Before the
Federal Communications Commission
Washington, D.C. 20554

In the Matter of

Amendment of the Commission's Rules with	)	
Regard to Commercial Operations in the	)	GN Docket No. 12-354
3550-3650 MHz Band	)	

Comments of Pierre de Vries

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J. Pierre de Vries
Senior Adjunct Fellow
Silicon Flatirons Center for Law, Technology, and Entrepreneurship
University of Colorado Law School
401 UCB, Wolf Law Building
Boulder, CO 80309

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I. Introduction and Summary

I, Pierre de Vries,[1] respectfully submit these comments on the Further Notice of Proposed Rulemaking (*FNPRM*)[2] in the above captioned proceeding.[3]

The Federal Communications Commission (FCC or Commission) has demonstrated its vision and pragmatism in developing rules for services in the 3550-3650 MHz (3.5 GHz) band. In particular, it should be applauded not only for adopting the three-tier framework proposed in the PCAST *Report*,[4] but also for implementing the Report's recommendations regarding receiver interference limits[5] as Reception Limits to be observed by services with Priority Access (PA) Licenses (PALs).

In my opening comments[6] I argued that harm claim thresholds, an implementation of interference limits policy described in the recent *White Paper* published by the FCC Technological Advisory Council (TAC),[7] would facilitate intensive spectrum sharing by ensuring that all parties have an explicit, upfront understanding of their operating rights

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[1] Senior Adjunct Fellow at the Silicon Flatirons Center at the University of Colorado, Boulder, and co-director of its Spectrum Policy Initiative. Member of the FCC Technological Advisory Council. These comments reflect my views alone, and not those of the Silicon Flatirons Center or any other party.

[2] Amendment of the Commission's Rules with Regard to Commercial Operations in the 3550-3650 MHz Band, GN Docket No. 12-354, *Further Notice of Proposed Rulemaking*, 29 FCC Rcd 4273 (2014) *(FNPRM*). *See also* Commission Seeks Comment on Shared Commercial Operations in the 3550-3650 MHz Band, 79 Fed. Reg. 31247-31282 (June 2, 2014).

[3] Amendment of the Commission's Rules with Regard to Commercial Operations in the 3550-3650 MHz Band, GN Docket No. 12-354, *Notice of Proposed Rulemaking*, 27 FCC Rcd 15594 (2012) (*NPRM*).

[4] *See* PCAST, Report to the President: Realizing the Full Potential of Government-Held Spectrum to Spur Economic Growth (rel. July 20, 2012) (PCAST *Report*), http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast_spectrum_report_final_july_20_2012.pdf.

[5] *Id.* at 33-38, App. D.

[6] Comments of Pierre De Vries, Senior Fellow, Silicon Flatirons Center for Law, Technology, and Entrepreneurship in response to *NPRM* in GN Docket No. 12-354 (filed February 20, 2013).

[7] FCC Technological Advisory Council, Receivers and Spectrum Working Group, Interference Policy - The Use of Harm Claim Thresholds to Improve the Interference Tolerance of Wireless Systems, *White Paper* (February 6, 2013) (2013 TAC *White Paper*) at 6, http://transition.fcc.gov/bureaus/oet/tac/tacdocs/WhitePaperTACInterferenceLimitsv1.0.pdf. For an accessible introduction, *see* FCC Technological Advisory Council, Receivers and Spectrum Working Group, Interference Limits Policy and Harm Claim Thresholds: An Introduction, *White Paper* (Version 1.0, March 5, 2014), http://transition.fcc.gov/oet/tac/tacdocs/reports/TACInterferenceLimitsIntrov1.0.pdf.

and responsibilities. Harm claim thresholds establish the "in-band and out-of-band interfering signals that must be exceeded before a radio system can claim that it is experiencing harmful interference."[8] Among other things, harm claim thresholds would give incumbents confidence in the level of protection they will receive, give new users a better understanding of the radio environment they are entering, and facilitate enforcing service rules and determining liability in the event of interference. Since the Reception Limits proposed in the *FNPRM* are an instance of interference limits policy and resemble harm claim thresholds, they offer these same benefits.

I noted in my reply comments[9] that there is widespread support for harm claim thresholds in the record and explained how harm claim thresholds could address several concerns raised in the opening comments.

In this filing, I commend the introduction of Reception Limits to facilitate coexistence among PALs. This Comment:

- Observes that the proposed value is reasonable though conservative;
- Discusses refinements to the definition of a Reception Limit, including specifying whether the probability applies to time and/or space, specifying the region over which it is to be measured, stating the statistical confidence level at which a violation needs to be demonstrated, adding a narrower resolution bandwidth, limiting the out-of-channel frequency range over which it applies, and adding antenna gain in order to define the limit as a field strength;
- Address questions raised by the Commission about the determination and enforcement of Receiver Limits;

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[8] 2013 TAC *White Paper, supra* note 7, at 3.

[9] Reply Comments of Pierre De Vries, Senior Fellow, Silicon Flatirons Center for Law, Technology, and Entrepreneurship in response to *NPRM* in GN Docket No. 12-354 (filed April 5, 2013).

- Argues that the use of Reception Limits could be extended to protect Incumbent Access tier users and optimize coexistence between Priority Access and General Access users.

II. Reception Limits will facilitate productive coexistence among Priority Access Licensees

I commend and strongly support the Commission's proposal to introduce Reception Limits, the "RF field strength … that PAL receivers would need to accept from nearby licensed transmitters … not to be exceeded with greater than 99 percent probability"[10] since, as the *FNPRM* states so eloquently, it "would effectively define the spectrum rights between PALs, and enable the SAS to assign these rights with clear obligations between respective licensees."[11] This proposal builds on the interference limits approach proposed in the PCAST *Report*, developed in the 2013 TAC *White Paper,* and supported by comments on the Public Notice requesting feedback on the *White Paper*.[12]

A. Dynamic frequency assignment requires an explicit statement of the interference rights and responsibilities of receivers

The optimum arrangement of transmitters and receivers requires not only the definition of transmission parameters but also the protection that should be afforded to receivers. A feasible but inefficient method to protect services with priority rights (e.g. incumbents vs. PAL, or PAL vs. General Authorized Access (GAA)) is to define conservative exclusion zones in frequency and space, as is proposed in the *FNPRM* to protect incumbents,[13] and/or to place limits on the maximum transmit power of potential interferers, as is done for PAL and GAA operations.[14]

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[10] *FNPRM* at ¶¶ 85-86.

[11] *Id.* at ¶ 85.

[12] *See* Office of Engineering and Technology Invites Comments on Technological Advisory Council (TAC) White Paper and Recommendations for Improving Receiver Performance, ET Docket No. 13-101, DA 13-801 (April 22, 2013), *available at* https://apps.fcc.gov/edocs_public/attachmatch/DA-13-801A1.pdf.

[13] *FNPRM* at ¶ 140.

[14] *Id.* at ¶¶ 74-76.

A more flexible method that will lead to more productive operation is to specify the interference protection that higher-ranking services are entitled to as co-channel and out-of-channel[15] Reception Limits, and then to calculate on a case-by-case basis the allowed interfering transmit power based on the transmitter's location and antenna pattern, path loss to the receiver, and the receiver's co-channel and out-of-channel Reception Limits. An explicit statement of the rights and responsibilities of receivers regarding out-of-channel interference protection, achieved through Reception Limits as proposed in the *FNPRM*, will also facilitate the close spectral packing of services thus increasing the value of shared operation in the 3.5 GHz band.

The *FNPRM* proposes that a Spectrum Access System (SAS) would dynamically assign PAL and GAA bandwidth in real time to promote efficient spectrum use.[16] In order to ensure the optimal protection of services with priority rights (incumbents vs. PAL, PAL vs. GAA), an SAS therefore needs to be given explicit data on the interference protection rights of higher-ranking services.[17] Reception Limits provide the required information on receiver protection without the FCC specifying receiver masks. [18]

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[15] "Out-of-band" might be a more familiar term in this context. I have used "out-of-channel" since there will be sharing between services within the 3.5 GHz band, and not just at the 3.5 GHz band boundary; following the terminology of the *FNPRM*, service assignments in the band are by "channel," e.g. 10 megahertz PAL channels. By the same token, "out-of-channel emission" might be a more accurate term than "out-of-band emission," but I have retained the latter since it has become customary.

[16] *FNPRM* at ¶¶ 28, 33.

[17] Comments of Google, Inc. on the Proposed Revised Framework in GN Docket No. 12-354 (filed December 5, 2013) at 10-11 ("SAS can take into account the actual size and shape of the adjoining emitter and receiver masks for devices at a given area and assign blocks to maximize efficient use."); Comments of IEEE Dynamic Spectrum Access Network Standard Committee in GN Docket No. 12-354 (filed February 4, 2013) at 5 ("spectrum consumption models (SCMs) … are used to capture the boundaries of RF spectrum use by devices and systems of devices … SCMs could be a core component of any future national Spectrum Access System (SAS)"); *id.* at 7 ("the modeling method uses 13 constructs that can collectively capture transmission power, spectral emissions, receiver susceptibility to interference, …").

[18] The receiver masks that guarantee adequate performance in an RF environment described by Reception Limits may, of course, be defined through industry standards and/or SAS protocols. *Cf.* Comments of Google, Inc. on the Proposed Revised Framework, *supra* note 17, at 11 ("[A]n SAS would have access to specific device performance characteristics based on information provided during the device's equipment certification process...") (citing Letter from Aparna Sridhar, Telecom Policy Counsel, Google Inc. to Marlene H. Dortch, Secretary, *ex parte*, GN Docket No. 12-354 (filed September 3, 2013) at 7).

