair of mechanisms is thus shown in Figure 9 for each setting of the parameters.

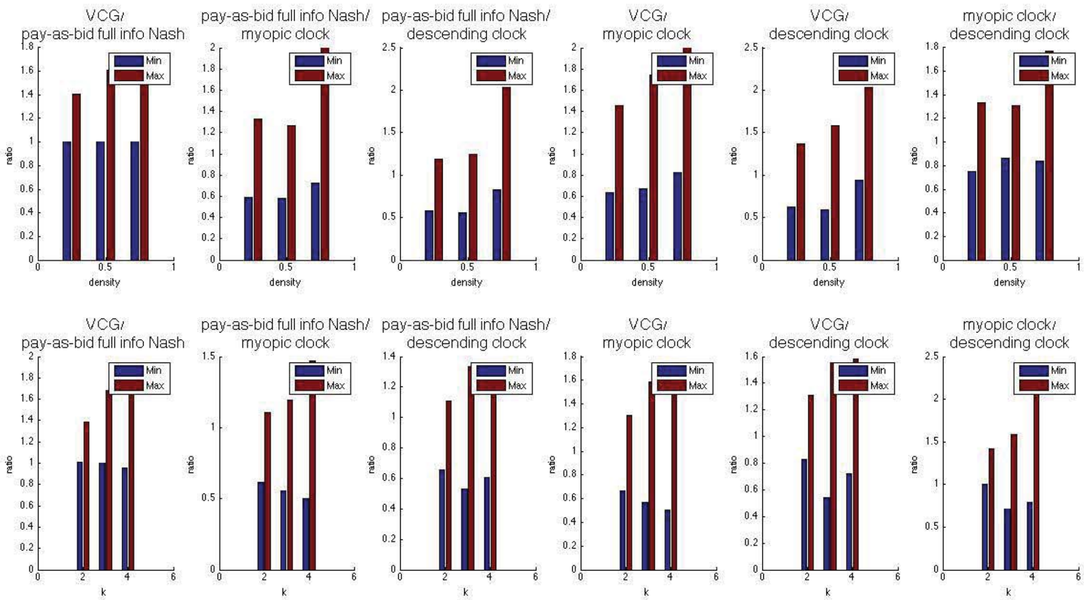


Figure 9: Maximum and minimum expenditure ratios for the mechanisms.

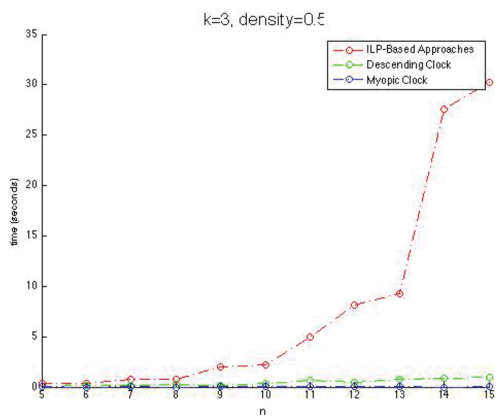


Figure 10: Computation time for the mechanisms as a function of problem size.

Finally, in addition to comparison of solution properties such as clearing expenditures, our simulation framework also permits computational comparisons and benchmarking. For example, in Figure 10, we show computation time (averaged over multiple trials) for each mechanism as a function of the number of broadcasters n. Since the computation time for both VCG and pay-as-bid sealed full-information Nash are dominated by calls to an ILP

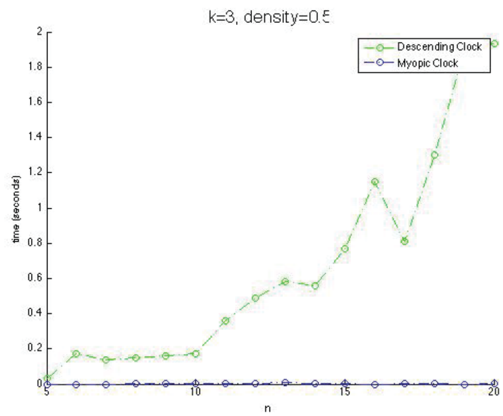


Figure 11: Computation time for the clock mechanisms as a function of problem size.

solver, we plot them together as “ILP-based approaches”. We can see that the computation for these approaches grows rapidly as n increases, much more so than for the two clock auction variants. Since the scale is dominated by the ILP running time, in order to compare the clock auctions we show their computational needs on a separate plot in Figure 11. Here we see the standard clock auction running time growing much more rapidly than the guaranteed-efficient myopic clock auction.