The *FNPRM* has adopted this approach for PAL-to-PAL adjacent channel coexistence through its proposed (out-of-channel) Reception Limits approach.[19] The PAL Reception Limit also provides a basis for assigning GAA operations to bands adjacent to PALs (see Section IV.C below). There is no reason, in principle, why Reception Limits may not also be used to protect incumbents (see Section IV below). Once the Commission and service operators have developed confidence in the Reception Limit approach, it could therefore also be used to determine secondary assignments within exclusion zones that protect federal and non-federal incumbents.

B. The -30 dBm per 10 MHz limit is reasonable though conservative

The proposed -30 dBm per 10 MHz limit proposed in the *FNPRM* appears to be rather conservative in the light of drive test data collected by CRFS for Ofcom in the United Kingdom in 2011.[20] (Similar US data is collected regularly by the operators of cellular services and infrastructure but is not in the public domain.) A paper presented at the CrownCom 2014 conference (attached to this document as Appendix A and hereinafter referred to as Riihijärvi et al.[21]) analyzed this data, focusing on the measured received downlink signal power of a 3G operator in the 2 GHz band in a 2 km x 2 km area of downtown Exeter; this subset consists of over 35,000 samples.

Riihijärvi et al. reported that the 99th percentile received power level[22] was -38 dBm.[23] Since this power was measured in a 5 MHz channel, it is equivalent to -35 dBm per

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[19] *FNPRM* at ¶¶ 85-86.

[20] CRFS Ltd., *Mobile "Not-Spot" Measurement Campaign - Final Report* (February 7, 2011), www.ofcom.org.uk/static/research/CRFSreport.pdf.

[21] J. Riihijärvi, A. Achtzehn, P. Mähönen, & P. De Vries, *A study on the design space for harm claim thresholds*, in 9th International Conference on Cognitive Radio Oriented Wireless Networks (CrownCom 2014) (2014) (Riihijärvi et al.), http://www.inets.rwth-aachen.de/fileadmin/templates/images/PublicationPdfs/2014/2014-CrownCom-Harm-Claim-Thresholds.pdf, *infra* Appendix A.

[22] CRFS reported measured power, not power at the antenna input or signal strength. I assume that the combination of antenna gain and cable loss was approximately 0 dB. The measurement equipment used high-gain antennas (CRFS Ltd., *supra* note 20, at 2 fig.1), and unless very high loss cables were used, this is a conservative assumption; the power at the antenna input was most likely at least a few dB less that the reported figure.

10 MHz. Since these are observations of a 2 GHz UMTS macro-cell downlink deployment, not a small-cell deployment with more time variation in a higher frequency band, this is a generous an upper bound for the received signal power in a 3.5 GHz small-cell scenario.

It is therefore quite possible that experience in the field will show that the -30 dBm per 10 MHz, 99th percentile Reception Limit could safely be dropped by 5 dB or more, leading to more operational flexibility for licensees.

III. Refining the definition of Reception Limits

The *FNPRM* proposes that the Reception Limit is "not to be exceeded with greater than 99 percent probability."[24] Such a statistical formulation is a necessary consequence of the essentially variable nature of the radio frequency environment, and the fact that requiring that a maximum value that should not be exceeded under any circumstances (i.e. with 100 percent probability) would require operators to deploy systems with unnecessary safety margins.

However, further refinement will be required to provide clarity about the kind of measurements that would be required to show that the limit has been exceeded. Such refinements could be done in more detailed service rules, through an OET Bulletin, or by delegating the task to a properly constituted multi-stakeholder organization.[25]

A. Specifying the region over which the probability should be calculated

The "99 percent probability" formulation is a reasonable high-level statement of a probability threshold. However, it is not clear whether this means 99% of measurements,

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[23] *See* Riihijärvi et al., *supra* note 21 and *infra* Appendix A, figs. 2(b) & 3(b). The vertical red line is the value for the full set of 35,000 samples in the 2 km x 2 km area, with a 95% confidence interval of less than ±1 dB.

[24] *FNPRM* at ¶ 86.

[25] FCC Technological Advisory Council, Receivers and Spectrum Working Group, Multi-stakeholder Organization to Develop Interference Limits Policies, *Recommended Charter* (June 17, 2014), http://transition.fcc.gov/bureaus/oet/tac/tacdocs/meeting61014/InterferenceLimitsMulti-stakeholderOrganization-RecommendedCharter.pdf.

99% of locations, or perhaps 99% of locations and times by analogy to the F(50, 90) field strength curves that specify television contours.[26] I assume that the *FNPRM* is referring to 99% of observations at suitably distributed locations over some specified measurement area.

The region in space (and perhaps time) over which the 99% limit should apply, and the steps required to ensure that measurements are sufficiently independent of each other, should also be defined.

A straightforward approach would be to enforce it over the licensing area, i.e. a transmitting PAL would only be in violation if the limit is exceeded when measured over the entire licensing area.[27] However, if the licensing area chosen for the final rule is considerably larger than the operating area of a typical PAL small-cell, then measuring the signal strength distribution over this large area may underestimate the interference potential in its immediate vicinity;[28] in this case it may be advisable to define the measurement area to be a smaller region, e.g. a 100 meter x 100 meter square or the coverage area of the transmitting PAL system.

Guidance should also be provided about an acceptable set of measurement locations. Following the precedent of the AT&T/Sirius XM coordination agreement reflected in 47 C.F.R 27.64(d)(2), the rule could specify that it should be met on all public roadways, or along a drive route agreed by all parties.[29] In order to avoid deadlock in deciding a route among many service operators, the Commission might specify all public

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[26] 47 C.F.R. § 73.625 (2001); R.A. O'Connor, *Understanding television's grade A and grade B service contours*, IEEE Transactions on Broadcasting, vol. 47, no. 3, pp. 309-14 (September 2001), *available at* http://dx.doi.org/10.1109/11.969381.

[27] I assume that licenses in all channels are defined over the same geographical areas so that adjacent channel measurements are done over the same area.

[28] This may be particularly relevant in rural areas where small cells are deployed over only a small part of a census tract.

[29] Amendment of Part 27 of the Commission's Rules to Govern the Operation of Wireless Communications Services in the 2.3 GHz Band, WT Docket No. 07-293, Establishment of Rules and Policies for the Digital Audio Radio Satellite Service in the 2310-2360 MHz Band, IB Docket No. 95-91, *Order on Reconsideration*, FCC 12-130 (October 17, 2012) at ¶18; 47 C.F.R. 27.64(d)(2) (2013) ("A WCS ground signal level … for more than 1 percent of the cumulative surface road distance on that drive route…").

roads as a default that can be changed by agreement among all affected parties. Current drive test equipment can take from one to dozens of measurements per second per 5 MHz channel, depending on the number of channels being measured concurrently;[30] ten measurements per second equates to 1,200 measurements per mile at 30 mph, thus providing adequate sample sizes for calculating the observed Reception Limit at a 95% confidence level (see Section B below).

It is also advisable to specify a height at which the Reception Limit would have to be met since resulting field strength levels vary by altitude, generally increasing as height increases. Specifying that the limit should be met at 1.5 meters above ground level would facilitate verification by drive tests and/or handset measurements.[31]

B. Providing confidence intervals[32]

The specification of the Reception Limit as a value not to be exceeded with greater than 99% probability is necessary but not sufficient; there should also be a statement of the statistical confidence level at which a violation should be demonstrated.

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[30] Rohde & Schwarz, R&S®TSMx Radio Network Analyzers: Powerful scanner family for mobile applications, Test & Measurement, Data Sheet 01.00, http://cdn.rohde-schwarz.com/dl_downloads/dl_common_library/dl_brochures_and_datasheets/pdf_1/TSMx_dat_en.pdf. Agilent Technologies, Agilent W1314A Multi-band Wireless Measurement Receiver, Data Sheet, https:..www.electrorent.com/products/search/pdf/AT-w1314a-e09.pdf.

[31] 47 C.F.R. § 27.55(a)(4)(ii) (2008) (specifying that "field strength is to be measured at 1.5 meters above the ground"). TV contours are defined by a field strength "9.1 meters (30 feet) above the roadbed." 47 C.F.R. § 73.686(b)(2) (2010). FCC rules typically do not specify the height at which field strengths (or equivalently, power flux densities) are measured, typically just qualifying them as "at the Earth's surface" or "on the ground." *See* 47 C.F.R. § 25.208(a) (2012); 47 C.F.R. § 25.252(a)(5) (2005); 47 C.F.R. § 27.55(b) and (c) (2008); 47 C.F.R. § 101.105(a)(4)(i) and (ii) (2005). Ofcom SURs are specified at 1.5 meters above ground level. Ofcom, *Spectrum Usage Rights: A guide describing SURs* (June, 4 2008), http://stakeholders.ofcom.org.uk/binaries/spectrum/spectrum-policy-area/spectrum-management/spectrum-usage-rights/sursguide.pdf. The 2013 TAC *White Paper* mentioned 1.5 m above ground level as a "typical measurement height … for interference into hand held or fixed user equipment…" 2013 TAC *White Paper*, *supra* note 7, at 9 Section 3.1.

[32] The material in this section was developed with the generous help of Dr. Janne Riihijärvi of the Institute for Networked Systems at RWTH Aachen University. Any errors are mine. *See* Our People, Janne Riihijärvi, Institute for Networked Systems RWTH Aachen University, NETS, http://www.inets.rwth-aachen.de/jar_personal.html.

The following thought experiment outlines the critical role of confidence when drawing conclusions from measurement-based evidence. Suppose we seek to establish whether a given coin is fair, that is, equally likely to yield heads or tails on each toss. Suppose further that we first toss the coin ten times, obtaining heads seven times and tails three times. Based on every-day experience, one would not take this as strong evidence for the coin being biased, even though we have measured it to yield heads 70% of the time. The situation would clearly be different if we had tossed the coin 1,000 times and had obtained 700 heads. In the latter case, our *confidence* of the 70% bias towards heads would be much higher.

The situation with establishing compliance with Reception Limits is similar to the biased coin case, with the only difference being the level of bias: for a coin one has to prove a deviation from a 50/50 split between heads and tails, whereas for Reception Limits the affected system has to prove a deviation from a 99/1 split between a signal level below and above the limit.