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ATTACHMENT

3

Reply Analysis of Yeon-Koo Che and Phil Haile

March 12, 2013

Introduction

We have been asked by counsel for AT&T to respond to comments filed on the FCC's proposed incentive auctions. In Che, Haile and Kearns (2013) (henceforth "CHK") we provided extensive comments on the FCC's Notice of Proposed Rulemaking and accompanying proposal of Milgrom, Ausubel, Levin and Segal (2012) (henceforth "MALS"). Many of our responses to comments by others will refer to our discussion and proposals in CHK.

In general, most commenters agreed with our assessment that the MALS proposals regarding the forward and reverse auctions offered a strong baseline from which to build a final auction design.

For the forward auction, commenters generally agreed with us that the MALS design offers substantial improvements over the Simultaneous Multi-Round ("SMR") auction used in prior FCC spectrum auctions. There was general support for the use of clocks and generic licenses in particular. Many commenters also agreed with our concern about the exposure problem, which remains unaddressed by the MALS auction design and is likely to limit auction revenue, distort bidding behavior, and lead to inefficient allocations. On the other hand, some commenters expressed concern about the potential for introducing package bidding to the forward auction. In particular, some view package bidding as inherently too complex. Others object to package bidding based on a view that package bidding discriminates against small bidders.

A specific package bidding proposal was offered in CHK. As we explain below, our Clock Package Auction (CPA) proposal avoids the pitfalls underlying the commenters' concerns. The CPA alters the MALS design in only three ways:

1. It expands the set of objects offered to include a small number of packages, using a geographically driven hierarchical package design closely tied to the actual structure of bidder complementarities between spectrum licenses.
2. It specifies how excess demand can then be properly calculated.
3. It provides a rule governing price clocks that ensures that package prices are additive in the prices of the package components when possible.

The CPA proposal does not add significant complexity to the MALS auction design. Indeed it simplifies bidding for most bidders by providing a means of addressing the exposure problem, thereby avoiding the need for schemes by package bidders to reduce their exposure risk and schemes by component bidders seeking to take advantage of others' exposure risk. Such strategies have been prevalent in past FCC spectrum auctions and create high demands on the sophistication of bidders seeking to bid optimally.

While this yields an important reduction in complexity relative to the MALS proposal, the MALS proposal's use of price clocks and generic licenses already simplifies bidding substantially relative to the SMR auction design. Thus, the CPA proposal design in fact offers substantial reductions in complexity relative to past FCC auctions.

We also explain below that the CPA does not discriminate against small bidders. Indeed the opposite is true. Contrary to common assertions that package bidding creates a threshold problem, we show that the threshold problem already arises in the MALS auction. Further we see no new potential for a threshold problem in the CPA that does not already exist in the MALS auction. In fact, there is at least one sense in which the CPA reduces the severity of the threshold problem relative to the MALS design and, *a fortiori*, the SMR auction design.

On the other hand, the CPA offers two specific corrections of biases against small bidders that exist in the MALS auction design. One is the exposure problem. Although exposure risk would affect almost all bidders in the MALS auction, it is important to recognize that small bidders seeking to enter the market are among those that need protection against this risk. The CPA offers such protection. The second bias against small bidders in the MALS auction design is the "overflow problem."   As discussed in CHK, this flaw in the MALS design will tend to force small bidders out of the market by raising the prices they must pay even when their demands are not a source of scarcity. The CPA eliminates the overflow problem.

We also discuss general concerns that the CPA would be susceptible to incentives for manipulative bidding.   We consider several specific types of manipulative strategies that might be attractive in other types of auctions and demonstrate that the specific rules of the CPA make these strategies unattractive to bidders. We see no new incentives for bid manipulation in the CPA relative to the MALS auction. Indeed, because elimination of the exposure problem eliminates the need for bidding schemes aimed at minimizing exposure risk (or exploiting that of other bidders), there may be fewer incentives for manipulative bidding in the CPA than in the MALS auction.

Finally, we address concerns about the reverse auction design raised by some commenters. We argue below that from a bidder's perspective there is no significant difference between a sequential and interleaved auction design. We also point out that, unlike other "sealed bid" auction designs, the option to conduct the reverse auction by proxy bidding would not complicate bidding but would simplify it.

Complexity of Package Bidding

Several commenters suggest that package bidding is inherently complex.[1] This is true if one assumes that package bidding implies allowing bidding on all possible combinations of licenses. Even with generic licenses, with 172 EAs in the 50 states, this gives roughly 6 sexdecillion (6×10^{51}) different packages on which bids might be made!

However, package bidding is not synonymous with unrestricted packages. Further, the Clock Package Auction (CPA) proposed in CHK specifies a limited set of packages. Although in principle complementarities between objects in a multi-object auction could be arbitrary, in the case of spectrum licenses complementarities depend primarily on geographic contiguity and population distribution. This makes it possible to restrict the set of packages severely in terms of the number of packages considered while still allowing bidders to effectively express the relevant complementarities in their valuations.

Under the specific CHK proposal to offer EA licenses, MEA licenses, REAG licenses, and nationwide licenses, the number of objects for sale would increase from 172 to 229. Aside from the introduction of these objects, the CPA is essentially identical in complexity (indeed, in almost all dimensions) to the MALS ascending clock auction. The generic treatment of licenses keeps decision making as simple as possible and avoids the possibility that identical licenses sell at different prices. And bidders need not choose what to bid but only which objects they wish to demand at the current clock prices. Even with the proposed addition of package objects, the ascending clock auction design is in fact much simpler than the SMR auction that has been used in previous FCC spectrum auctions.

Further, the addition of packages will actually *reduce* complexity for most bidders, who will seek multiple complementary licenses. Unlike auctions that exclude packages, bidders in the CPA need not develop strategies for managing the severe exposure risk that would otherwise be involved in bidding for combinations of licenses needed by an entrant to build a viable network or by an existing provider seeking to make effective use of the newly offered spectrum band. Even bidders seeking single licenses face a simpler bidding problem in the CPA, as they

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[1] *See, e.g.*, Comments of United States Cellular Corporation, *Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Docket No. 12-268, at 51 (Jan. 25, 2013) ("USSC Comments"); Comments of Cellular South, Inc., *Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Docket No. 12-268, at 5 n.11 (Jan. 25, 2013) ("Cell South Comments").

have no incentives to manipulate their bids in attempts to exploit the exposure risk of package bidders.

Package Bidding and Treatment of Small Bidders

The Threshold Problem

Several commenters express concern about the effect that package bidding would have on small bidders, due to the "threshold problem" (Bykowsky, Cull and Ledyard (2000)).[2] The threshold problem can arise when a package bidder is competing against multiple bidders, each seeking a component of the package. In such situations, the "small bidders'' may have incentives to free ride by holding back their demand in the hope that others will contribute more toward pushing the sum of the component prices past the willingness to pay of the package bidder. Such free riding can result in allocation of licenses to the package bidder even when the small bidders together place greater value on them than the package bidder does.

The threshold problem is a potential concern in most types of auctions in which some bidders view licenses as complements. This includes the MALS auction, the SMR auction used in previous FCC auctions, and the CPA variation of MALS. However, we see no potential for a threshold problem in the CPA that does not also exist in the MALS auction (or in an SMR auction). And, as we show below, there is at least one sense in which the threshold problem can be more severe in the MALS auction than in the CPA.

The threshold problem has been discussed extensively since the first FCC spectrum auctions. Unfortunately, this discussion has often been imprecise. It is commonly asserted that the threshold problem is introduced by combinatorial auction designs. This is incorrect. The threshold problem exists in many auctions without combinatorial bidding, including the SMR auction used in previous FCC auctions and the MALS clock design proposed for the forward auction in the upcoming incentive auctions.

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[2] *See, e.g.*, USSC Comments at 53; Comments of the MetroPCS, *Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Docket No. 12-268, at 14 (Jan. 25, 2013) ("RTG Comments"); Comments of the Rural Telecommunications Group, Inc., *Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Docket No. 12-268, at 9 (Jan. 25, 2013) ("RTG Comments").