Since it is obviously impractical to conduct fine-grained measurements over the entire region in which systems operate,[33] one is forced to draw conclusions from a limited number of measurements, and thus dealing with the uncertainties this creates becomes crucial. Each of these measurements effectively corresponds to a toss of a biased coin with the sides (say) "below limit" and "above limit," and we seek to determine whether the coin lands with the "above limit" side showing more than 1% of the time, corresponding to a violation of the Reception Limit that specifies that it should land "below limit" 99% or more times. The key statistical concepts for quantifying uncertainty, and thus our confidence in measurement-based estimates, are *confidence intervals* and the related *confidence levels*.

[33] That is, at the scale of the coherence distance that represents the fading environment of a particular region.

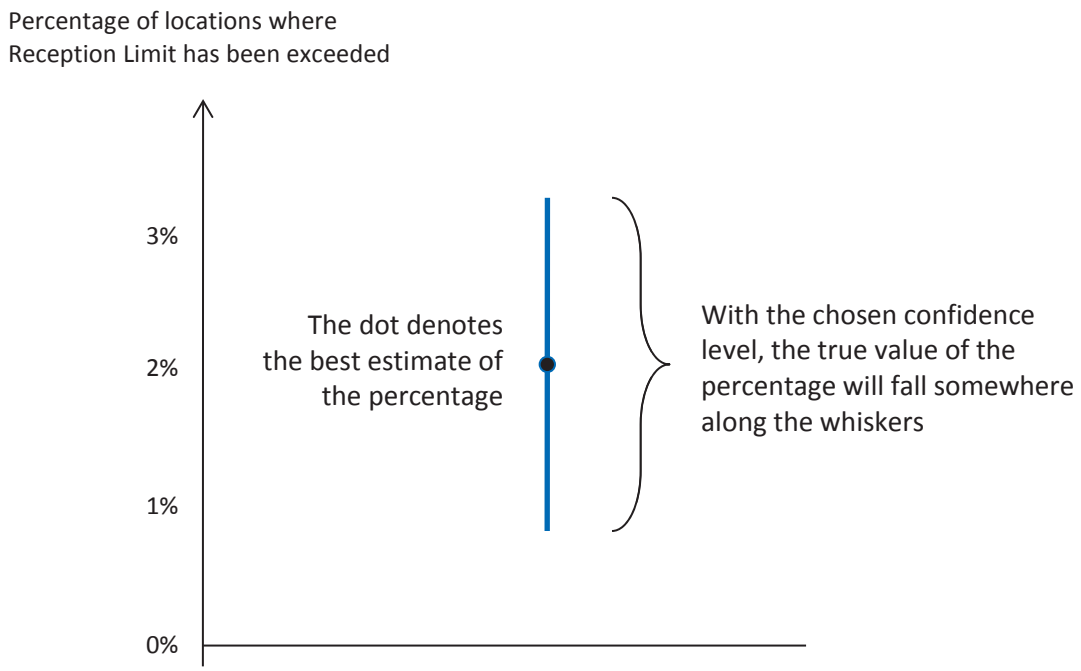

Figure 1. The structure and meaning of confidence intervals.

Figure 1 shows an example of a typical estimate with the associated confidence interval. For the application discussed here, the dot corresponds to the observed fraction of locations where the Reception Limit has been violated. The whiskers on either side of the dot show the corresponding confidence interval. These are ranges of percentages inside which the *true* value (which we cannot know precisely) lies with the confidence level chosen when computing the confidence interval. For example, if a confidence level of 95% is used, the true value will lie within the confidence interval 95% of the time if a similar measurement procedure is repeatedly applied. Choosing a higher confidence level will result in a larger confidence interval, and vice versa. In addition to the confidence level chosen, the number of measurements collected plays a key role in determining the length of the confidence interval, as we will see in the Examples 1 and 2 below.

Figure 2below illustrates the different combinations of the estimates and the corresponding confidence intervals that can arise when measuring Reception Limits. Let us assume that a 95% confidence level is required when showing that the Reception Limits

has been exceeded. In case (a), the system is compliant with the Reception Limit, since both the measured percentage (marked by the dot) and the 95% confidence interval (marked by the whiskers) lie entirely below the 1% threshold.

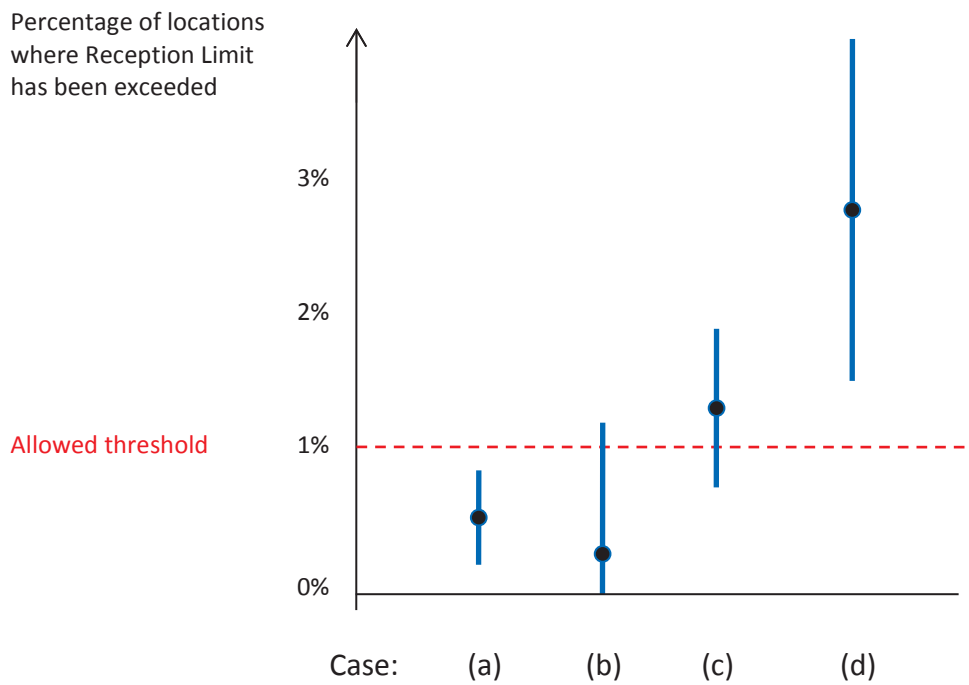

Figure 2. Illustration of different combinations of percentage estimates and confidence intervals that can arise in Reception Limit determination.

In case (b), the measured percentage falls below the threshold, but because the confidence interval is so wide (perhaps because of a limited number of observations), it is possible that the threshold may actually be exceeded. That is, it is possible that the "true" value, which we are 95% confident falls between the whisker ends, lies above the threshold. Case (c) is similarly ambiguous; in this case, the measured value (the dot) lies above the threshold, but because the lower end of the 95% confidence limit falls below the threshold, it is possible that the system is, in fact, compliant.

Finally, case (d) is an example where the system is unambiguously in violation at the 95% confidence level, since both the measured value and the confidence interval lie

above the threshold. Note that in this case the confidence interval is very wide (presumably because of small number of measurements), but since the threshold value is exceeded much more often than in the previous cases, there is no ambiguity over whether the system is in violation of the rule despite the high uncertainty on the exact degree of violation.

The service rules should be clear about whether cases (b) and (c) should be counted as violations or not. I read the proposed rule to imply that only case (d) would be a violation, since I assume that the interfered-with party has the burden of proof and would need to show convincingly (e.g. at the 95% confidence level) that the Reception Limit has been exceeded.

With this groundwork laid, one can now illustrate how the estimates (the location of the dot) and the confidence intervals (the length of the whiskers) can be computed in practice[34] and how they depend on the number of measurements available. Let us study two illustrative cases by way of example. In the first example, assume that 100 measurements have been made, with 2 measurements exceeding the threshold. In the second example, a larger number of 1,000 measurements have been taken, this time with 20 measurements exceeding the threshold. In both examples, we estimate that the threshold is exceeded in 2% of the cases, as 2/100 = 20/1,000 = 2%. The confidence intervals turn out to be very different, however. For both examples they can be computed using the formula[35]

$$p \pm Z\sqrt{p(1-p)/n}$$

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[34] The following discussion uses a simplified approach (based on a normal approximation for the confidence interval) that is, strictly speaking, not valid for extreme percentages (e.g. 1% or 99%) and the smaller of the sample sizes in the example. For the purposes of regulation it may be advisable to use the Wilson confidence interval that has good properties even for a small number of trials and/or an extreme probability. *See* A. Agresti & B.A. Coull, *Approximate is better than "exact" for interval estimation of binomial proportions*, The American Statistician, vol. 52, no. 2, at 119-26 (May 1998), *available at* http://www.sci.csueastbay.edu/~esuess/Statistics_65016502/Handouts/2013/6502/papers/p.hat/agresti&coull.pdf; E.B. Wilson, *Probable inference, the law of succession, and statistical inference*, Journal of the American Statistical Association, vol. 22, no. 158, at 209-12 (June 1927), *available at* http://psych.stanford.edu/~jlm/pdfs/Wison27SingleProportion.pdf; Wikipedia, *Binomial proportion confidence interval*, http://en.wikipedia.org/wiki/Binomial_proportion_confidence_interval.