We will illustrate this with a simple example. Let there be two licenses, A and B. Suppose there are three bidders with valuations for the objects as follows:

	Object:	A	B	A+B
Bidder:				
1		3	0	3
2		0	3	3
3		0	0	4

The efficient allocation awards license A to bidder 1 and license B to bidder 2.

Consider the MALS auction and suppose that the current clock prices are $p_A = p_B = 1$. If all bidders follow straightforward bidding, bidder 1 will demand A as long as $p_A < 3$, bidder 2 will demand B as long as $p_B < 3$, and bidder 3 will demand a unit of each as long as $p_A + p_B < 4$.[3] The auction will end at prices $p_A = p_B = 2$ and the efficient allocation will be obtained.

However, consider a deviation by bidder 1. Instead of continuing to demand A at prices $p_A = p_B = 1$, he instead drops his demand for A, maintaining his eligibility by bidding on some other license whose price is rising in every round. If bidders 2 and 3 continue to follow straightforward bidding,[4] p_B will rise while p_A will not. Once p_B reaches $3-\varepsilon$, bidder 1 returns to demand A. The auction then ends at prices $p_A = 1+\varepsilon, p_B = 3$ where bidder 3 drops out. By withholding his true interest for A, bidder 1 is able to free ride on the straightforward bidding behavior of bidder 2 and obtain license A at a lower price. Of course, there is no reason for only bidder 1 to think this way. But if bidders 1 and 2 both attempt to free ride, bidder 3 may win. This is the threshold problem. Bidders 1 and 2 must both contribute to the effort to displace the package bidder, but each would prefer the other to contribute more.

This illustrates an important point. The threshold problem is not the result of package bidding (there is none in the MALS auction). Rather, it is the result of "package valuations,'' i.e., of complementarities that bidders act on to at least some degree. The same problem would arise in the SMR auction.

It is easy to see both that the threshold problem arises in the CPA as well and that there is no new threshold problem relative to the MALS auction. At prices

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[3] Note that we are assuming here that bidder 3 is ignoring the exposure problem, but as we explain below, the conclusion does not hinge on this assumption.

[4] Under "straightforward bidding'' each bidder demands the set of objects that would maximize his profit if the current prices turned out to be final prices.

$$p_A = p_B = 1$$
$$p_{AB} = 2$$

bidder 1 is again tempted to withhold his demand for A, waiting until prices reach

$$p_A = 1$$
$$p_B = 3 - \varepsilon$$
$$p_{AB} = 4$$

to again demand A. When prices rise to

$$p_A = 1 + \varepsilon$$
$$p_B = 3$$
$$p_{AB} = 4 + \varepsilon$$

bidder 3 exits. However, once again, if bidders 1 and 2 both try to free ride this way, bidder 3 may win.

Although these examples suggest an identical threshold problem in the MALS auction and CPA, one might be concerned that we have been too pessimistic about the MALS auction. In particular, we have assumed above that bidder 3 ignores the exposure problem entirely when bidding. This may indeed be unrealistic. As discussed in CHK, the threshold problem is likely to suppress the bids of package bidders, likely leading to reduced revenue and inefficient allocations. However, less aggressive behavior by the package bidder does not necessarily soften the threshold problem. Above we assumed that bidder 3 continued to demand license j as long as

$$p_j \leq 4 - p_{-j}\,.$$

Suppose instead that he exposes himself to only half the risk, demanding j only as long as

$$p_j \leq \frac{4 - p_{-j}}{2}\,.$$

For example, if the price of A remains zero but that of B rises, this means that bidder 3 is willing to demand B only until its price reaches 2, thus putting on the line half of his package value.

Now the free riding incentive exists at the beginning of the auction. Suppose that the auction begins at reserve prices of ε. Hoping that bidder 2 bids straightforwardly, bidder 1 might, for example, plan to wait until p_B reaches $2 - \varepsilon$ to demand A for the first time with the auction then proceeding as follows

p_A	p_B	Demands
ε	ε	0,B,A+B
$\vdots$	$\vdots$	"
ε	$2-\varepsilon$	A,B,A+B
2ε	2	A,B,0

Thus, it is not generally true that the threshold problem softens when package bidders respond more conservatively to the threshold problem.[5] Here, the incentive to free ride begins immediately: each component bidder has an incentive to withhold its demand throughout the auction because, depending on how the other component bidder behaves, it may be possible to obtain the license at the reserve price. This contrasts with the CPA where the lowest price at which a component bidder could win is 1 (for example, bidder 3 is willing to pay 4 for AB while bidder 2 will pay no more than 3 for B; thus bidder 1 cannot win A at a price below 1).[6] Thus, there is at least one sense in which the threshold problem may be viewed as more severe in the MALS auction than in the CPA.