[35] *See* A. Agresti & B.A. Coull, *supra* note 34; *Binomial proportion confidence interval*, *supra* note 34.

where p is the estimated percentage (2% in both of our examples), n is the number of available measurements (100 and 1,000 in the two examples, respectively), and Z is a constant multiplier depending only on the chosen confidence level. Its values can be readily found for any confidence level of interest from mathematical tables or computed using standard numerical tools. If one assumes the 95% confidence level, Z turns out to be 1.96. Substituting these numbers into the above equation yields for the first example

$$2\% \pm 2.744\% \quad ,$$

and for the second example

$$2\% \pm 0.868\% \quad .$$

Since percentages below zero are not possible, the final 95% confidence intervals for the two examples range from 0% to 4.744% for Example 1, and from 1.132% to 2.868% for Example 2 (see Figure 3 below).[36]

From these numbers we can estimate that the system exceeds the Reception Limit in 2% of the locations in both examples. However, with only 100 measurements one is unable to state with 95% confidence that the system exceeds the Reception Limit in more than 1% of locations. With 1,000 measurements the entire 95% confidence interval lies above the 1% threshold value, indicating that the system is indeed non-compliant.

In order to have confidence that an observed estimate of the Reception Limit percentage is reliable, observations need to be uniformly distributed over the measurement area in an unbiased manner, and not just (say) in high level emission regions near known transmitters. Using uncorrelated observations also leads to a more rapid convergence of the estimated percentiles of the received power values, as can be seen by comparing Figures 5 (a) and (b) in Riihijärvi et al.

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[36] The corresponding 95% Wilson confidence intervals are from 0.550% to 7.001% for Example 1, and from 1.298% to 3.069% for Example 2, in other words, not substantially different from the normal approximation used in the text.

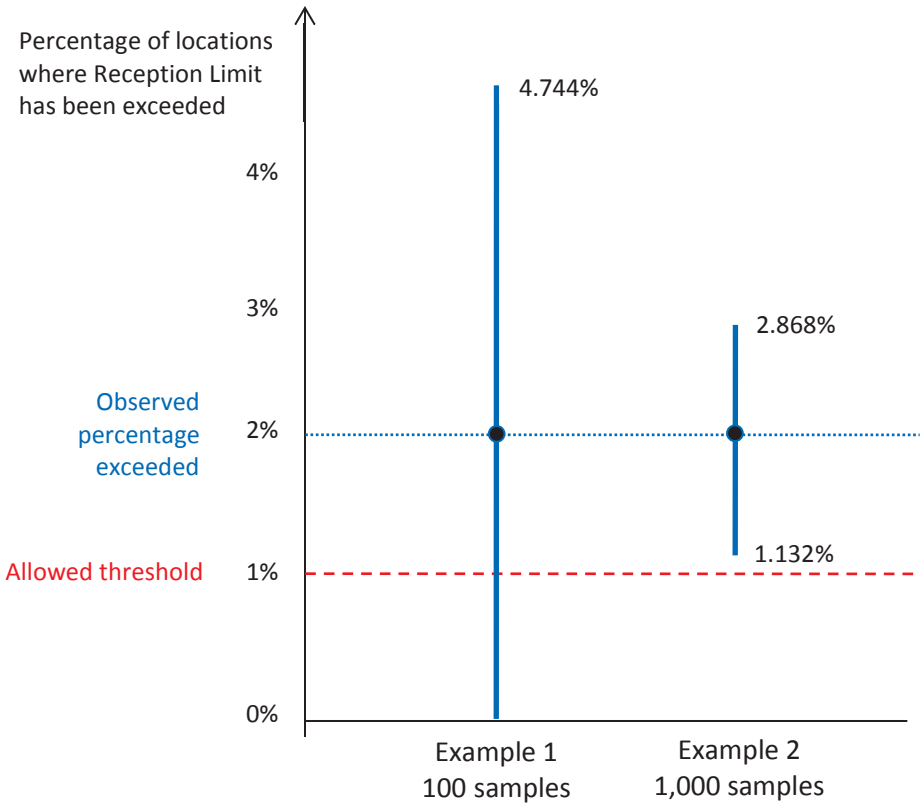

Figure 3. Illustration of 95% confidence intervals for 2% examples discussed in the text.

The use of a 99th percentile is defensible and accounts for all but the most extreme deviations from the norm. However, the larger the percentile, the larger the number of observations that are required to show that the limit is met (or exceeded) at a given confidence level. This is illustrated in Figure 5 (b) of Riihijärvi et al.,[37] where one can observe that the value for the 90th percentile interference power stabilizes after about 700 samples, whereas the observed 99th percentile value is still somewhat unstable after 2,000 observations. Thus, the higher the probability percentage chosen, the more measurements will have to be taken to establish that the Reception Limit has been exceeded for a given

[37] Riihijärvi et al., *supra* note 21 and *infra* Appendix A.

confidence level. Similarly, the higher the confidence level required, the more measurements will be needed for a given probability percentage.

C. Specifying Receiver Limit over a narrower resolution bandwidth, and specifying the frequency range over which it applies

The Commission's choice to specify Receiver Limits as a power flux density over 10 MHz presumably reflects its choice to assign PALs as 10 MHz channels.[38] However, the FNPRM explicitly states that the Commission does not intend to create a fixed bandplan,[39] and it is possible that operators may elect to use less than the full 10 MHz of a channel. It is therefore advisable to define the Receiver Limit spectral density over a narrower bandwidth, e.g. as -40 dBm per 1 MHz measured by a 0 dBi antenna in addition to, or in place of, defining it as -30 dBm per 10 MHz measured by a 0 dBi antenna.

The *FNPRM* does not specify the bandwidth on either side of an assigned channel over which devices operating on a Priority Access basis must accept interference. For the avoidance of doubt I recommend that the Commission follows the approach outlined in the 2013 TAC *White Paper* and defines the out-of-channel Reception Limit over a limited bandwidth on either side of the assigned channel,[40] for example 20 or 30 MHz above and below the assigned channel.

D. Adding antenna gain to fully define Receiver Limits as a field strength

In a minor oversight, the *FNPRM* correctly describes Reception Limits as field strengths but then specifies them in units of power spectral density rather than power *flux* spectral density. [41] Field strength (e.g. in units of microvolt/meter or dB(μV/m)) and power flux density (i.e. power per area, e.g. milliwatts/meter^2 or dB(mW/m^2)) are equivalent

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[38] *FNPRM* at ¶ 6.

[39] *Id.* at ¶ 6 n.12.

[40] 2013 TAC *White Paper, supra* note 7, at 10 fig.1 ([A] "harm claim threshold profile … extends over … a limited frequency range beyond the assignment boundary"), and at 15 ("will be given no protection against interfering signals beyond this point").

[41] *FNPRM* at ¶ 85.

quantities.[42] However, if only a power (e.g. milliwatts or dBm) is specified, the frequency and antenna gain also needs to be given in order to calculate the equivalent field strength.[43]

Indeed, the Received Signal Strength Limits are specified in exactly this way as received signal power level of -80 dBm measured by a 0 dBi antenna (the frequency is set by the scope of the rule),[44] and the proposed protection for FSS earth stations is given as a power flux density.[45]

I therefore assume that the proposed rule of "a power spectral density limit of -30 dBm / 10 megahertz" should be read as "a power *flux* spectral density limit of -30 dBm / 10 megahertz *measured by a 0 dBi antenna*." (Equivalently, and perhaps more succinctly, the Reception Limit could be specified as a field strength of 118 dB(μV/m) per 10 megahertz.[46])

E. The determination and enforcement of Receiver Limits

I will now address some questions raised in the *FNPRM*. Some of the questions posed about Received Signal Strength Limits, i.e. how and where should the signal level be determined,[47] also apply to Reception Limits, with the proviso that Received Signal Strength Limits are defined at a service area boundary, while Reception Limits are defined over a service area. I will also address questions raised in the *FNPRM* about the use of SASs to administer Reception Limits.[48]

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[42] The conversion between them is dB(μV/m) = dB(W/m^2) + 145.76. F.H. Sanders, *Derivations of relationships among field strength, power in Transmitter-Receiver circuits and radiation hazard limits*, NTIA Technical Memorandum TM-10-469, at 7 Equation (28) (June 2010) (NTIA *TM-10-469*), *available at* http://www.its.bldrdoc.gov/pub/ntia-rpt/10-469/10-469.pdf.

[43] *Id.* at 5-7, § 2.2, Equation (20).

[44] *FNPRM* at ¶ 79.

[45] *Id.* at ¶ 159.

[46] The conversion is made using Equation (20) in NTIA *TM-10-469*, using f = 3600 MHz and G = 0 dBi. NTIA *TM-10-469*, *supra* note 42, at 6 Equation (20). The field strength resulting in -30 dBm of power after a 0 dBi antenna is not very sensitive to frequency, varying between to 118.2 dB(μV/m) at 3550 MHz and 118.6 dB(μV/m) at 3700 MHz .

[47] *FNPRM* at ¶ 79.

[48] *Id.* at ¶ 86.

How should this signal level be determined?[49] The signal level can be determined by measurement, modeling, or modeling calibrated through measurement. The customary measurement technique, used in the example mentioned above, is a drive test.[50] In addition, or alternatively, field strength levels could be reported to the SAS using technology like the Minimization of Drive Tests (MDT), a feature introduced in 3GPP Release 10 that enables operators to utilize end users' equipment to collect radio environment information.[51]

However, if the location, transmit power and antenna pattern of transmitters are known, the resulting field strength can also be estimated using propagation models. Each method has limitations; drive test data only measures signal levels along public roads, while propagation models provide better coverage but suffer from various errors e.g. due to lack of terrain detail. Performance can be improved by combining the two methods, e.g. using drive test data to calibrate model results, and conversely using models to fill in sampling gaps.[52]

How feasible would it be for the SAS to calculate and enforce such a limit?[53] An SAS could calculate and enforce, that is impose on devices, the application of such a limit,

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[49] *Id.* at ¶ 79.

[50] In a drive test, measurement equipment is mounted in or on a vehicle that is driven along a drive test route; the information is typically analyzed offline. Drive test equipment typically collects data on received signal levels as well as network services, as well as GPS information to provide location logging.

[51] J. Johansson, W. A. Hapsari, S. Kelley, & G. Bodog, *Minimization of drive tests in 3GPP release 11*, IEEE Communications Magazine, vol. 50, no. 11, at 36-43 (November 2012), *available at* http://dx.doi.org/10.1109/mcom.2012.6353680.

[52] C. Phillips, M. Ton, D. Sicker, & D. Grunwald, *Practical radio environment mapping with geostatistics*, 2012 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN), at 422-33 (October 16-19, 2012), *available at* http://dx.doi.org/10.1109/dyspan.2012.6478166; J. Riihijarvi, P. Mahonen, M. Wellens, & M. Gordziel, *Characterization and modeling of spectrum for dynamic spectrum access with spatial statistics and random fields*, IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), at 1-6 (September 15-18, 2008), *available at* http://dx.doi.org/10.1109/pimrc.2008.4699912.

[53] *FNPRM* at ¶ 76. The term "enforcement" has many meanings, including (1) imposing a rule (ensuring through various ex ante means that a rule is observed, including through device certification, education, and device control database protocols); (2) monitoring and investigating (observing and collecting data on the RF and service environment, determining the source of interference/degradation, and raising a flag when interference/degradation is observed); (3) adjudication (deciding the merits of allegations of violations); and

but it may not be necessary: since the Reception Limit is derived from standard operating parameters for LTE networks,[54] and since the CRFS measurements discussed in Section II.B above suggest that it is roughly in line with actual deployment, it is likely to be observed in the ordinary course of deployment. If an affected PA licensee believes it is exceeded, a quick drive test will readily provide evidence to that effect.

That said, the calculation capabilities proposed by some commenters in this proceeding would support the calculation and application of Reception Limits by SASs. For example, the dynamic spectrum management functionality described by Google requires that the SAS is able to calculate the resulting field strength for a transmitter in the database at all locations in order to ensure the protection of higher-ranking services.[55] The question of whether the SASs could enforce these limits is an instance of the larger question of enforcement by SASs;[56] this topic has been raised by the FCC TAC, [57] and further work is required before the answer is clear.

Could the SASs track, manage, and enforce agreements between different users?[58] Just as in proposed rules for received signal strength limits at the border of service areas and the custom for out-of-band emissions (OOBE),[59] the FCC proposes that affected parties could agree to higher or lower Reception Limits and asks whether it would be

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(4) enforcement action (applying penalties and ex post other measures to bring parties into compliance and deter future violations). I take the primary intended meaning here to be "impose" or "apply" ex ante.

[54] *Id.* at ¶ 86.

[55] Comments of Google, Inc. on the Proposed Revised Framework, *supra* note 17, at 12 ("…Google has developed a prototype SAS … The Google prototype includes the following capabilities: 1. Managing a mix of Priority Access and GAA devices, ensuring non-interference to Priority Access devices…2. Dynamic protection of C-band satellite users … based on the actual antenna elevation and elevation angles of the C-band dish and the distance between the dish and secondary users.… 3. Reflecting a wide range of device characteristics—including but not limited to power, out-of-band emissions, bandwidth, and directionality…").

[56] *FNPRM* at ¶ 162.

[57] FCC Technological Advisory Council, Receivers and Spectrum Working Group, Introduction to Interference Resolution, Enforcement and Radio Noise, *White Paper*, at 19 (June 10, 2014), http://transition.fcc.gov/bureaus/oet/tac/tacdocs/meeting61014/InterferenceResolution-Enforcement-Radio-Noise-White-Paper.pdf; FCC Technological Advisory Council, Receivers and Spectrum Working Group, Report of Advanced Sharing and EWT WG, at slide 46 (June 10, 2014), http://transition.fcc.gov/bureaus/oet/tac/tacdocs/meeting61014/TACmeetingslides6-10-14.pdf.

[58] *FNPRM* at ¶ 86.

[59] *Id.* at ¶ 79.

feasible for the SASs to administer such a scheme.[60] It would not be difficult for SASs to record and track such agreements once they have been made, since it such agreements can be stored as one of the data fields that have to be synchronized among them; it would use whatever mechanism works for everything else.

As noted above, the question of whether and how SASs could enforce agreements about Reception Limits needs to be addressed in the larger context of SAS enforcement authority and capability.

IV. Reception Limits could be extended to protect Incumbent Access tier users, and optimize coexistence between Priority Access and General Access users

Using Reception Limits to facilitate PAL coexistence is a valuable and laudable step, and a good way to build confidence in this important regulatory tool. However, Reception Limits have benefits that apply not only to interaction between PAL systems but also to the optimal protection of Incumbent Access (IA) operations from secondary and tertiary services (i.e. PAL and GAA), and coexistence between PAL and GAA.

Since interference limits policy is has only seen limited use to date,[61] I am not proposing that Reception Limits should be applied immediately to these other cases, especially incumbent services; however, their application should be a regulatory goal since the benefits they bring to optimizing PAL-to-PAL coexistence would then also be brought to PAL-to-IA and GAA-to-PAL interference management.

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[60] *Id.* at ¶ 86.

[61] Current implementations of interference limits include the minimum receiver performance criteria 800 MHz case (Pierre De Vries, *800 MHz receiver criteria as harm claim thresholds*, DEEP FREEZE, at 9 (December 5, 2012), http://deepfreeze9.blogspot.com/2012/12/800-mhz-receiver-criteria-as-harm-claim.html), and the WCS/SDARS rules (47 C.F.R. § 27.64(d)(2) (2013)). The RTCA/DO-229 test interference levels at the antenna port for aviation GPS receivers are referenced e.g. in FAA Technical Standard Order (TSO) - C145 and C-146. Christopher J. Hegarty, *Civil Aviation GNSS Standards*, MITRE, at slide 5 (June 20, 2014), http://transition.fcc.gov/oet/prd/GPS-WORKSHOP_6-20-14/PANEL_3/3-2_Hegarty_MITRE.pdf (presented at FCC Workshop on GPS Protection and Receiver Performance (June 20, 2014), http://www.fcc.gov/events/workshop-gps-protection-and-receiver-performance).

A. Reception Limits would facilitate the protection of federal users, and their adoption for this purpose should be a long-term goal

The Commission has opted to frame the protection of federal users in terms of exclusion zones, at least for the moment.[62] A more resilient long-term solution would be to use the interference limits policy / harm claim thresholds approach recommended by the PCAST *Report* and 2013 TAC *White Paper*, implemented in part in this *FNPRM* as out-of-channel Reception Limits.

An exclusion zone, including exclusions at frequency offsets outside a service's assigned bandwidth,[63] are the result of a calculation that combines (1) assumptions about acceptable levels of service; (2) data on the sensitivity of a service's receivers and antennas; (3) assumptions about propagation and terrain models; and (4) assumptions about the energy radiated by the interferer in the direction of the affected service, including transmitted power, antenna height, and antenna gain. There are uncertainties in all of these assumptions and they compound when combined, leading to over-conservative protection radii (i.e. exclusion zones).

Harm claim thresholds allow one to factorize the problem; the thresholds embody the combination of assumptions (1) and (2), both for co-channel exclusion zones and, by specifying an out-of-channel frequency profile for the threshold, for the frequency off-set. Once established, back-off distances and frequency off-sets to avoid interference can be calculated ad hoc depending on terrain and interfering service, i.e. factors (3) and (4). If the interfering service parameters change – as might happen if new or additional services are deployed[64] – the harm claim thresholds (i.e. the co-channel and out-of-channel Reception Limits) of the IA service do not have to change; adjustments in back-off factors and off-sets can be done in an SAS.

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[62] *FNPRM* at § III.B.1 ¶ 137-42.

[63] *Id.* at ¶ 138.

[64] *Id.* at ¶ 140 ("… the rules proposed in this *FNPRM* contemplate additional uses other than small cells, with varying maximum transmit power levels and antenna gains, which must factor into the consideration of Exclusion Zones.").

Reception Limits as an instance of harm claim thresholds are therefore a more fact-based, transparent, robust, and future-proof way to protect incumbent uses than exclusion zones. As a first step, I would suggest that co-channel and out-of-channel Reception Limits are provided to the FCC by NTIA as part of their "continuing … dialogue",[65] and that the FCC then uses this information to calculate exclusion zones based on the characteristics of the interfering service. Eventually, new services could be accommodated dynamically with these calculations implemented in the SASs.

Reception Limits also provide a quantitative and transparent basis for eventually authorizing coordinated operations for GAA and PAL users inside proposed exclusion zones through SASs.[66] SASs would use the Reception Limits of primary incumbents to calculate the allowed transmit power of PA and GAA users based on their locations and intervening terrain.

B. Reception Limits would facilitate the protection of Fixed Satellite Earth stations

I agree with the Commission's assessment that an analytic model of expected aggregate power-flux density could be used by SASs to authorize operations that would protect FSS earth stations.[67] The amount of protection that FSS earth stations require would be best expressed as a co-channel and out-of-channel Reception Limit that would specify the resulting impinging field strength along various vectors.[68] With this information in hand, along with data on the transmit power and antenna pattern (or EIRP) and location of interfering devices, and suitable terrain data and propagation models, an aggregate resulting field strength (or equivalently, power flux density) can be calculated by an SAS.

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[65] *Id.* at ¶ 141.

[66] *Id.* at ¶ 142.

[67] *Id.* at ¶ 150.

[68] 2013 TAC *White Paper, supra* note 7, at 23-24. If the Reception Limit (i.e. the harm claim threshold described in the *White Paper*) is specified as a pattern in altitude and azimuth at the FSS earth station, antenna characteristics as suggested in *FNPRM* are redundant since they are included in the calculation of the Reception Limit. *FNPRM* at ¶ 151.

C. Reception Limits could be used to ensure that General Authorized Access operation does not interfere with Priority Access use

Since Reception Limits state the out-of-band interference protection rights of PAL receivers, they could – and I recommend, should – be used to specify GAA operation that does not degrade the performance of PA systems.

The *FNPRM* proposes a variety of maximum transmit power levels to accommodate a range of Citizens Broadband Radio Service use cases.[69] These limits are designed to protect Priority Access and Incumbent Access services.