Note, however, that the use of price clocks in both the MALS auction and the CPA may reduce the need for bidders to "coordinate" in overcoming the threshold problem relative to an SMR auction. In the SMR auction, the same incentives arise. But the SMR auction provides no "suggested'' prices that might be used to coordinate. Ausubel, Cramton and Milgrom (2006, p. 134) have previously argued (in the context of the clock phase of the CCA) that "the price adjustment process is effectively resolving the threshold problem by specifying who should contribute what as the clock ticks higher." This may overstate the effectiveness of price clocks, but they may indeed reduce the need for bidders to "coordinate" to unseat a package bidder. One way to see this is to observe that a component bidder does not need to form a winning coalition with other component bidders to win his desired component license. All he needs is to demand the desired component in each round.

Another feature of the CPA is its use of additive package pricing (except in the case of excess demand for the package itself). Without this feature, large gaps could arise between the standing price for a package and the sum of the standing prices for the components of the package. This could be a serious concern when packages are added to the SMR auction, for example. When such gaps arise, bidders for the components must coordinate to overcome this gap in order to displace the package bidder. With additive package pricing, clock prices reveal to the component bidders a set of prices which, if accepted, would unseat the package bidder

[5] Of course, if the package bidder acts as if he has *no* complementarity, the threshold problem will not arise. However, given the substantial complementarities between spectrum licenses, the FCC should not hope for this outcome.

[6] Bidder 1 could still begin withholding demand at the beginning of the auction in the CPA. But unlike the MALS auction, he has no strict incentive to do so.

unless that bidder also agrees to a higher price. This may substantially mitigate the severity of the threshold problem.

This observation has been made previously by Goeree and Holt (2010, p. 148) in the context of their Hierarchical Package Bidding (HBP) extension of the SMR auction design, which uses additive package pricing to determine minimum acceptable bids: "bidders on individual licenses in that region would know how high they have to bid to unseat the provisional regional winner. In this sense, prices help these bidders solve a coordination or 'threshold problem'."[7]
Note that "package prices" are effectively additive in the MALS auction design as well: a bidder seeking the package AB must offer a total of $p_A + p_B$. Thus, we do not claim that the use of additive clock pricing in the CPA offers superior mitigation of the threshold problem relative to the MALS design. Rather, we point out that the CPA retains the substantial advantages of the MALS design without introducing any new threshold problem. Nonetheless, the previous example suggests that the CPA, by mitigating the exposure problem that exists in the MALS design, may in fact soften the severity of the free-riding incentives (threshold problem) that exist in the MALS auction design.

In practice, the threshold problem and its impact are less likely to be significant in clock auctions (e.g., MALS or CPA) than in the SMR auction. For strategic withholding of demand to be profitable, a component bidder must know that he is facing a package bidder and another component bidder to free-ride on. The presence of such opponents may be identifiable in an SMR auction, which reveals provisional winners in each round. But clock auctions do not reveal which opponents are demanding which objects. All a bidder can see are the prices quoted on different items; the sources of price movements are not revealed. A price increase on an individual license could just as likely be triggered by a component demand as by a package demand. This anonymity feature also makes ineffective any attempts by a bidder to strategically exploit a potential threshold problem, e.g., a component bidder pretending to be a package bidder or an individual bidder pretending to be a package bidder. The uncertainty about the free-riding potential does not of course mean that bidders will not attempt to free-ride. They may if they perceive a sufficient likelihood of an opportunity and expect a sufficient gain from it. But free-riding is not without risk. When a bidder holds back his demand at a price significantly below his value, he is risking a sizable profit since the auction could end in the next round. Uncertainty about even the existence of the free-riding incentive lowers the potential gain that could tempt a bidder to take on such risk.