The use of the PAL Receiver Limit along with propagation loss calculations – implemented in an SAS – may reveal that higher power GAA operation would not cause interference to the affected PAL receivers. In other words, the FCC would simply specify the maximum allowed resulting signal strength at the protected receiver and let an SAS calculate the allowed GAA transmit power. The FCC should allow higher power GAA operation in such cases, or at least allow potentially affected PAL services to permit such operation.

Even if the FCC should elect not to do this, it would serve the public interest for it to disclose the assumptions about Reception Limits and path loss that led to its final rules. This disclosure would serve as a way to validate whether the assumptions are fulfilled in practice as services are deployed and operating experience grows. That, in turn, would serve as a transparent basis for deciding whether operating rules need to be relaxed or tightened, whether in new FCC service rules or in operating protocols in SASs.

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[69] *FNPRM* at ¶ 74.

Respectfully submitted,

/s/ Pierre de Vries

J. Pierre de Vries
Silicon Flatirons Center for Law, Technology, and Entrepreneurship
University of Colorado Law School
401 UCB, Wolf Law Building
Boulder, CO 80309

Appendix A. Riihijärvi et al. (2014)

A study on the design space for harm claim thresholds

J. Riihijärvi, A. Achtzehn, P. Mähönen, & P. De Vries

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A Study on the Design Space for Harm Claim Thresholds

Janne Riihijärvi, Andreas Achtzehn, Petri Mähönen
Institute for Networked Systems, RWTH Aachen University
Kackertstrasse 9, D-52072 Aachen, Germany
email: {jar, aac, pma}@inets.rwth-aachen.de

Pierre de Vries
Silicon Flatirons Center
University of Colorado, Boulder
email: pierredv@hotmail.com

***Abstract*—Harm claim threshold policies will offer more flexibility in radio regulations. By defining bounds on the acceptable field strength over time and space, these policies will introduce quantitative measures to inter-system coexistence questions. However, this requires tangible and sufficient means of proving claims of interference through spectrum measurements. In this paper we study drive test requirements in terms of necessary sample set sizes and decorrelated sampling. We present order statistics and extreme value theory as powerful mathematical tools for describing necessary confidence intervals, and test their practical viability with data from an extensive measurement campaign. Furthermore, we discuss by means of example necessary extensions to planning tools for wireless networks operating under an interference threshold policy. Our results emphasize that future regulations will need to be accompanied by a rigorous specification of evidence collection requirements in order to compensate for bias and correlation structure in the spectrum data.**

I. Introduction

Existing radio regulation often relies on qualitative and subjective statements when it comes to specifying coexistence rules such as interference tolerance levels between wireless systems. For example, the ITU defines harmful interference as "interference which endangers the functioning of a radionavigation service or of other safety services or seriously degrades, obstructs, or repeatedly interrupts a radiocommunication service" [1]. Such statements are problematic since they are difficult to reason about objectively, and lack clarity when it comes to the rights and responsibilities of wireless operators regarding harmful interference. Because of this there has recently been significant interest in reforming inter-system interference policies in terms of *interference limit policies*. Such policies aim to describe the radio environment a receiver must be able to cope with and the limits to emissions of a transmitter using *objective* and *measurable* criteria instead.

A key example of an interference limit policy is the *harm claim threshold* approach [2] described in the 2012 PCAST report on spectrum, and developed by an FCC TAC Working Group in 2012. A harm claim threshold essentially consists of a field strength profile defined over a frequency range, associated with a percentage of locations and times where the field strength must be exceeded at some confidence level to qualify as harmful interference. The threshold, which defines the interference a wireless operator needs to accept from others, would be low within his own band, higher for adjacent bands, and highest for bands far away in frequency. Under a harm claim threshold policy, disputes on coexistence issues would become a matter of conducting spectrum use measurements to determine whether the parties involved cause interference exceeding the given threshold.

While interference limit policies in general and the harm claim threshold approach specifically are conceptually attractive and would seem to offer numerous benefits when compared to current regulatory approaches, several open questions remain on how they should be applied in practice. The key design space for harm claim threshold consists of (1) the field strength (or received interference power) threshold at a given frequency range, (2) the percentage of locations and times this threshold must be exceeded, and (3) the confidence level at which the exceedance must be shown. Whereas those parameters are ultimately a matter of policy, the design of measurement campaigns to reliably and sufficiently substantiate claims under harm claim threshold policies will necessarily need to build on knowledge of the statistical principles and practical constraints of extensive spectrum data collection. In this paper we first discuss by means of data analysis and simulations minimum requirements for the design of such campaigns. After introducing mathematical tools from *order statistics* and *extreme value theory*, we use data from a large measurement campaign in the UK to assess their feasibility and prediction capabilities in the process of deriving the actual interference statistics. We found that high thresholds require a substantial sampling density, and that bias from low randomization of measurement locations significantly affects the measurement results.

In the second part of the paper, we take the role of an operator deploying a network subject to a harm claim threshold policy. We discuss simulation requirements of relevant real-world propagation effects (in particular correlated shadowing) and quantify their effects on the convergence rate of the presented estimators. We show that, due to sampling correlations, significantly lower confidence is reached for the same sampling set sizes, i.e., deployment studies will become more complex. These results are highly relevant for the policy community, because they highlight the necessity for a discussion on establishing best practice rules for measurement campaigns for interference limit claims, and for operators wishing to incorporate these policies into their network planning tools.

II. A Primer on Order Statistics

In order to reason statistically about the exceedance of a signal level threshold, we adopt the following simple probabilistic model. Let X be a random variable, yielding the received power or field strength at a randomly picked location, measured over a given range of frequencies. Denote then by $F_X(x)$ the cumulative distribution function (CDF) of X. If F_X were somehow known, computing different statistics of interference power for harm claim thresholds becomes trivial. For example, $1 - F_X(-40\,\text{dBm})$ would yield the fraction of locations at which the interference exceeds $-40\,\text{dBm}$, whereas $F_X^{-1}(0.95)$ would give the interference threshold that is exceeded only at 5% of locations. Unfortunately in any realistic scenario the "true" CDF $F_X(x)$ is unknown, and different properties related to it must be estimated through measurements (in case of, say, policy enforcement) or simulations (in case of planning a network roll-out under given harm claim threshold constraints). We shall focus in this and the following section on the former problem, and discuss simulation-based inference further below in Section IV.

We assume that we have conducted n measurements of the interference power X, denoted by $X_1, \ldots, X_n$ with the purpose of establishing a claim on exceedance of a given claim threshold. The *order statistics* of these measurements would simply be the indexed measurement values sorted in ascending order, and denoted by $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$. Thus $X_{(1)} = \min\{X_1, \ldots, X_n\}$, $X_{(n)} = \max\{X_1, \ldots, X_n\}$, with the rest of the order statistics carrying information about the *percentiles* $F_X^{-1}(p)$ of X. Note that due to the limited sample size, $X_{(n)}$ is *not* in general the true maximum of X, but some necessarily smaller value we may use to estimate the true maximum. Similarly, for large enough sample we use $X_{(p\times(n+1))}$ as the estimate for the pth percentile of interference power. The probability distribution associated to the inherent error of this estimate is called the *sampling distribution* of the corresponding percentile.

Given the limited sampling size, we need to state the uncertainty of $X_{(p\times(n+1))}$ for any claim. The measure of the estimation error $X_{(p\times(n+1))}$ is, when used as a proxy for $F_X^{-1}(p)$, given by the associated *confidence interval*. Following the result of Conover [3], [4], for unknown F_X the confidence interval for the pth percentile can be constructed from n measurements as follows. First, let Z_p denote the pth percentile of the standard normal distribution $N(0,1)$. Then the upper limit of the $100(1-\alpha)\%$ confidence interval is given by $X_{(u)}$, where

$$u = p(n+1) + Z_{1-\alpha/2}\sqrt{np(1-p)}, \tag{1}$$

and the lower limit by $X_{(l)}$, where

$$l = p(n+1) - Z_{1-\alpha/2}\sqrt{np(1-p)}. \tag{2}$$

If u or l turns out to be non-integer, the corresponding endpoint of the confidence interval can be determined by interpolating between the nearest adjacent order statistics. In addition to these *two-sided* confidence intervals we will need the *one-sided* confidence intervals, giving only the upper or lower bound for the error. These are obtained simply by replacing $Z_{1-\alpha/2}$ by $Z_{1-\alpha}$ in the corresponding formula above. These upper and lower bounds yield a necessary extension to the regulation policy, as they define the certainty level of the measurement results, and thus the likelihood that the harm claim threshold indeed has or has not been exceeded for measurement results near the threshold. Most importantly, for these bounds to hold, measurements must be gathered in a manner that is *unbiased* in time and space, which must be taken into account when specifying the allowable *measurement plans* in the regulation.

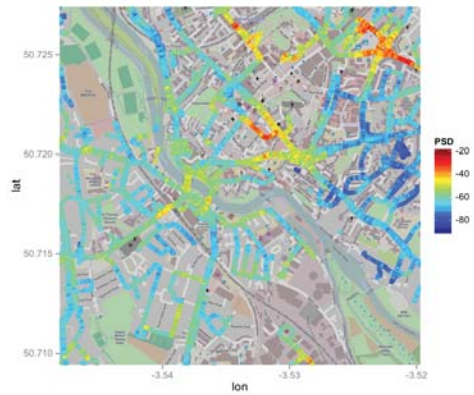


Fig. 1. Example drive test data set used for studying the order statistics and extreme values of received power profiles.

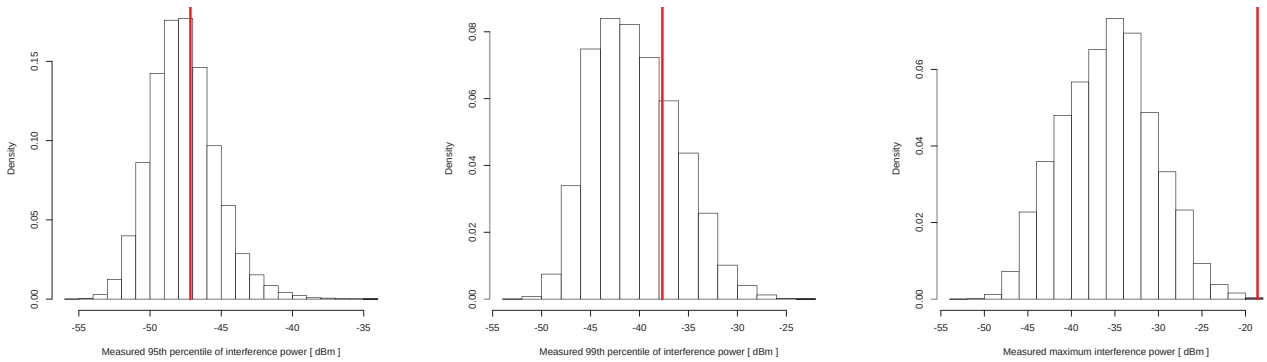
III. Estimating Order Statistics from Drive Tests

We shall now apply the methods outlined in the previous section to extensive drive test data set, and study in detail the properties of both true extreme values of power levels in the data set, as well as those of the different estimators for them based on smaller subsets of the entire measurement data set. These results can be directly used to shed light on adjacent channel interference statistics for real-world wireless systems.

A. Measurement Data Set

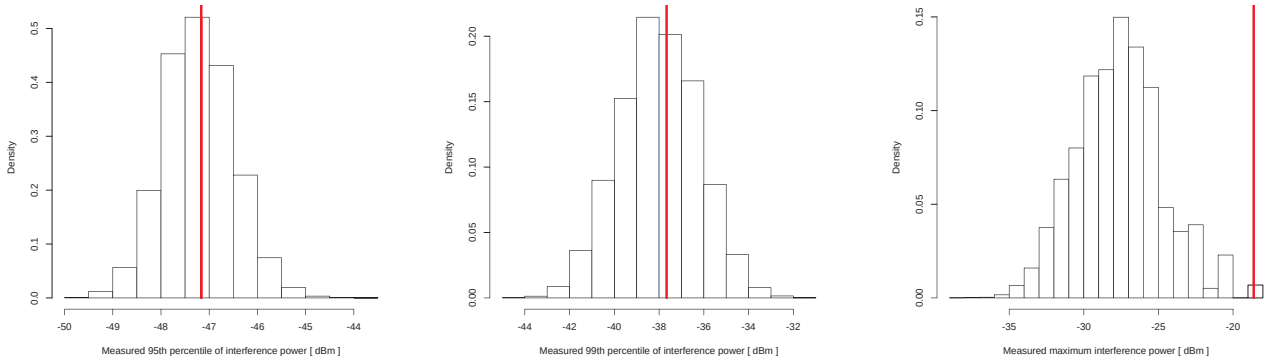
As a basis for our study we use drive test data gathered by CRFS on behalf of the UK regulator Ofcom as a part of the "not-spot" study for coverage of mobile broadband. The measurements were conducted using CRFS RFeye nodes, which were set up on rooftop boxes of measurement vehicles together with omnidirectional antennas. We refer the reader to [5] for more details on the measurement campaign.

Since the majority of the geographic region covered by the measurements is sparsely populated and thus with low deployment density of wireless services, we limit the study in this paper on a small region corresponding to the urban downtown area of the city of Exeter. Further, we focus on the UMTS downlink band, as this removes the impact of time domain dynamics which are present in TDMA, OFDMA and random access based systems. The resulting data set shown in Figure 1 consists of over 35 000 samples, covering a square



(a) 95th percentile. (b) 99th percentile. (c) Maximum.

Fig. 2. The sampling distributions (histograms) for 95th and 99th percentiles and the maximum with 100 randomly selected measurement locations as well as the underlying true values (red line).



(a) 95th percentile. (b) 99th percentile. (c) Maximum.

Fig. 3. The sampling distributions and true values for 95th and 99th percentiles and the maximum with 1000 randomly selected measurement locations. Notice that the variability of the estimates has reduced significantly compared to the 100 measurement case shown in Figure 2.

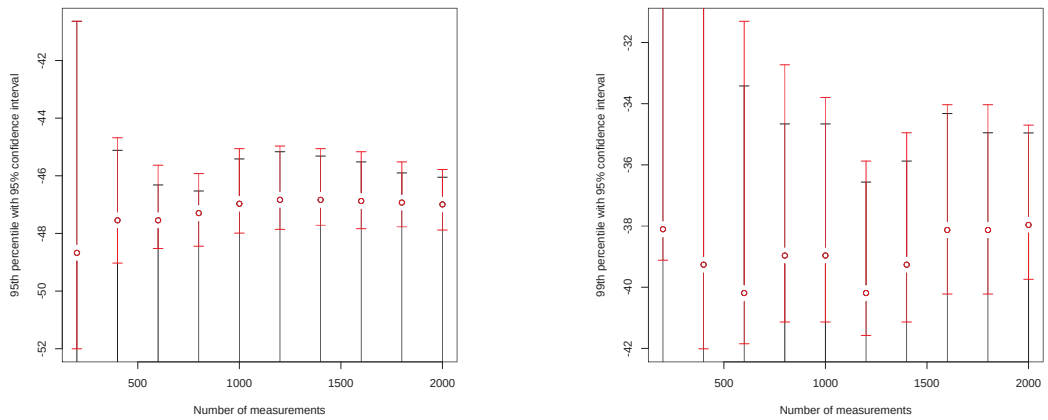
of $2\,\text{km} \times 2\,\text{km}$. Reported powers are the sum over a selected carrier of a local 3G operator. Such a data set would e.g. be used for an interference claim for a service operating in an adjacent band relative to the UMTS downlink frequencies.

B. Sampling Distributions and Confidence Intervals

We begin by studying the sampling distributions for the different percentiles of the received power levels for different number of measurement points. Figure 2 shows these distributions for $n = 100$ measurements together with the "true" values of these percentiles computed over the entire data set. These plots were obtained by repeatedly selecting random measurements from the entire data set, computing the given percentage, and finally plotting the histogram of all the percentage estimates obtained. We see that in all the cases the sampling errors are substantial as can be expected for such a small measurement count. However, we note that the sampling errors increase rapidly as the percentile is increased, and in the case of maximum ($p = 1$) the errors are significantly biased, resulting in major underestimation of the maximum observable interference level. From Figure 3 we see that increasing the measurement count to $n = 1000$ significantly reduces the errors for the 95th percentile and to some extent also for the 99th percentile. However, the estimates for the maximum are still highly biased and variable.

Several important conclusions can be drawn from these results. First is that regulating for the *worst case interference* is untenable. The high estimation errors for the maximum interference power would force carefully acting operators to vastly restrict transmit powers, and make definite conclusions in harm claim threshold disputes difficult to achieve due to the high level of uncertainty. Second is that also for the lower percentages, the number of measurements conducted has major impact on the estimation accuracy. Further, the choice of percentile in a harm claim threshold entitlement has a significant impact on the number of measurements that the holder has to make to demonstrate a harm claim at a given confidence level. While such statements are qualitatively trivial, we will now see that they can be quantified successfully by using the appropriate confidence intervals for the estimates as outlined in the previous section.

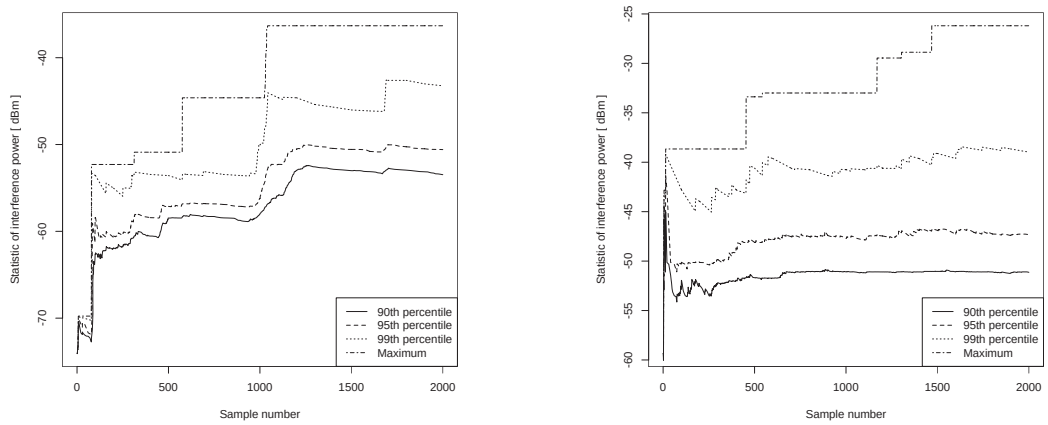
Figure 4 shows the convergence of the estimated received power percentiles together with the one-sided and two-sided confidence intervals. We see that while a relatively small number of measurements suffices to obtain stable estimates



(a) 95th percentile with 95% CI.

(b) 99th percentile with 95% CI.

Fig. 4. The convergence of estimated order statistics and their confidence intervals for the UMTS data set.



(a) Sampling along a drive path.

(b) Randomized sampling.

Fig. 5. The convergence of order statistics with and without randomisation. In the latter case the convergence to true values is substantially slowed down by spatial correlations in the data.

with bounded confidence intervals for the 95th percentile, for the 99th percentile a much larger number of measurements is needed to obtain finite confidence intervals to begin with, and in any case convergence is very slow. Note that these results are not contradicting those shown in Figures 2 and 3, as the true underlying sampling distributions require the entire data set for their estimation. We conclude that, for a given number of measurements, regulators will need to be very conservative and cautious on their expressiveness in terms of high percentiles of interference levels.