[7] Goeree and Holt (2010) also provide evidence from the laboratory that, even in an SMR auction with package bidding, restricting packages to a hierarchical structure (as we proposed for the CPA) helps bidders overcome the threshold problem by eliminating ambiguity about which component prices must rise to displace the package bidder.

The Overflow Problem

In CHK we pointed out that the MALS auction design introduces a new type of bias against bidders seeking single licenses or small groups of licenses. Bidders seeking packages of licenses will be constrained by the fact that unequal quantities of spectrum will be cleared in different markets. Consequently there will be EAs in which the number of licenses available exceeds the maximum number of the encompassing regional packages that can be allocated. The auction design needs to account for this. Otherwise bidders for single EA licenses can face rising prices even when their demands are not a source of scarcity. This will push such bidders out of the market unnecessarily and lead to misallocation and/or undersell.

We discussed the overflow problem in greater detail in CHK, where we showed that the CPA eliminates the overflow problem. This is possible because the CPA provides a bidding language that allows the price clocks to ensure that EA license prices do not rise when the excess demand is for the package rather than for the EA license itself. This is possible only when bidders can express package bids and when the clock adjustment process properly accounts for feasibility constraints, making sure that demand for packages flows down to the EA licenses only when the demands for single EA licenses are themselves a source of scarcity at the current prices.

Entrants

Exposure risk has been neglected in most prior FCC auction designs.[8] In some cases this has likely led to withdrawal of potential entrants from the auction. In other cases, this has forced package bidders to bid strategically, deviating from straightforward bidding in hopes of resolving uncertainty about closing prices of some licenses before committing to other licenses in the package. Bidders competing against package bidders unprotected against exposure risk have incentives for strategic bidding designed to maximize their competitors' exposure risk, potentially softening competition. Such incentives for bid manipulation are undesirable in themselves, as they can lead to poor price discovery and inefficient allocations. Further, the

[8] An exception is the 700MHz Upper C Block auction of (regulatorily) impaired spectrum, where a SMR auction with limited hierarchical package bidding was used following the design of Goeree and Holt (2010). See also Rothkopf, Pekec and Harstad (1998) for an early proposal to use hierarchical packages in an ascending auction. Another exception is Auction 66 of spectrum for Advanced Wireless Services, where licenses for Regional Economic Areas were offered in addition to licenses for Economic Areas and Cellular Market Areas.

need to understand such bidding strategies places a heavy burden on entrants seeking to bid effectively in the auction.

Although the MALS forward auction design introduces a number of important improvements on the SMR auction, it does nothing to address the problems discussed above. The MALS design neglects the exposure problem entirely.

The CPA design substantially mitigates exposure risk by using a geography-based hierarchical package structure we understand to be closely tied to the actual structure of complementarities between licenses in the mobile wireless industry. By minimizing exposure risk, the CPA eliminates bias in the MALS design (and SMR) against bidders seeking packages by eliminating (or substantially reducing) the risk of exposure. This is likely to improve both the efficiency of the spectrum allocation and auction revenues. One can see evidence of the latter in the FCC's Auction 66, where bids for REAG licenses were on average 37 percent higher per MHz than those for EAs covering the same population.[9] The exposure problem is likely to affect all bidders, due to the economies of scale from horizontal spectrum contiguity (see CHK for additional discussion) and the fixed costs associated with introduction of a new frequency band to an existing wireless deployment. Smaller firms seeking to enter the wireless market certainly are not immune to this exposure risk. And they face, in addition, the need to establish a sufficient geographic footprint to enable service to consumers who now expect coverage to extend outside small areas like EAs. As argued by Cramton et al. (2007, p. 23), "Package bidding levels the playing field and removing it would seriously damage the prospects for new entry."

Manipulative Bidding

It has been suggested that the CPA design may introduce incentives for strategic bid manipulation. No specific manipulations have been articulated, and we speculate that the concerns may arise from a misunderstanding of the CPA.

The CPA changes no rules of the MALS auction design. The CPA design modifies the MALS design only by (1) adding new objects (packages), (2) explaining how excess demand can then be properly calculated, and (3) defining rules for how price clocks are to adjust to maintain additive package pricing when possible. All else is the same. This includes, for example, the rule regarding reductions in expressed demand: as in the MALS proposal, in the CPA a bidder may reduce his quantity demanded of an object only when its price increases.

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[9] If one compares price per MHz-Pop, the premium paid for REA licenses was more than 100%.

We have already pointed out that, by eliminating (or substantially reducing) the exposure problem, incentives for strategic manipulation of bids that are present in the SMR and MALS design will be eliminated (or substantially reduced) by the CPA design. Thus, there is at least one way in which the CPA *reduces* incentives for manipulative bidding. Furthermore, we do not see any new opportunities for profitable bid manipulation that result from the modification of the MALS auction embodied in the CPA design proposal.

An example may help to illustrate the protection against profitable manipulation offered by the specific design of the CPA. Suppose there is one license available in each of two EAs, A and B. One bidder seeking the AB package competes against two component bidders demanding A and B, respectively. Call the first bidder the package bidder and the latter two bidders the A-bidder and B-bidder. There are several ways one might imagine a bidder manipulating his bids in hopes of improving his profit:

(i) *The package bidder may pretend to be a component bidder in order to divide and conquer the component bidders:* One possibility is for the package bidder to run up the price clock for A by repeatedly demanding it until the A-bidder drops out, then switch to demanding the AB package he actually desires. This strategy would not help the package bidder to lower the price for the package, since any increase in the price of A triggers a commensurate price increase in the package price. The package bidder can do no better than by demanding the desired package straightforwardly (i.e., until its price reaches his valuation for the package). Nor would either component bidder be preempted by such a manipulation by the package bidder. A component bidder can never lose by demanding its desired component straightforwardly (i.e., until price rises to his valuation for the component). This is in contrast to the SMR in which a package bidder could indeed lower the price of the package, or weaken the competition, by driving out a component bidder.[10] Thus, **due to the additive package pricing rule of the CPA, there is no gain from this type of manipulation**.

(ii) *A component bidder, say the A-bidder, may seek to shift the competitive burden to the B-bidder:* He may try to do this by either (i) withholding his demand on A (parking eligibility on some other licenses with rising prices), (ii) demanding B to push up its price, or (iii) demanding the package AB. We extensively discussed the first possibility earlier in considering the threshold problem, and the second possibility involves a risk of winning a license that the A-bidder does not desire. Importantly, these two options are also available in MALS with the same consequences. The only new option made available by package bidding in CPA is the last option. And this option is totally ineffective as a means of lowering the price for A, leading only to a risk of ending up with an unwanted item. Demanding the package AB creates excess demand for this

[10] This was the main motivation of the FCC 700 MHz auctions which limited the withdrawal of component bids even when they were no longer provisionally winning. Limiting bid withdrawal has a serious side effect, however, for it ties up the budget of the bidder and thus constrains his ability to move across different licenses as prices change.

package and leads to the same increase in the price of A that would have resulted if the A-bidder had just bid for A.[11] Thus, **the CPA's introduction of packages to the MALS auction introduces no new opportunity for manipulation of this type**.

(iii) *There are in fact no package bidders, but a component bidder may pretend to be a package bidder by demanding a package of licenses:* One can imagine a bidder engaging in such behavior in an attempt to cause his opponents to free-ride on each other (withhold their demands) and allow him to win. As mentioned earlier, such an attempt to "signal" package preferences might be effective in the SMR setting but will not work in a clock auction in which bidders see only prices rather than demands. The anonymity of demands in the CPA means that the opponents will not know the source of an observed price increase, so the bidder cannot communicate the message he wishes to send. At the same time, such an attempt will entail a risk of the A-bidder ending up with a package of licenses he does not want. Thus, **this strategy has cost but no benefit**.

We acknowledge (see also CHK) the possibility that some licenses may go unsold in the CPA. This can happen, for example, when a component bidder drops out but remaining component bidders manage to displace the package bidder. This kind of undersell potential should be familiar from the clock phase of the Combinatorial Clock Auction ("CCA"). However, the magnitude of undersell in the CPA will tend to be far less than that under the clock phase of the CCA. This is because in the clock phase of the CCA, a bidder may drop his demand for an object whose price has not risen, as long as the price has risen for some other object he also demanded. This is not permitted in the CPA.[12] Bidders may reduce demand only on objects whose prices have increased. This rule is natural in an auction and offers a compromise between the extremely lax bid withdrawal rule of the CCA clock phase and the severe MALS rule, which discriminates against package bidders by prohibiting such a bidder from reducing his demand for a package unless the price of *every* component has increased.