C. Impact of Spatial Correlations

We conclude this section by highlighting the impact of the used *measurement plan* on the results. Measured field strength or received interference power values are highly correlated spatially, resulting in measurements at nearby locations mainly yielding redundant information on the region-wide statistics. This makes is important to "spread out" the measurements used to draw conclusions on harm claim threshold violations. The confidence intervals used above were also derived assuming uncorrelated data, which can only be achieved through keeping the measurement points sufficiently far apart from each other.

The impact of spatial correlations on the convergence of the different statistics considered until now is illustrated in Figure 5 for two extreme cases of measurement plans. First we look at a non-randomized plan where 2000 measurements are taken consecutively along a path covered by the drive tests. Second is a completely randomized plan, in which the same number of measurements are selected completely at random among all the locations available in the data set. We see that in the first case the convergence of all the estimates is very slow, whereas in the second case all the percentiles except the maximum converge quickly. This highlights the necessity of randomization of the measurement locations: A mischievous operator may select drive paths as to cover particularly high-level emission regions (e.g. close to known base stations), thereby ignoring the smoothing effects of lower power levels

at the coverage edges of a region. Similarly, measurement campaigns only in low coverage areas will yield fundamentally lower interference levels. Policy makers will thus need to decide how to balance between results from heterogeneous coverage regions.

IV. Impact of Model Errors on Network Design with Interference Limits

The discussion in the previous section focused on the measurement-driven determination of interference limits for already deployed systems. However, such approaches are not feasible for the network deployment stage, where an operator must decide on the feasibility of deploying a system under an agreed upon harm claim threshold regime. In this section we show through a simulation example that many of the features seen in measurements, such as the high variability of maximum interference power and highest percentiles, are present in the network planning problem as well. Lower percentiles on the other hand result in more predictable behavior under uncertainties in the propagation environment.

Our simulated network is composed of 13 UMTS base stations in the previously discussed $2\,\text{km} \times 2\,\text{km}$ square region in downtown Exeter. We have acquired transmitter locations $\nu_{\text{TX},i}$ and transmit powers $P_{\text{TX},i}$ from a public database of UK regulator Ofcom [6]. Base stations in the scenario transmit at power levels between 40.6–60.3 dBm in the 2.1 GHz downlink UMTS band. Whereas in reality the base stations are owned by different operators and consequentially operate on different carrier frequencies, noting that the transmitter information reflects the initial rollout situation of UMTS in the UK, we have decided to combine all operator base stations to approximate a presumed intermediate network densification.

Received signal strength values $P_{\text{RX},j}$ for the different measurement locations $\gamma_{\text{RX},j}$ in the region are calculated for a fixed receiver antenna height of $h_m = 1.5\,\text{m}$ as

$$P_{\text{RX},j} = 10 \log_{10} \left(\sum\nolimits_i 10^{(P_{\text{TX},i} - L_{i,j} + \chi_{i,j})/10} \right), \tag{3}$$

where $L_{i,j}$ is the mean pathloss at distance $d_{i,j} = |\nu_{\text{TX},i} - \gamma_{\text{RX},j}|$ from the transmitter. For calculating $L_{i,j}$ we employ the Okumara-Hata model [7] for urban areas.

For a single transmitter setup, the order statistics may be easily derived analytically, assuming that the overall selection of measurement locations is uniform. For multiple transmitters, this approach is infeasible, because the *combined* geometry of link distances will need to be taken into account. Furthermore, local structures such as a buildings, trees, etc. obstruct the individual propagation path and result in deviations from the mean pathloss, modelled through a correction term $\chi_{i,j}$ in (3). It is precisely this term through which uncertainties in network planning under an interference limit policy are captured. Large-scale measurement campaigns, e.g. [8], have shown that this *shadowing term* is locally correlated, i.e. nearby measurement locations are likely to be similarly affected by obstacles. The authors in [9] provide an extensive survey of models of these shadowing correlations, and we recommend their paper for further discussion on the topic. For space reasons we limit ourselves in this paper to only the most popular model for shadowing, namely Gudmundson's exponential distance model [10][1], which defines the correlation coefficient between any two locations $\gamma_{\text{RX},j}$ and $\gamma_{\text{RX},k}$ as $\rho_{j,k} = \exp\big(-d_{j,k}/(\ln(20) \times d_c)\big)$. The parameter d_c is called the *decorrelation distance* and describes the distance at which the correlation approaches 0.5.

A. Simulation Results

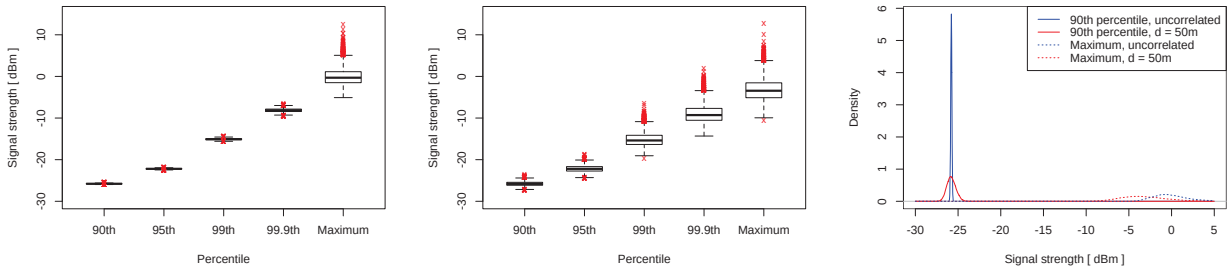
We have developed a custom MATLAB toolchain to create realizations from the correlated shadowing model and to compute the aggregate power levels over the Exeter area. Each simulation was repeated $5\,000$ times. Our results for different percentile values are shown in Figure 6, where we compare the uncorrelated shadowing and the correlated shadowing case with a decorrelation distance of 50 m.

Figure 6a presents the results over the iterations of the simulation for the uncorrelated case. We see that the percentiles exhibit low variations, e.g. the minimum and maximum deviate by only 0.5 dB, which means that already very few iterations suffice in order to find tangible signal strength thresholds. Only between 39–44 iterations (less than 1 percent) produced statistical outliers, i.e. values that were either larger than $\zeta_u = Q_3 + 1.5 \times (Q_3 - Q_1)$ or smaller than $\zeta_l = Q_1 - 1.5 \times (Q_3 - Q_1)$, where Q_1 and Q_3 are the first and third quartile of the samples, respectively. These results come as little surprise, because for multiple transmitters the variability over repetitions of the simulations in each measurement location solely originate from a sum of i.i.d. lognormal variables. While there is no closed-form analytical solution to describe this distribution, several approximations thereof exist [11]. The maximum encountered value shows a larger spread over the different runs, which complies with our earlier discussion on its statistics. Furthermore, its long right tail becomes apparent from the larger number of outliers (approximately 2.5 percent).

Our simulations with correlated shadowing produce significantly different results, see Figure 6b. Whereas the median values for the various percentiles approximate the uncorrelated case, the spread of the outcomes is significantly larger. These results originate from the decreased "randomness" of the shadowing process, where, due to the interdependencies of the local shadowing terms, deviations from mean signal strength values do not equally even out. We note that for high percentiles, a significant bias of positive outliers becomes apparent, and approximately 2% of all runs produced such irregular results.

Finally, we show the probability density functions (PDFs) of realizations of the 90th percentile and the maximum value for the uncorrelated and correlated case, respectively, in Figure 6c. As discussed earlier, the uncorrelated case results in a strong mode at the mean estimated percentile value, whereas the pdf of the correlated case is significantly flatter and exhibits a heavy right tail. We emphasize again that the uncertainties

[1]The authors in [9] suggest an earlier origin of this model, see their discussion.



(a) Uncorrelated shadowing. (b) Correlated shadowing, d_c = 50 m. (c) Probability density functions of measures.

Fig. 6. Simulation results for the correlated shadowing environment.

involved in these estimates need to be absorbed by the network operator in their power budget. Thus, high degrees of estimation error, will require high safety margins.

V. Discussion and Conclusions

In this paper we have studied the use of rigorous statistical methods for measurement and estimation problems related to operating wireless systems under interference limit policies, and in particular harm claim threshold policies. We have seen that the percentiles involved in the definition of the harm claim threshold play a crucial role in both reliability with which harm claims can be verified or refuted through measurements, as well as in the uncertainties in network planning an operator must take into account when operating or deploying a wireless system. For enforcement, a policy *must* specify not only the harm claim threshold itself, but the confidence at which exceedances must be measured to occur. Further, since the confidence intervals are always derived under simplifying assumptions, additional safety margins are needed to account for deviations from these assumptions.

Random sampling would in theory be a powerful tool for conducting measurements on harm claim threshold violations. This is needed to overcome the impact of spatial correlations in interference as seen in Section 3. Since conducting measurements at randomly chosen locations is logistically difficult, alternative requirements need to be implemented. One option would be to use stratified sampling to obtain a simple and practical measurement plan design that has most of the benefits of the random design. In this approach the drive test is overprovisioned to obtain larger number of measurements than would be needed in the completely random case, after which the measurement data is culled to retain only measurements that are separated by a certain minimum distance. We plan to advance our research to study the feasibility of this approach.

Finally, we note that the shadowing effects in wireless propagation are still ill-understood. Designing a wireless network under the harm claim threshold approach requires extensive simulations because of the uncertainties in radio propagation and in particular the correlation statistics of shadow fading. Again, if the percentile in the policy is chosen to be too high, the uncertainty associated with the simulation results will force the network operator to vastly reduce the transmit powers of their network, potentially severely hampering the deployment of new services. However, this paper is not to be misunderstood as arguing against the use of interference limit policies. On the contrary, the authors believe that with careful design, a more efficient spectrum allocation and a more flexible policy environment will be created. A further study into the required policy tools under real-world constraints, e.g. the measurement campaigns setups, will be highly beneficial for the community.

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