If some items are unsold at the end of the auction, we proposed in CHK that the FCC should retain the option of reoffering them via another clock auction. As discussed in CHK, the prospect of unsold items being available in such a supplementary resale auction, possibly at lower prices, could create incentives for "small" bidders to withhold demand in the primary auction. However, such a strategy would be highly risky, since such a bidder could not be

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[11] The only case in which the A-bidder's added demand for AB would not "flow down" to object A under the CPA rules is when (due to unequal spectrum clearing in markets A and B) this additional unit of demand for AB does not conflict with any existing demand for one of the components. If that is component A, then the A-bidder does not need the price of B to rise for him to win A. If that is component B, then the A-bidder's manipulative demand for AB does not flow down to B. Thus, manipulative bidding for AB cannot help the A-bidder.

[12] In the CPA, only a bidder for a nationwide license has the same flexibility for demand reductions that exists in the CCA clock phase. If that is a significant concern, the national license could be excluded from the specific package structure proposed in CHK. The resulting structure would remain a multitree, as the CPA requires for unambiguous determination of excess demand. We understand (see for example the Comments of T-Mobile) that a substantial share of the geographic license complementarities that will exist in these auctions could be captured by having only MEA and REAG packages.

certain that his desired license would go unsold, or that the FCC would indeed choose to reoffer the license. Thus we do not expect such a strategy to be an attractive option for bidders.

Single Pass Reverse Auction

Several commenters expressed concern about separation between the forward and reverse auctions.[13] In CHK we proposed the option of conducting the reverse auction in a single pass. This option offers a number of advantages, including simplifying bidder participation in the reverse auction. Reverse auction bidders would not need to be reconvened to establish prices for each new closing target. And accepted bids in the reverse auction would be made on the same day with the same information. This contrasts with an interleaved design in which clock prices for the reverse auction resume from the closing prices obtained for the previous (unsuccessful) clearing target. In that case, bidders demanding high prices might effectively commit themselves to being repacked days or weeks ahead of the final offers that are made by other stations.

Further, the single-pass option would not introduce significant complexity for reverse auction bidders. From a bidder's perspective, the single-pass auction would be virtually identical to one of sequencing the reverse and forward auctions under an interleaved design. Just as with the interleaved design proposal, a bidder merely responds to a sequence of price offers by selecting its preferred option at each set of prices. The single pass option would traverse a wider range of prices than any single reverse auction stage under the interleaved design, but the bidder's options during the auction would be identical under both designs, as would the rules determining allocations and prices paid to bidders for each relinquishment option.

Proxy Bidding in the Reverse Auction

Several commenters expressed concerns about the demands on reverse auction bidders if a sealed bid mechanism or proxy bidding system is used. We share the concern that many types of sealed bid mechanisms would introduce unnecessary complexity in the reverse auction. For example, we discussed practical limitations of the Vickrey-Clarke-Groves (VCG) auction in CHK. We also discouraged the use of a discriminatory ("pay-as-bid") design in the reverse auction. A discriminatory auction requires substantial sophistication from bidders. To bid optimally, they must develop a clear understanding not only of their own valuations, but of the competition

[13] *See, e.g.*, MetroPCS Comments at 8.

they are likely to face. Such an auction design may be well suited to environments in which bidders have substantial bidding expertise, but not a one-off auction in which firms with little or no bidding experience are asked to bid to sell large assets.

Nevertheless, these limitations are not inherent to all sealed bid auctions, nor to clock auctions with proxy bidding. A clock auction with proxy bidding was one option we discussed in CHK, where we pointed out that proxy bidding may offer a substantial benefit to the well functioning of the reverse auction by allowing repacking problems to be solved offline rather than in real time. The single-pass reverse auction proposed in CHK adds no significant complexity to the reverse auction relative to the MALS clock auction proposal. Indeed, proxy bidding *simplifies* participation (this is why it is used by eBay, for example). Rather than standing by waiting to see whether prices reach a level where a bidder's preferred relinquishment option would switch, a bidder can simply report the "switch points" to the proxy system with assurance that his plan will be executed automatically.

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